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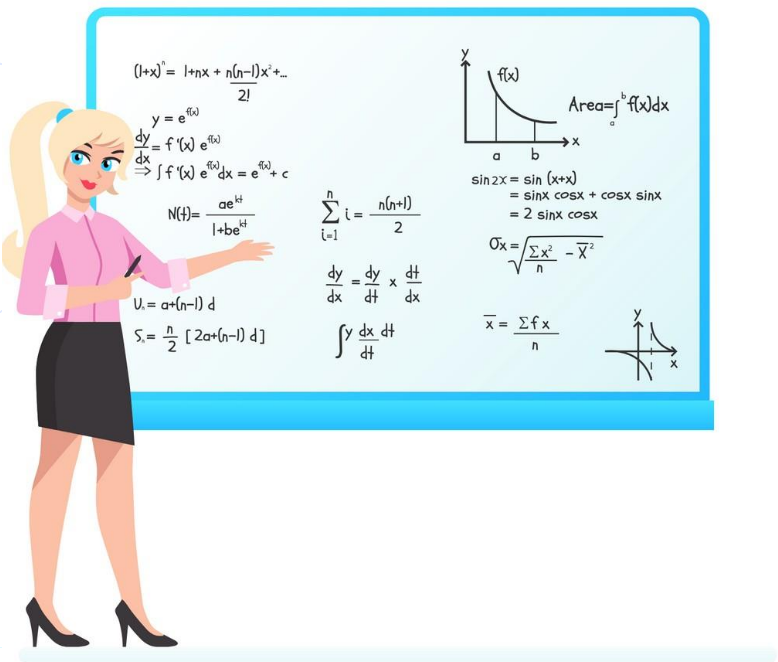
Topology

Textbook and Course Advice



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Topology:

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1 Branches of Topology & Pre-Requisites

There are 3 branches to topology - point set, algebraic and differential. They should be studied in this exact order.

1.1 Point Set

Point set generally has no prerequisites apart from real analysis, but it is pretty much unheard of to take topology before linear algebra or real analysis. You should have a semester of real analysis under your belt so that you can understand why you are doing all the boring point set stuff (I am actually fond of point set topology, but it is not what most people think of when they think of topology - no coffee mugs turning into donuts!)

Real analysis is a sufficient background to get started, and topology is the natural next step as much of point-set topology generalizes ideas from real analysis. Logically, point-set topology depends only on the basics of set theory: sets and operations thereon, functions, sequences, that sort of thing. Familiarity with the real numbers, real vector spaces, metric spaces, and continuity in those contexts would be helpful for drawing examples and as a basis for generalization and abstraction. More advanced real analysis is not really required. In fact, most real analysis courses will start with some basic point-set topology, at least in the special case of metric spaces. There is not much to point-set topology by itself; it's mostly definitions, with a few simple theorems to build off. Really, the basics of point-set topology should be known to all Mathematics students, and it is required for all the rest of topology, as well as analysis and geometry.

Going into more detail, you'll find continuity restated in terms of open sets so that it can be defined for functions between spaces where a metric doesn't exist (but this generalized definition agrees with analysis' epsilon-delta definition when one does). Likewise, there is a generalized definition for sequence convergence that agrees with analysis in a metric space, but bizarre things can happen outside of one, such as every sequence converging to every point in the space. You'll also explore specifically which hypotheses we put on a space give rise to different theorems. For instance, in a metric space, we have compactness $\Longleftrightarrow$ sequential compactness $\Leftrightarrow$ limit-point compactness. Why is this so? How do these implications change when we remove hypotheses (e.g. when we assume our topological space isn't a metric space, or when we remove the assumption that two points are guaranteed disjoint neighbourhoods. Two theorems you'll recognize from analysis, the Bolzano-Weierstrass theorem and the Heine-Borel Theorem, are central to these considerations. So, having taken real analysis and encountering things like compactness, continuity, and convergence in a specific kind of topological space (a metric space) makes encountering these concepts in a more general setting easier.
Basically, topology tries to define the basics of what a continuous function is in a generalized space. It turns out that this revolves around the notion of open sets (like open intervals in the real numbers). Other restrictions are introduced to get rid of degenerate spaces but without knowing why we are doing this it may be difficult to follow along.

Meeting the formal prerequisites will not necessarily give you the motivational prerequisites. In my opinion, mathematical maturity is the only real pre-requisite, though as mentioned introductory real analysis is helpful. I say mathematical maturity, because it's quite a bit more abstract than even your algebra course might have been. Examples are often quite difficult to visualize and develop intuition for, especially the odd counterexamples. Having said this, as mentioned above, although real analysis not technically a necessity, it would be much better if you get exposed to some mathematical analysis before starting point set topology. One is deprived from many concrete examples if he goes into topology without analysis background, and concrete examples are a really useful mental crutch.

Have fun though - topology is really cool and leads in lots and lots of different directions!

1.2 Algebraic Topology

Algebraic topology requires some, surprise, abstract algebra and point set topology. It's pretty much unheard of to take algebraic topology before learning abstract algebra as algebraic topology is no cakewalk.I think you will find that to learn algebraic topology well, you will need a good grounding in point-set topology. It's probably worth the investment of time because point-set topology is a language which is pervasive throughout mathematics.

As already mentioned, algebraic topology will require algebra. At the very least, you need to know about groups, Abelian groups, and vector spaces. More general modules could be helpful. Know about homomorphisms, isomorphisms, quotients, products of these objects. At some point, you will want to learn about tensor products, but you don't need them to get started. There are funkier algebraic structures you can use, like Hopf algebras, but you don't need to know them in advance; learn when you get to them. You'll hit some nice motivations for category theory. At some point, algebraic topology heads back into pure algebra as homological algebra, but again, don't study that until you come at it from the topological side. If you have some experience with the combinatorics of graphs or simplicial complexes or similar structures, that can be helpful, but it is not required.

1.3 Differential Topology

Differential topology is somewhat of a different beast. In order to be able to read, "intro to smooth manifolds", you'll just need some very basic general topology and a solid understanding of multi-variate calculus. Differential topology studies smooth manifolds, settings where you can do calculus. (Actually, there exist things called topological manifolds, which lack the differential structure and could be considered part of point-set topology. I don't know of any reason to study them on their own, except as a source of well-behaved examples for other parts of topology.) Prerequisites will include linear algebra and vector calculus. You should know linear maps backward and forward: how they relate to matrices, images, kernels, rank and nullity, determinants. Know your multivariable derivatives, implicit function theorem and Stokes' theorem. Tensor products will eventually rear their head, in a different guise than in algebraic topology. In my experience, many schools have an undergraduate differential geometry course which would make a good starting point before differential topology. Differential topology will lead back to a more advanced differential geometry course. You'll also likely see some partial differential equations. This is another case where a basic PDE course could feed into a differential topology course which feeds into a more advanced PDE course.

2 Further Topology Courses

Most undergraduate programs don't focus much on topology. There may be a single point-set topology course, a differential geometry course, or these may only be available as special topics. Further study is mostly at the graduate level. (If you self-study, this doesn't apply as much to you, but consider the level that study materials will be written to). I would say that an undergrad topology course could be taken concurrently with real analysis, assuming you are sufficiently comfortable with reading and writing proofs. Differential geometry could be taken earlier, as part of a transition to upper division along with linear algebra and differential equations. (Or possibly just after finishing those courses.) An undergraduate topology course might cover homotopy, part of algebraic topology, which would require some knowledge of group theory. It might also define manifolds and shade into differential topology. From there, you could move into either differential or algebraic topology, but algebraic is usually treated as more advanced. Eventually you will learn about tensors, either in algebra, topology, or, heaven help you, a physics class. Most people struggle with these and have to turn the concepts around a few times before they can fit them in their head, so you may benefit from seeing them in multiple classes. Really, at this point you are starting to get to graduate level courses, and the options there will depend on your particular interests.

3 Textbooks

Topology is also a huge area, so it's quite easy to get lost at the beginning!

3.1 Point Set

- **Introduction to Topology, Bert Mendelson**
  Begin with this to get a grasp on the big picture.

- **Schaum's Outlines for General Topology**
  This is a great very straight forward book.

- **Topology: A first course, Munkres**
  Munkres is the best for a beginner since it has lots of examples and makes it easy to self-teach. It is the best general (point set) topology book and goes through all the set theory pre-requisites you need very quickly. So, I recommend reading Intro to Topology by Mendelson first. Anyone curious about topology should then pick up this book as it is a great read! This books also covers a basic introduction to homotopy; although no homology. You should follow the MIT OCW Topology course which is based on this book by James Munkres.

- **Introduction to Topological Manifolds, Lee**
  If you find you're not liking Munkres, you maybe prefer Lee's "Topological Manifolds" (I actually found Lee's exposition to be a nicer read).

- **Introduction to Topology and Modern Analysis, George F. Simmons**
  This is a wonderful book.

- **Topology: An Introduction, Stefan Waldmann, published by Springer**
  This is a short and readable book.

- **Topology Without Tears, Morris**
  https://www.topologywithouttears.net
  This is not the most rigorous, but the explanations are nice and readable.

- **Topology Through Inquiry, Michael Starbird and Francis Su**
  This is very good for the thought developing processes regarding topology.

- **Introduction to Topology, Tej Bahadur Singh**

- **Introduction to Topology: Pure and Applied, Colin Adams, Robert Franzosa**

- **A Course in Point-Set Topology by John B. Conway**

- **Principles of Topology, Fred H. Croom**

- **Counterexamples in Topology, Dover**
  This will give you all the tables you can ever want and more.

- **General Topology, J. L. Kelley**
  This is also a great book. You should read this and Munkres.

- **General Topology, Engelking**
  Kelley and Engelking are good advanced books.

3.2 Algebraic Topology

- **Algebraic Topology, Hatcher**
  This is the standard reference, but to be honest, I don't really like it. Bredon might be a good alternative, and of course Bott and Tu, once you learn some differential topology.

- **Elements of Algebraic Topology, Munkres**
  This covers homology which Munkres' Topology: "A first course," doesn't;. That book is perhaps a little old-fashioned, though. Algebraic topology has moved on and the old language of (co)homology theories being defined by complexes is being eschewed in terms of the more modern language of spectra and derived functors.

3.3 Differential

- **Introduction to Smooth Manifolds, Lee**
  This is great for differential geometry. It is in the same class with Munkres.

- **Differential Topology, Guillemin and Pollack**
  This is self-contained in a way since it contains a chapter about advanced multivariable calculus. This is more elementary than Lee's book.

- **Differential Topology, Shastri**
  This is great for differential geometry. It is in the same class with Munkres.

- **An Introduction to Manifolds, Tu**
  This is a fantastic book and a favourite amongst many students.