5 Basic Indices Rules/Laws

Multiplication

Rule 1: Add the powers when multiplying

$x^a \times x^b = x^{a+b}$

The bases must be the same to use this rule and notice how they do not change (x stays x)

Why is this rule true?

$x^2 \times x^3 = (x \times x) \times (x \times x \times x)$

We have five x's

$= x^5$

(the power simply tells us how many of the base we have in total)

Hence, we add the powers

> Simplify $5x^4 \times x^3$
>
> $5x^4 \times 1x^3$
>
> $= (5 \times 1)x^{4+3} = 5x^7$

Rule 2: Multiply the powers with a bracket

$(x^a)^b = x^{ab}$

Notice how the base does not change (x stays x)

Why is this rule true?

$(x^2)^3$

We have x^2 three times

$= x^2 \times x^2 \times x^2$

Now use rule 1 to add the powers

$= x^6$

Hence, we multiply the powers when we have a bracket

Rule 3: Rule 2 can be extended for when we have more than 1 term inside the bracket

$(cx^ay^b)^d = (c)^d(x^a)^d(y^b)^d$

Now apply rule 2 for each bracket

$= c^dx^{ad}y^{bd}$

Or: Think of this rule as keep the bases the same, multiply the powers

$(c^1x^ay^b)^d = c^dx^{ad}y^{bd}$

> Simplify $(4x^2y^3)^4$
>
> Colour code as $(4x^2y^3)^4$
>
> $= (4)^4(x^2)^4(y^3)^4$
>
> $= 256x^8x^{12}$

Mistake 1: The base DOES NOT change

$2^3 \times 2^6$ doesn't equal 4^9

Instead, $2^3 \times 2^6 = 2^9$

Mistake 2: Don't ignore the power when it isn't written (it means power 1)

$2x^2 \times 3x$ doesn't equal $6x^2$

Instead, $2x^2 \times 3x^1 = 6x^3$

Mistake 3: The power affects the first number term also

$(2x^2y^4)^3$ doesn't equal $2x^6y^{12}$

Instead, $(2x^2y^4)^3 = 8x^6y^{12}$

Mistake 4: We raise the first number to the power, we don't multiply it

$(5x)^3$ does not equal $15x^3$

Instead, $(5x)^3 = 5^3x^3 = 125x^3$

VERY COMMON Mistakes

Mistake 5: Don't mistake rule 3 when there is a sign $(+ \text{ or } -)$ in the middle.

$(2x)^2$ is not the same as $(2+x)^2$

$(2x)^2 = 4x^2$

whereas $(2+x)^2 = 4 + 4x + x^2$

The latter is expanding brackets

Mistake 6: Don't confuse addition/ subtraction with multiplication. We can only add/subtract "like" terms and when we add/subtract the algebra part doesn't change

- $2x + 3x$ is not the same as $2x \times 3x$
  $2x + 3x = 5x$ by collecting like terms
  $2x \times 3x = 6x^2$ using indices rule 1
- $2x^2 + 3x^2$ is not the same as $2x^2 \times 3x^2$
  $2x^2 + 3x^2 = 5x^2$ but $2x^2 \times 3x^2 = 6x^4$
- $2x^2 + 3x^3$ cannot be done/simplified
  but $2x^2 \times 3x^3 = 6x^5$

Division

Rule 1: Subtract the powers when dividing

$x^a \div x^b$ or $\frac{x^a}{x^b} = x^{a-b}$

The bases must be the same to use this rule and notice how they do not change (x stays x)

Why is this rule true?

$$\frac{x^7}{x^4} = \frac{x \times x \times x \times x \times x \times x \times x}{x \times x \times x \times x}$$

$$= \frac{x \times x \times x \cancel{\times x \times x \times x \times x}}{\cancel{x \times x \times x \times x}}$$

$= x^3$

Hence, we subtract the powers

> Simplify $16x^2y^5 \div 4x^6y^3$
>
> $16x^2y^5 \div 4x^6y^3$
>
> $= (16 \div 4)x^{2-6}y^{5-3}$
>
> $= 4x^{-4}y^2$

Common Mistakes

Mistake 1: The base DOES NOT change

$2^9 \div 2^6$ doesn't equal 1^3

Instead, $2^9 \div 2^6 = 2^3$

Mistake 2: Don't ignore the power when it isn't written (it means power 1)

$6x^2 \div 3x$ doesn't equal $2x^2$

Instead, $6x^2 \div 3x^1 = 2x$

Mistake 3: Don't let the fraction division notation confuse you

$\frac{24x^6y^2}{32x^4y^3}$

Deal with each part separately

$\frac{24x^6y^2}{32x^4y^3}$

$= \frac{3x^2}{4y}$

How did we get this?

Think of it as simplifying $\frac{24}{32}$ which is $\frac{3}{4}$ and there are 6 x's and 2 y's in the numerator and 4 x's and 3 y's in the denominator

$\frac{3xxxxxxyy}{4xxxxyyy}$

We cross off the corresponding matching pairs

$\frac{3xx\cancel{xxxx}\cancel{yy}}{4\cancel{xxxx}\cancel{yy}y}$

We have 2 x's left in the numerator and 1 y left in the denominator

$= \frac{3x^2}{4y}$

OR:

Just think when we move the powers between numerator and denominators we subtract them

$$\frac{24x^6y^2}{32x^4y^3} = \frac{24x^{6-4}}{32y^{3-2}} = \frac{3x^2}{4y}$$

Raising Numbers to Powers

Rule 1: Raising to a power of zero:

Anything to the power of 0 is always 1

$(\text{ANYTHING non zero})^0 = 1$

> $2^0 = 1$
>
> $x^0 = 1$
>
> $(2x)^0 = 1$
>
> $\left(\frac{2}{3}\right)^0 = 1$

Rule 2: Raising a fraction to a power:

$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

Apply the power to both the numerator and denominator

> Simplify $\left(\frac{2}{3}\right)^3$
>
> $\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3}$
>
> $= \frac{8}{27}$

Note: If more than 1 "element" inside the bracket we then use multiplication rule 3

> Simplify $\left(\frac{2x}{3y^2}\right)^3$
>
> $\left(\frac{2x}{3y^2}\right)^3 = \frac{(2x)^3}{(3y^2)^3}$
>
> $= \frac{(2)^3(x)^3}{(3)^3(y^2)^3}$
>
> $= \frac{2^3x^3}{3^3y^6}$
>
> $= \frac{8x^3}{27y^6}$

Rule 3: Raising negative numbers to a power:

$(\text{positive number})^{\text{even power}} = +$

$(\text{posiitve number})^{\text{odd power}} = +$

but

$(\text{negative number})^{\text{even power}} = +$

$(\text{negative number})^{\text{odd power}} = -$

> Example 1:
>
> Simplify $(-2)^4$ versus -2^4
>
> $(-2)^4 = -2 \times -2 \times -2 \times -2 = 16$
>
> $-2^4 = -(2 \times 2 \times 2 \times 2) = -16$
>
> They are not the same thing!
>
> $(-2)^4 = 16$ and $-(2)^4 = -16$
>
> Example 2:
>
> Simplify $(-2)^3$ versus $-(2)^3$
>
> $(-2)^3 = -2 \times -2 \times -2 = -8$
>
> $-2^3 = -(2 \times 2 \times 2) = -8$
>
> Here they are the same thing!
>
> $(-2)^3 = -8$ and $-(2)^3 = -8$

Common Mistakes

ab^x versus $(ab)^x$

> Simplify $2(3)^2$
>
> $2(3)^2$ does not equal 6^2
>
> We must do the power 3^2 first
>
> (because of BIDMAS/BODMAS)
>
> $2(3)^2$
>
> $=2(9)$
>
> $= 18$

Negative Powers

Rule 1: $x^{-n} = \frac{1}{x^n}$ and $(ab)^{-n} = \frac{1}{(ab)^n}$

The easiest way to think of this rule is that if we move terms between the numerator and denominator, the POWER of what is being moved changes(swaps/reverses) its sign (a positive becomes a negative and vice versa)

> Simplify 2^{-3}
>
> $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
>
> We moved the power -3 from the numerator down to the denominator and reversed the sign (in other words $-$ became $+$).
>
> Note: 2^{-3} means $\frac{2^{-3}}{1}$ hence the power was in the numerator originally and negative. We then moved it to the denominator and it became positive. There is a 1 in the numerator since the

> Get rid of the negative powers in $\frac{2x^2y^{-3}}{3z^{-4}}$
>
> $\frac{2x^2y^{-3}}{3z^{-4}}$
>
> The constants 2 and 3 stay where they are since they are positive and so can the x^2 term since it doesn't have a negative power. Remember for terms with negative powers that anything that moves between numerator and denominator changes the sign of its power
>
> $\frac{2x^2z^4}{3y^3}$

Rule 2: Raising fractions to negative powers

$$\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n = \frac{y^n}{x^n}.$$

We flip the fraction and change the sign

Why is this rule true?

Way 1: Flip the fraction and the power becomes positive. $\left(\frac{y}{x}\right)^n$. Now raise both to the power n giving $\frac{y^n}{x^n}$

Way 2: Apply the power to both the numerator and denominator first to get $\frac{x^{-n}}{y^{-n}}$.

Now deal with the negative powers which gives $\frac{y^n}{x^n}$

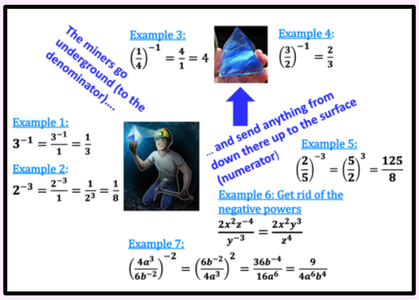
Way 3: Get rid of the negative power first by writing over 1

$$\frac{1}{\left(\frac{x}{y}\right)^n} = \left(\frac{y}{x}\right)^n \quad \text{or} \quad \left(\frac{1}{\frac{x}{y}}\right)^n = \left(\frac{y}{x}\right)^n$$

Now raise both to the power n giving $\frac{y^n}{x^n}$

Notice how writing a fraction over 1 just flips the fraction and hence just leads to way 1

> Simplify $\left(\frac{64}{125}\right)^{-\frac{2}{3}}$
>
> $\left(\frac{64}{125}\right)^{-\frac{2}{3}}$. Flip the fraction $\left(\frac{125}{64}\right)^{\frac{2}{3}}$
>
> $\left(\frac{125}{64}\right)^{\frac{2}{3}} = \frac{125^{\frac{2}{3}}}{64^{\frac{2}{3}}} = \frac{25}{16}$

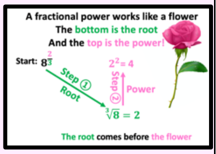


Rational Powers (Fractional Powers)

$a^{\frac{n}{m}} = \left(\sqrt[m]{a}\right)^n$

"ROOT AND THEN POWER

Note: $x^{\frac{1}{m}} = \sqrt[m]{x}$



> Simplify $27^{\frac{2}{3}}$
>
> $27^{\frac{2}{3}}$
>
> Root
>
> $\left(\sqrt[3]{27}\right)^2$
>
> $(3)^2$
>
> Power
>
> $= 3^2$
>
> $= 9$

> Simplify $\left(\frac{64x^6z^{12}}{27y^3}\right)^{\frac{1}{3}}$
>
> $\left(\frac{64x^6z^{12}}{27y^3}\right)^{\frac{1}{3}} =$
>
> $= \frac{(64x^6z^{12})^{\frac{1}{3}}}{(27y^3)^{\frac{1}{3}}}$
>
> $= \frac{64^{\frac{1}{3}}(x^6)^{\frac{1}{3}}(z^{12})^{\frac{1}{3}}}{27^{\frac{1}{3}}(y^3)^{\frac{1}{3}}}$
>
> $= \frac{64^{\frac{1}{3}}x^2z^4}{27^{\frac{1}{3}}y}$
>
> $= \frac{4x^2z^4}{3y}$

Common Mistakes

$\sqrt{x} = x^{\frac{1}{2}}$

We drop the 2 for square root. When nothing is written to the left of the root it means square root

© mymathscloud

Getting Like Bases In Order to Simplify

$\frac{1}{2\sqrt{2}}$ Write as a power of 2 $=\frac{1}{2\times 2^{\frac{1}{2}}}$ $=\frac{1}{2^{\frac{3}{2}}}$ $=2^{-\frac{3}{2}}$	$(5\sqrt{5})^3$ Write as a power of 5 $=\left(5\times 5^{\frac{1}{2}}\right)^3$ $=\left(5^{\frac{3}{2}}\right)^3$ $=5^{\frac{9}{2}}$	$\frac{y^4\times y^n}{y^2}$=$y^{-3}$ Find the value of n $\frac{y^{4+n}}{y^2}$=y^{-3} $y^{4+n-2}=y^{-3}$ $y^{2+n}=y^{-3}$ $2+n=-3$ $n=-5$	$3^a=\frac{1}{9}$, $3^b=9\sqrt{3}$ $3^c=\frac{1}{\sqrt{3}}$ Find the value of a+b+c $3^a=3^{-2}$ $3^b=3^2\times 3^{\frac{1}{2}}$ $3^b=3^{\frac{5}{2}}$ $b=2.5$ $3^c=3^{-\frac{1}{2}}$ $c=-\frac{1}{2}$	Find the value of $\sqrt[3]{8\times 10^6}$ without a calculator Let's make all the same base $\sqrt[3]{2^3\times(2\times 5)^6}$ $\sqrt[3]{2^3\times 2^6\times 5^6}$ $\sqrt[3]{2^9\times 5^6}$ $(2^9\times 5^6)^{\frac{1}{3}}$ $2^{9\times\frac{1}{3}}\times 5^{6\times\frac{1}{2}}$ $2^3\times 5^2$ 8× 25 200	Express 8^{2x+3} in the form 2^y $(2^3)^{2x+3}$ $=2^{3x+9}$	$32^{\frac{3}{2}}\times 8^3\times 2^{-\frac{5}{2}}$ Leave answer in the form 2^a We need to make all a base of 2 $(2^5)^{\frac{3}{2}}\times(2^3)^3\times 2^{-\frac{5}{2}}$ $2^{\frac{15}{2}}\times 2^9\times 2^{-\frac{5}{2}}$ 2^{14}	simplify $\sqrt{2}(x^3)\div\sqrt{\frac{32}{x^2}}$ $2^{\frac{1}{2}}x^3\div\left(\frac{32}{x^2}\right)^{\frac{1}{2}}$ $2^{\frac{1}{2}}x^3\div\frac{2^{\frac{5}{2}}}{x}=2^{\frac{1}{2}}x^3\times\frac{x}{2^{\frac{5}{2}}}$ $2^{-2}x^4=\frac{1}{4}x^4$
Write $\frac{1}{4\sqrt{2}}$as a power of 2 Write $8^{\frac{1}{2}}$ as a power of 2 Write $\sqrt{3}$ as a power of 9 Write 5^{20} as a power of 5 Write $\frac{1}{16}$ as a power of 2		$y=\frac{1}{27}x^3$ Find the following in the form kx^n i. $3y^{-\frac{1}{3}}$ ii. $\sqrt{27y}$ i. $3y^{-\frac{1}{3}}=3$ $\left(\frac{1}{27}x^3\right)^{-\frac{1}{3}}$ ii. $\sqrt{27y}=$ $\sqrt{27\left(\frac{1}{27}x^3\right)}$			$2^{3x}\times 8^{4x}$ $2^{3x}\times(2^3)^{4x}$ $2^{3x}\times 2^{12x}$ 2^{3x+12x} 2^{15x}		$10^{-\frac{1}{3}}$ x $25^{\frac{2}{3}}\div 2^{\frac{5}{3}}$ Leave answer in the form $2^a\times 5^b$ $(2\times 5)^{-\frac{1}{3}}$ x $(5^2)^{\frac{2}{3}}\div 2^{\frac{5}{3}}$ $2^{-\frac{1}{3}}\times 5^{-\frac{1}{3}}\times 5^{\frac{4}{3}}\div 2^{\frac{5}{3}}$ $2^{-\frac{1}{3}-\frac{5}{3}}\times 5^{-\frac{1}{3}+\frac{4}{3}}$ $2^{-2}\times 5$

Solving with unknown powers

We need to make the bases the same and then equate the powers

We might need to simplify before equation the powers using multiplication and division indices rules

Solve $16^{2x-4}=32^{x-5}$ We need to make the bases the same $(2^4)^{2x-4}=(2^5)^{x-5}$ Use indices rule $(x^a)^b=x^{ab}$ $2^{8x-16}=2^{5x-25}$ $8x-16=5x-25$ $x=-3$	Solve $27^x\times 3^{2-x}=\frac{1}{9^{2x}}$ $(3^3)^x\times 3^{2-x}=\frac{1}{(3^2)^{2x}}$ $3^{3x}\times 3^{2-x}=\frac{1}{3^{4x}}$ $3^{3x+2-x}=3^{-4x}$ $3^{2+2x}=3^{-4x}$ $2+2x=-4x$ $6x=-2$ $x=-\frac{1}{3}$

Type 1:
2 terms only. Write the bases the same and equate powers

Type 2:
3 terms which are a hidden quadratic. Might need to use the two indices multiplication rules first:
$x^{a+b}=x^a\times x^b$ and $x^{ab}=(x^a)^b$

If you can solve all the following you have understood this well

Type 1:

- $16^{2x-4}=32^{x-5}$
  $(2^4)^{2x-4}=(2^5)^{x-5}\Leftrightarrow 2^{8x-16}=2^{5x-25}\Leftrightarrow 8x-16=5x-25\Leftrightarrow x=-3$

Type 2:

- $5^{2x}+4(5^x)-5=0$
  $y^2+4y-5=0\Longrightarrow y=-5,1\therefore 5^x=-1,5^x=1\Longrightarrow x=0$
- $3^{2x+1}-26(3^x)-9=0$
  $3(3^{2x})-26(3^x)-9=0$ etc
- $2^{2x-5}-3(2^{x-3})+1=0$
  $\frac{1}{32}2^{2x}-\frac{3}{8}(2^x)+1=0\Leftrightarrow 2^{2x}-12(2^x)+32=0$ etc
- $4^x-2^{3+x}+16=0$ $(2^2)^x-2^3 2^x+16=0\Leftrightarrow 2^{2x}-8(2^x)+16=0$ etc
- $2^{2x+3}-3(2^{x+1})+1=0$

Solving with known powers

Integer

- $$x^a=c\Longrightarrow x=\begin{cases}\pm c^{\frac{1}{a}} & if\ a\ even\\ c^{\frac{1}{a}} & if\ a\ odd\end{cases}$$

Note: There is no sol when c is neg and a is even

Rational:

$$x^{\frac{a}{b}}=c\Longrightarrow x=\begin{cases}\pm c^{\frac{b}{a}} & if\ a\ even\\ c^{\frac{b}{a}} & if\ a\ odd\end{cases}$$

Note: This is not defined when c is negative
You can remember this by: **"Raise other side to reciprocal power"**

$x^{\frac{3}{5}}=a\Longrightarrow x=a^{\frac{5}{3}}$ since reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$

Why is this true? Write as $\left(\sqrt[5]{x}\right)^3=a$. Then undo 1 layer at a time: $\sqrt[5]{x}=a^{\frac{1}{3}}\Longrightarrow x=\left(a^{\frac{1}{3}}\right)^5\Longrightarrow x=a^{\frac{5}{3}}$

If you can solve all the following you have understood this well:

$x^2=16$ $x=\pm 4$	$x^2=-16$ no sol	$x^3=8$ $x=2$	$x^3=-8$ $x=-2$	$x^{\frac{1}{2}}=3$ $x=9$	$x^{\frac{1}{2}}=-3$ no sol
$x^{\frac{1}{3}}=3$ $x=27$	$x^{\frac{1}{3}}=-3$ $x=-27$	$x^{-\frac{1}{2}}=3$ $x=\frac{1}{9}$	$x^{-\frac{1}{2}}=-3$ no sol	$x^{\frac{3}{2}}=27$ $x=27^{\frac{2}{3}}$ $x=9$	$x^{\frac{3}{2}}=-27$ no sol
$x^{-\frac{3}{2}}=27$ $x=27^{-\frac{2}{3}}$ $x=1/9$	$x^{-\frac{3}{2}}=-27$ no sol	$x^{\frac{2}{3}}=16$ $x=16^{\frac{3}{2}}$ $x=\pm 64$	$x^{\frac{2}{3}}=-16$ $x=(-16)^{\frac{3}{2}}$ no sol	$x^{-\frac{2}{3}}=16$ $x=16^{-\frac{3}{2}}$ $x=\pm\frac{1}{64}$	$x^{-\frac{2}{3}}=-16$ $x=(-16)^{-\frac{3}{2}}$ no sol

Adding/Subtracting Algebra

Simplify $2a + 3b - 4a + 5b$

Circle the like terms and deal with each separately. Remember the letters just stand for an object

$$\boxed{2a} \boxed{+3b} \boxed{-4a} \boxed{+5b}$$




If I had 2 apples and took away 4 apples then I would have negative two apples
If I had 3 bananas and got 5 more bananas then I would have 8 bananas

$-2a + 8b$

Multiplying/Dividing Algebra (Indices)

This does not include negative and fractional powers

Multiply/divide the numbers together

Stick the algebra terms together (like glue)
Note: when we **multiply** we add the powers of the like letters if there are like/common
when we **divide** we subtract the powers of the like letters if there are like/common l

Multiplication
Simplify $3x \times 2y$

$3x \times 2y$

Simplify $5x^4 \times 2x^3$

$5x^4 \times 2x^3 = (5 \times 2)x^{4+3} = 10x^7$

Simplify $3x^3 \times 4x^2y$

$3x^3 \times 4x^2y = (3 \times 4)x^{3+2}y = 12x^5y$

Division
Simplify $8x^8 \div 4x^4$

$8x^8 \div 4x^4 = (8 \div 4)x^{8-4} = 2x^4$

With brackets
Simplify $(3x^4y^2)^3$

$3x^4y^2 \times 3x^4y^2 \times 3x^4y^2 = (3 \times 3 \times 3)x^{4+4+4}y^{2+2+2} = 27x^{12}y^6$

Multiplication and division together
Simplify $\frac{5x^7 \times 4x^8}{2x^6}$

$$\frac{5x^7 \times 4x^8}{2x^6} = \frac{(5 \times 4)x^{7+8}}{2x^6} = \frac{20x^{15}}{2x^6} = (20 \div 2)x^{15-6} = 10x^9$$

Important:
Don't confuse addition/ subtraction with multiplication/division. We can only add/subtract "like" terms and when we add/subtract the algebra part doesn't change.

- $2x + 3x$ is not the same as $2x \times 3x$

$2x + 3x = 5x$
$2x \times 3x = 6x^2$

- $2x^2 + 3x^2$ is not the same as $2x^2 \times 3x^2$

$2x^2 + 3x^2 = 5x^2$
$2x^2 \times 3x^2 = 6x^4$

- $2x^2 + 3x^3$ cannot be done/simplified but $2x^2 \times 3x^3 = 6x^5$

Exercises (9 types of questions):

There is not that much difficulty between bronze, silver, gold or diamond until type 6 as the questions only get hard once several types are combined

Type 1: Multiplying

Level 1: Bronze

1) $x \times x^3$
2) $x + x$
3) $2x \times 3x^3$
4) $2x + 3x$
5) $2x^2 + 3x^2$
6) $5x^4 \times x^3$
7) $\boldsymbol{4y^5 \times 3y^4}$

Level 2: Silver

1) $3x^2y \times 6x^5y^4$
2) $2x^3y \times 3x^4$
3) $\mathbf{2^4} \times \mathbf{2^2}$. Leave your answer as a power of 2

Level 3: Gold

1) $5x^{\frac{1}{3}} \times 4x^{\frac{2}{3}}$
2) $3^4 \times 3^2 \times 3^3$. Leave your answer as a power of 3

Type 2: Multiplying With Brackets

Level 1: Bronze

1) $(2x)^2$
2) $(2+x)^2$
3) $(2x^3y)^4$
4) $\boldsymbol{(3x^5y^2)^3}$

Level 2: Silver

1) $4x^{-3}(2x^2 + 5x^3)$
2) $\boldsymbol{x\left(2x^{-\frac{1}{4}}\right)^4}$

Level 3: Gold

1) $3x^2(x+5)^2$
2) $\boldsymbol{2(x^2-4)(x+2)}$

Level 4: Diamond

1) $\left(3x^4 - 2x^{\frac{5}{2}}\right)\left(5x - 2x^{\frac{2}{3}}\right)$

Type 3: Dividing

Level 1: Bronze

1) $x^5 \div x^{-2}$
2) $\boldsymbol{8x^8 \div 4x^4}$
3) $2^9 \div 2^6$. Leave your answer as a power of 2
4) $3^4 \div 3^2 \div 3^{-3}$. Leave your answer as a power of 3

Level 2: Silver

1) $\frac{16x^5y^2}{24x^2y^4}$
2) $\frac{18x^7y^4z^3}{9x^5yz^2}$
3) $\frac{x^2+3x}{x}$
4) $\frac{3x^2+6x^3}{3x}$
5) $\frac{12x^3+16x^5}{4x^2}$

Level 3: Gold

1) $\frac{2x^3\sqrt{y}z^2}{4x^4y^5z}$
2) $\frac{\left(2x^{\frac{1}{2}}\right)^3}{4x^2}$
3) $\frac{2x^3\times5x^5}{2x^4}$
4) $\frac{y^4\times y^n}{y^2}=y^{-3}$. Find the value of n

Level 4: Diamond

1) $\frac{(x-2)^2}{x}$
2) $\frac{(2x^4y^3)^3}{2x^4y^6}$
3) $\frac{(2x^2)^5}{(4x^3)^6}$
4) $\frac{(3x^4y^2)^3}{(4x^3y^5)^3}$
5) $\left(\frac{2a^3b^5}{3a^5b^2}\right)^2$

Type 4: Raising To Powers

Level 1: Bronze

1) 2^5
2) 3^3

Level 2: Silver

1) $(-2)^3$
2) $(-2)^4$
3) $\left(\frac{2}{3}\right)^3$
4) -6^2
5) -2^3
6) 5^0

Level 3: Gold

1) x^0
2) $(2x^2)^0$
7) $\left(-\frac{2}{3}\right)^3$

Type 5: Negative Powers

Level 1: Bronze

1) 8^{-3}
2) $\left(\frac{2}{3}\right)^{-2}$

Level 2: Silver

1) $(-4)^{-2}$

2) $(-3)^{-3}$

3) $\left(\frac{2}{3}\right)^{-2}$

Level 3: Gold

1) $144^{\frac{1}{2}} \times 64^{-\frac{1}{3}}$

2) $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

Level 4: Diamond

1) $\left(\frac{25x^4}{4}\right)^{-\frac{1}{2}}$

Type 6: Fractional Powers

Level 1: Bronze

1) $8^{\frac{1}{3}}$

2) $\mathbf{27^{\frac{2}{3}}}$

Level 2: Silver

1) $\left(\frac{81}{16}\right)^{\frac{3}{4}}$

2) $\left(\frac{64}{27}\right)^{\frac{2}{3}}$

Level 3: Gold

1) $-\left(\frac{64}{125}\right)^{-\frac{2}{3}}$

2) $\left(\frac{64x^6z^{12}}{27y^3}\right)^{\frac{1}{3}}$

3) $\left(\frac{16w^8}{y^{20}}\right)^{-\frac{3}{4}}$

Type 7: Writing As the Same Base - Simplifying

Level 1: Bronze

1) Express 8^2 as a power of 2

2) Express 25^4 as a power of 5

3) Express $8^{\frac{1}{3}}$ as a power of 2

Level 2: Silver

1) Write $\frac{1}{16}$ as a power of 2

2) Write $8\sqrt{8}$ as a power of 8

3) Write $\frac{1}{4\sqrt{2}}$ as a power of 2

4) Write $\left(5\sqrt{5}\right)^3$ as a power of 5

4) Express the following in the form 5^k

   i. 25^4

   ii. $\frac{1}{\sqrt[4]{5}}$

   iii. $\left(5\sqrt{5}\right)^3$

Level 3: Gold

1) Express 9^{2x+1} as a power of 3

2) Express $\left(\frac{1}{3}\right)^x$ as a power of 3

3) Express $\left(\frac{1}{27}\right)^{x+2}$ as a power of 3

4) Given that $y = \frac{1}{27}x^3$. Express each of the following in the form kx^n, where k and n are constants

i. $y^{\frac{1}{3}}$
ii. $3y^{-1}$
iii. $\sqrt{27y}$

5) Given that $y = \frac{1}{8}x^5$, express each of the following in the form ax^b where a and b are constants to be found
i. y^5
ii. $24y^{-2}$
iii. $\sqrt{8y}$

6) **Simplify $2^{3x} \times 8^{4x}$. Leave answer in the form 2^a**
7) **Express 8^{2x+3} in the form 2^y, stating y in terms of x**

Level 4: Diamond

1) Simplify $\sqrt{2}(x^3) \div \sqrt{\frac{32}{x^2}}$ as much as possible
2) $32^{\frac{3}{2}} \times 8^3 \times 2^{-\frac{5}{2}}$. Leave answer in the form 2^a

Type 8: Writing As the Same Base – Solving with Unknown Papers (equating powers)

Level 1: Bronze

1) $9^{x-2} = 27$
2) $8^{2x+1} = 16$
3) $3^{x+4} - 27 = 0$
4) $16^{2x-4} = 32^{x-5}$

Level 2: Silver

1) $16^{\frac{1}{5}} \times 2^x = 8^{\frac{3}{4}}$
2) $2^x \times 4^{x+1} = 8$
3) $16^{\frac{1}{5}} \times 2^x = 8^{\frac{3}{4}}$
4) $25 \times 5^{-x} = 125^x$
5) $2^{x^2} = 8^x \times 16$
6) $3 \times 4^{2x+8} = 24$
7) $27 \times 81^{2-x} = 9^{-3x}$
8) $2^n = 2^{x^2} \times 16^x \times 8$

Level 3: Gold

1) $7^{x2} - 49^{6-2x} = 0$
2) $3^{2x} = \frac{1}{81}$
3) $4^{3x-2} = \frac{1}{2\sqrt{2}}$
4) $16^{4x-1} = \frac{1}{4\sqrt{2}}$
5) $\frac{9^{x-1}}{3^{y+2}} = 81$ **Express y in terms of x**
6) $2^x \times 4^y = \frac{1}{2\sqrt{2}}$. Express y as a function of x.
7) $27^x \times 3^{2-x} = \frac{1}{9^{2x}}$
8) $3^a = \frac{1}{9}$, $3^b = 9\sqrt{3}, 3^c = \frac{1}{\sqrt{3}}$. Find the value of $a + b + c$

Level 4: Diamond

1) $x + 2y = 5$ and $4^x = 8^y$
2) Express $\frac{\sqrt{27^x}}{3^{2x-1}}$ in the form 3^y, where y is an expression in term of x
3) Hence solve $\frac{\sqrt{27^x}}{3^{2x-1}} = \sqrt[3]{81}$

Type 9: Writing As the Same Base – Solving With Unknown Powers (Substitutions)

Level 1: Bronze

1) $5^{2x} + 4(5^x) - 5 = 0$
2) $2^{2x} - 12(2^x) + 32 = 0$
3) $3(3^{2x}) - 26(3^x) - 9 = 0$

Level 2: Silver

1) $2^{2x+2} - 10 \times (2^x) + 4 = 0$
2) $3^{2x+1} - 26(3^x) - 9 = 0$

Level 3: Gold

1) Show that $2^{2x} + 3(2^{x+1}) + 8 = 0$ has no solutions
2) $2^{2x+3} - 3(2^{x+1}) + 1$
3) $2^{2x-5} - 3(2^{x-3}) + 1 = 0$
4) $3(3^{3y}) + 9(3^{2y}) - 9(3^y) - 81 = 0$
5) $(8^{x-1})^2 - 18(8^{x-1}) + 32 = 0$

Level 4: Diamond

1) $4^{x-1} = 2^x + 8$
2) $4^x - 12(2^x) + 32 = 0$
3) $4^x - 2^{3+x} + 16 = 0$
4) $2^{2x+3} - 3(2^{x+1}) + 1 = 0$

Challenge – CHALLENGE