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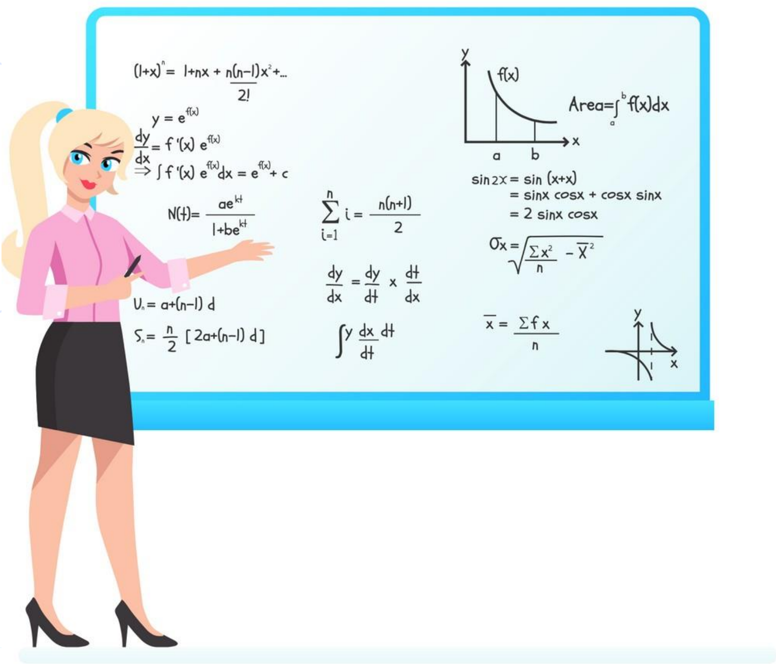
Linear Algebra

Textbook and Course Advice



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Linear Algebra:

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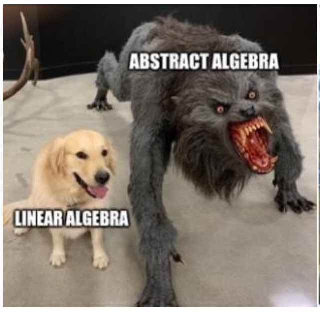

1 General Advice

Linear Algebra is a bit strange at first as it is so abstract, but once you get used it to it, it's not too bad. It's actually one of the easiest courses to get high marks in! If you like calculus III which is a non-maths degree module (integrals especially) you'll love differential equations, but if you love proofs and algebra, then you'll like linear algebra.

A crash course before you start really helps. First do a review or preview of matrices (maybe via videos) so that you can pause and do a deep dive into vocabulary/terminology so that you won't get lost during the lectures. Be sure to really focus on learning the definitions/terminology - don't memorize them, understand them.
Have a study group, do the homework right after you learn it and ask questions. I cannot stress enough how important it is to keep up with the homework. Whatever you do, don't fall behind!

At a fundamental level, linear algebra is based on sets of simultaneous equations. The following three simultaneous equations in $x, y,$ & z

$$ax + by + c\,z = d$$

$$ex + fy + g\,z = h$$

$$jx + ky + l\,z = m$$

This can be written compactly in "matrix" form as:

$$\begin{pmatrix} a & b & c \\ e & f & g \\ j & k & l \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} d \\ h \\ m \end{pmatrix}$$

And this can further by written more concisely as:

$$\mathrm{A}\boldsymbol{x} = \boldsymbol{v}$$

Where A is a matrix and x & v are vectors:

$$\mathrm{A} \equiv \begin{pmatrix} a & b & c \\ e & f & g \\ j & k & l \end{pmatrix}$$

$$\boldsymbol{x} \equiv \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\boldsymbol{v} \equiv \begin{pmatrix} d \\ h \\ m \end{pmatrix}$$

By studying the properties of *A* you can make deductions about the solutions to the equation set. This may seem trivial for the three equations above, but have significant implications for several fields of mathematics, science and engineering.

Matrices have an algebra associated with them. You can multiply them together, add them, invert them and transpose them. The elements can be complex values, not just reals.

One special property are the "eigenvalues" and "eigenvectors" of a matrix. By definition, these are solutions to:

$$Ax = \lambda x$$

Where λ , which is a scalar, is an eigenvalue and x is an eigenvector. Again, the eigenvectors & eigenvalues of a system are of significance.

Linear Algebra can get very abstract in nature.

2 Topic Advice

2.1 Pre-Topic Learning

- Basic Matrices:
  If you have no experience with matrices, you must start with the fundamentals. This is a very easy topic, but people that have a hard time differentiating between their "left" and "right" tend to have a difficult time remembering and understanding notation for columns and rows, and when you use one vs the other, and by extension, the basics of matrix multiplication (which involves simultaneously moving across rows and columns of 2 matrices). With matrix multiplication, the # row and # column you select combine to create the # position in the resulting matrix.

  If you've seen multivariable calculus (or at least the inner product section of linear algebra), doing matrix multiplication is just doing a ton of dot products. This is sometimes obscured by defining matrix multiplication using sigma notation. With so many subscripts and indices floating around, math majors often get confused. Once you can get past this notation, it's very easy.

  Make sure you recap the following:
    - Matrix terminology (rows, columns, index, size)
    - Matrix arithmetic/basic operations (adding, subtracting and multiplying matrices)
    - Context (matrices as arrays of numbers, matrices as ways of writing down systems of equations)
    - Determinants
    - Vectors and dot product/Cross product
    - Linear Systems

2.2 Deeper Topic Learning

Knowing the topics mentioned above is enough to start the course. However, having a base level of knowledge of the topics listed will give you an even better head start. If you have more time you should go into the exact specifics mentioned.

Normally courses are set up into the following 4 parts:

- Matrices in solving systems
    - Writing a linear system as a matrix system
    - Finding a determinant
    - Definition of an overdetermined or underdetermined and classifying a system as such (learn how the determinant fits into this)

- Vector spaces
    - Definition of a vector space
    - Vector space axioms
    - Definition of a subspace
    - Determining whether a subset is a subspace
    - Definition of linear independence and why it is important
    - Definition of a spanning set of vectors, basis and transformation
    - Linear algebra is all about a basis!

      Consider this:
      We can describe all of the coordinate plane using only (a,0)+(0,b). We just need two variables, that's why it's called 2-dimensional. one variable, like (a,a) wouldn't be enough since we can't make (2,3) with that setup, and having more than two is overkill. As long as the two components are fundamentally different, we can even use weird combos, like (a,0)+(b,b) to create any coordinate pair.

      A tip: When studying vector spaces and linear transformations, you can't do anything without a basis. It might sound silly, but when you realize this you will start to understand linear algebra!

- The spaces of a matrix
    - Definition of fundamental spaces of a matrix
    - Defining row space, null space, column space and knowing how these spaces are related
    - Knowing how the spaces are important in solving systems of equations and how the determinant is related to these spaces

- Eigenvalues and eigenvectors
    - Eigenvalues of a matrix
    - Eigenvectors of a matrix
    - How determinants are related to eigenvectors and eigenvalues
    - Definition of an eigenspace
    - How the above objects are related to the fundamental spaces of a matrix
    - Useful applications for eigenvalues (there are a lot!)

3 Textbooks

3.1 Intuition

- **The Manga Guide to Linear Algebra, Takahashi.**
  This is a fun introduction to Linear Algebra!

- **Linear Algebra for Dummies, Sterling**

3.2 Computational/Technique based

- **Schaum's 3000 solved Problems**
  Doing all the exercises for every topic from this book will help enormously.

- **Discrete Maths, Kenneth Rosen**
  Discrete math and precalculus textbooks usually have a section on computing stuff with matrices - I wouldn't say these are linear algebra explicitly but they're fine for familiarizing yourself with the main tool used to compute things in a first linear algebra course (matrices)

3.3 Beginner

In addition to the books below, it is important to read the books recommended by your course to learn the abstract/rigorous part.

- **Linear Algebra Demystified, McMahon**
  If you're a beginner and want to go from scratch, this is a great book

- **Elementary Linear Algebra, Howard Anton**
  This is the go-to linear algebra textbook. If you want to learn linear algebra with a more geometric interpretation or intuitive aspect, then this book is a good choice.

- **Schaum's Linear Algebra**

- **Basic Linear Algebra, Blyth**

- **Guide to Linear Algebra, David A. Towers**

- **Linear Algebra: Step by Step, Kuldeep Singh**

- **Introduction to Linear Algebra, Gilbert Strang**

- **Linear Algebra, Jim Hefferson**

- **Elementary Linear Algebra, Ron Larson**

3.4 Advanced

The first thing I recommend checking when opening a Linear Algebra book is the determinant chapter (if there is one). Usually, this chapter tells you about the abstractness of the group.

- **Linear Algebra Done Right, Axler**
  This book is simply inappropriate for your first exposure to linear algebra. I recommend this as a "second time around" book. The problem with LADR as somebody's first linear algebra text is that it doesn't have anything about determinants until near the end and if you're about to take a course in linear algebra where the bulk of the course boils down to checking if a determinant is 0 or not, you'll really struggle.

The two main things tested in a linear algebra course are your ability to row reduce and calculate a determinant (and draw a conclusion from there), so even if this book is a better presentation of the material, it's not the one you want to look at for a first class.
You can also use the (Linear Algebra Done Right) website with videos from the author of this book.
https://linear.axler.net/LADRvideos.html

- **Linear Algebra, Serge Lang**

- **Linear Algebra and its Applications, Strang**

- **Linear Algebra, Friedberg**
  I really recommend this book. You can treat it as an easier version of Hoffman's.

- **Linear Algebra, Hoffman**
  Hoffman defines the determinant function over rings (amazingly) instead of fields. He begins by defining the determinant function as a map that satisfies some properties. Then proves existence (by the well-known formula using cofactors or permutations) and then proves uniqueness.
  This book is more abstract than other books and should be read in a detailed enjoyable way, whilst doing as many exercises as you can from it. The exercises are tough, but worth it!

4 Videos

- 3 Blue 1 Brown "Essence of Linear Algebra" playlist
  This playlist is wonderful to build intuition for the entire course and then you can just learn the actual calculations once the semester begins. However, it is not going to help you solve problems, compute things and write proofs. If you're prepping for a class, you're probably going to want to be able to do precisely this, so if that's the case, skip this playlist until later on when you've built up your own intuition from doing problems. It's nice to know why you're learning certain things if your professors omit it though!
  https://www.youtube.com/playlist?list=PLZHQObOWTQDPD3MizzM2xVFitgF8hE_ab&fbclid=IwAR2wyx90lPS90FwFhcg79a-bzfTUt9a9_JypG8X00VoCPdTjQ9BuG5PT6KQ

- Gilbert Strang from MIT
  This guy is one of the best teachers of Linear Algebra!
  https://www.youtube.com/playlist?list=PL221E2BBF13BECF6C&fbclid=IwAR2wyx90lPS90FwFhcg79a-bzfTUt9a9_JypG8X00VoCPdTjQ9BuG5PT6KQ

  https://ocw.mit.edu/resources/res-18-010-a-2020-vision-of-linear-algebra-spring-2020/

- Professor Aviv Censor
  https://www.youtube.com/watch?v=aefKXYYXT6I...

- Linear Algebra lecture playlist
  https://youtube.com/playlist...