Surds

1) Simplify the following:

a) $\sqrt{7} \times \sqrt{7}$

b) $\sqrt{3} \times \sqrt{3}$

c) $\sqrt{20}$

d) $\sqrt{24}$

e) $\sqrt{72}$

f) $\sqrt{200}$

g) $\sqrt{\frac{2}{25}}$

2) Simplify the following:

a) $\sqrt{2} \times \sqrt{18}$

b) $\sqrt{8} \times \sqrt{32}$

c) $\sqrt{99} \times \sqrt{22}$

d) $\sqrt{45} \times \sqrt{20}$

e) $\sqrt{18} \times \sqrt{128}$

f) $\sqrt{28} \times \sqrt{175}$

3) Expand and simplify where possible:

a) $\sqrt{3}(3-\sqrt{3})$

b) $\sqrt{2}(6+2\sqrt{2})$

c) $\sqrt{7}(2+3\sqrt{7})$

d) $\sqrt{2}(\sqrt{32}-\sqrt{8})$

4) Expand and simplify where possible:

a) $(1+\sqrt{2})(1-\sqrt{2})$

b) $(3+\sqrt{5})(2-\sqrt{5})$

c) $(\sqrt{3}+2)(\sqrt{3}+4)$

d) $(\sqrt{5}-3)(\sqrt{5}+1)$

e) $(2+\sqrt{7})(2-\sqrt{7})$

f) $(\sqrt{6}-3)^2$

5) Work out the following, giving your answer in its simplest form:

a) $\frac{(5+\sqrt{3})(5-\sqrt{3})}{\sqrt{22}}$

b) $\frac{(4-\sqrt{5})(4+\sqrt{5})}{\sqrt{11}}$

c) $\frac{(3-\sqrt{2})(3+\sqrt{2})}{\sqrt{14}}$

d) $\frac{(\sqrt{3}+1)^2}{\sqrt{3}}$

e) $\frac{(\sqrt{5}+3)^2}{\sqrt{20}}$

f) $\frac{(5-\sqrt{5})(2+2\sqrt{5})}{\sqrt{20}}$

1) $\sqrt{5} = 5^k$

a) Write down the value of k.

b) Expand and simplify $(2+\sqrt{5})(1+\sqrt{5})$

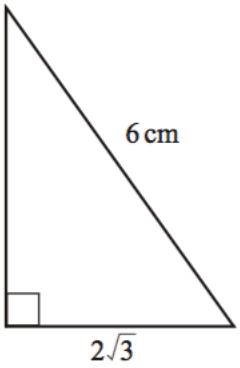
Give your answer in the form $a + b\sqrt{c}$

where a, b and c are integers.

2) The diagram shows a right-angled triangle with lengths of sides as indicated.

The area of the triangle is A cm^2

Show that $A = k\sqrt{2}$ giving the value of k.



3) Given that

$$\frac{8-\sqrt{18}}{\sqrt{2}} = a + b\sqrt{2}$$, where a and b are integers,

find the value of a and the value of b.

4) Work out $(2+\sqrt{3})(2-\sqrt{3})$

Give your answer in its simplest form.

Surds

1) Rationalise the denominator, simplifying where possible:

a) $\frac{3}{\sqrt{2}}$

b) $\frac{2}{\sqrt{2}}$

c) $\frac{3\sqrt{2}}{\sqrt{7}}$

d) $\frac{\sqrt{5}}{\sqrt{10}}$

e) $\frac{1}{4\sqrt{8}}$

f) $\frac{\sqrt{15}}{\sqrt{3}}$

g) $\frac{1}{\sqrt{27}}$

2) Rationalise the denominator of $\frac{1}{\sqrt{3}}$

3) Rationalise the denominator of $\frac{1}{8\sqrt{8}}$ giving the answer in the form $\frac{\sqrt{2}}{p}$

Manipulating Surds

Do these questions without using a calculator.
They're actually not as bad as they might look because you can leave $\sqrt{}$ or π in your answe

Q1 Simplify:

a) $\sqrt{5} \times \sqrt{3}$ **b)** $\frac{\sqrt{20}}{\sqrt{5}}$ **c)** $(\sqrt{x})^2$

d) $\sqrt{x^2}$ **e)** $\sqrt{8} \times \sqrt{8}$ **f)** $\frac{\sqrt{30}}{\sqrt{6}}$

Q2 A circle has a radius of $\sqrt{3}$ cm. What is its exact area? (Area of circle = πr^2.)

Q3 Simplify these expressions:

Remember — $\sqrt{a} \times \sqrt{b} = \sqrt{(ab)}$.

a) $\sqrt{4} - \sqrt{1}$ **b)** $2\sqrt{3} + 3\sqrt{3}$ **c)** $\sqrt{8}$ **d)** $(2+\sqrt{3})^2$

e) $4\sqrt{5} - \sqrt{5}$ **f)** $\sqrt{50}$ **g)** $\sqrt{8} - \sqrt{2}$ **h)** $\sqrt{18} - \sqrt{9}$

Q4 Are the following expressions rational or irrational?

a) $(1+\sqrt{5})(1-\sqrt{5})$ **b)** $\frac{1+\sqrt{5}}{1-\sqrt{5}}$

Q5 If $x = 1$ and $y = \sqrt{2}$, are the following expressions rational or irrational?

a) $(x + y)(x - y)$ **b)** $\frac{x+y}{x-y}$

Q6 Rationalise the denominators of the following expressions, and then simplify if necessa

a) $\frac{1}{\sqrt{2}}$ **b)** $\frac{2}{\sqrt{8}}$ **c)** $\frac{a}{\frac{\sqrt{40}}{2}}$ **d)** $\frac{x}{\sqrt{xy}}$

e) $\frac{1}{1+\sqrt{2}}$ **f)** $\frac{6}{3+\sqrt{3}}$ **g)** $\frac{2}{1+\sqrt{6}}$ **h)** $\frac{5+\sqrt{5}}{5-\sqrt{5}}$

Remember: rationalising the denominator means getting rid of the square root signs on the bottom of fractions.

1 Find the value of the integer k.

a $\sqrt{8} = k\sqrt{2}$ **b** $\sqrt{18} = k\sqrt{2}$ **c** $\sqrt{50} = k\sqrt{2}$ **d** $\sqrt{80} = k\sqrt{5}$

2 Simplify

a $\sqrt{200}$ **b** $\sqrt{32}$ **c** $\sqrt{20}$ **d** $\sqrt{28}$

3 Solve the equation $x^2 = 30$, leaving your answer in surd form.

4 Expand these expressions. Write your answers in the form $a + b\sqrt{c}$ where a, b and c are integers.

a $\sqrt{3}(2 + \sqrt{3})$ **b** $(\sqrt{3} + 1)(2 + \sqrt{3})$ **c** $(\sqrt{5} - 1)(2 + \sqrt{5})$

d $(\sqrt{7} + 1)(2 - \sqrt{7})$ **e** $(2 - \sqrt{3})^2$ **f** $(\sqrt{2} + 5)^2$

5 The area of a square is 40 cm^2. Find the length of one side of the square.
Give your answer as a surd in its simplest form.

6 The lengths of the sides of a rectangle are $(3 + \sqrt{5})$ cm and $(3 - \sqrt{5})$ cm.
Work out, in their simplified forms:

a the perimeter of the rectangle **b** the area of the rectangle.

7 The length of the side of a square is $(1 + \sqrt{2})$ cm. Work out the area of the square.
Give your answer in the form $(a + b\sqrt{2})$ cm^2 where a and b are integers.

Exercise 2G

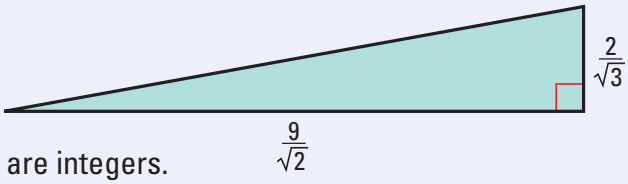
1 Rationalise the denominators and simplify your answers, if possible.

a $\frac{1}{\sqrt{2}}$ **b** $\frac{1}{\sqrt{5}}$ **c** $\frac{5}{\sqrt{10}}$ **d** $\frac{2}{\sqrt{2}}$ **e** $\frac{4}{\sqrt{12}}$

2 Rationalise the denominators and give your answers in the form $a + b\sqrt{c}$ where a, b and c are integers.

a $\frac{2+\sqrt{2}}{\sqrt{2}}$ **b** $\frac{6-\sqrt{2}}{\sqrt{2}}$ **c** $\frac{10+\sqrt{5}}{\sqrt{5}}$ **d** $\frac{12-\sqrt{3}}{\sqrt{3}}$ **e** $\frac{14+\sqrt{7}}{\sqrt{7}}$

3 The diagram shows a right-angled triangle.
The lengths are given in centimetres.
Work out the area of the triangle.
Give your answer in the form $a + b\sqrt{c}$ where a, b and c are integers.



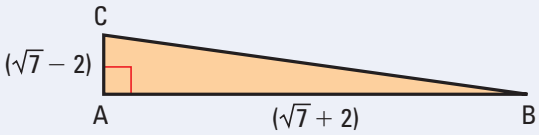
4 Solve these equations leaving your answers in surd form.

a $x^2 - 6x + 2 = 0$ **b** $x^2 + 10x + 14 = 0$

5 The diagram represents a right-angled triangle ABC.
AB $= (\sqrt{7} + 2)$ cm AC $= (\sqrt{7} - 2)$ cm.
Work out, leaving any appropriate answers in surd form:

a the area of triangle ABC

b the length of BC



Simplifying Square Roots

1. Simplify the following as far as possible:

(a) $\sqrt{18}$ (b) $\sqrt{8}$ (c) $\sqrt{12}$

(d) $\sqrt{50}$ (e) $\sqrt{45}$ (f) $\sqrt{44}$

(g) $\sqrt{75}$ (h) $\sqrt{63}$ (i) $\sqrt{72}$

2. Simplify the following as far as possible:

(a) $\sqrt{320}$ (b) $\sqrt{180}$ (c) $\sqrt{300}$

(d) $\sqrt{245}$ (e) $\sqrt{200}$ (f) $\sqrt{343}$

(g) $\sqrt{135}$ (h) $\sqrt{150}$ (i) $\sqrt{216}$

3. Find the following in the form $\sqrt{n}$:

(a) $7\sqrt{2}$ (b) $3\sqrt{3}$ (c) $2\sqrt{7}$

(d) $3\sqrt{7}$ (e) $2\sqrt{2}$ (f) $5\sqrt{5}$

4. Simplify the following as far as possible, leaving your answer in the form $a\sqrt{b}$:

(a) $2\sqrt{3}+5\sqrt{3}$ (b) $7\sqrt{2}-3\sqrt{2}$ (c) $\sqrt{3}+\sqrt{12}$

(d) $\sqrt{27}+2\sqrt{3}$ (e) $5\sqrt{5}+\sqrt{45}$ (f) $7\sqrt{2}+\sqrt{50}$

(g) $\sqrt{18}+\sqrt{200}$ (h) $\sqrt{60}+\sqrt{135}$ (i) $\sqrt{180}-\sqrt{20}$

5. Simplify the following as far as possible:

(a) $2\sqrt{3}\times 5\sqrt{3}$ (b) $5\sqrt{2}\times 2\sqrt{2}$ (c) $\sqrt{3}\times\sqrt{27}$

(d) $\sqrt{50}\times 2\sqrt{2}$ (e) $2\sqrt{7}\times 3\sqrt{28}$ (f) $3\sqrt{3}\times 5\sqrt{75}$

(g) $\dfrac{4\sqrt{2}}{\sqrt{8}}$ (h) $\dfrac{12\sqrt{3}}{\sqrt{48}}$ (i) $\dfrac{10\sqrt{6}}{\sqrt{150}}$

(j) $\dfrac{\sqrt{12}}{\sqrt{300}}$ (k) $\dfrac{3\sqrt{28}}{\sqrt{7}}$ (l) $\dfrac{\sqrt{8}+\sqrt{12}}{\sqrt{2}+\sqrt{3}}$

Simplifying Square Roots

1. Simplify the following as far as possible:

(a) $\left(\sqrt{5}+2\right)^2$ (b) $\left(\sqrt{2}+\sqrt{3}\right)^2$

(c) $\left(\sqrt{7}-\sqrt{3}\right)^2$ (d) $\left(\sqrt{5}+2\right)\left(\sqrt{5}-2\right)$

(e) $\left(\sqrt{11}+\sqrt{7}\right)\left(\sqrt{11}-\sqrt{7}\right)$ (f) $\left(\sqrt{5}+2\right)^3$

2. Write the following, in the simplest possible form:

(a) $\sqrt{50}+\sqrt{18}$ (b) $\sqrt{75}-\sqrt{12}$ (c) $\sqrt{300}-\sqrt{48}$

(d) $\dfrac{\sqrt{72}}{\sqrt{8}}$ (e) $\dfrac{\sqrt{45}}{\sqrt{20}}$ (f) $\dfrac{8}{\sqrt{2}}$

3. Write the following, in the simplest possible form (in the form $a\sqrt{b}$):

(a) $\dfrac{6}{\sqrt{2}}$ (b) $\dfrac{3}{\sqrt{3}}$ (c) $\dfrac{10}{\sqrt{5}}$

(d) $\dfrac{21}{\sqrt{7}}$ (e) $\dfrac{15}{\sqrt{5}}$ (f) $\dfrac{33}{\sqrt{11}}$

(g) $\sqrt{8}-\dfrac{2}{\sqrt{2}}$ (h) $\sqrt{20}+\dfrac{15}{\sqrt{5}}$ (i) $\sqrt{300}-\dfrac{15}{\sqrt{3}}$

(j) $\sqrt{28}-\dfrac{14}{\sqrt{7}}$ (k) $\sqrt{12}+\dfrac{9}{\sqrt{3}}$ (l) $\sqrt{44}+\dfrac{22}{\sqrt{11}}$

4. (a) Express $\dfrac{10}{\sqrt{2}}$ in the form $a\sqrt{b}$, where a and b are positive integers.

An isosceles right-angled triangle ABC has a right angle at B. The length of its equal sides is $\dfrac{10}{\sqrt{2}}$ cm.

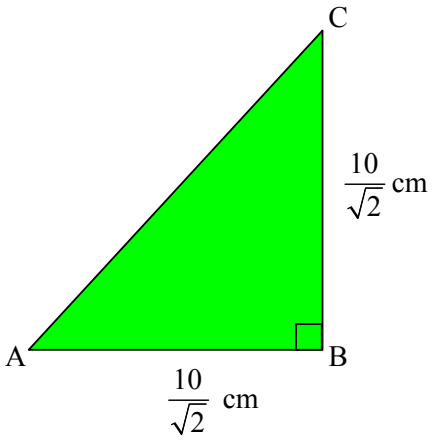


Diagram **NOT** accurately drawn

(b) Find the area of the triangle. Give your answer as an integer.

(c) Find also the hypotenuse of the triangle. Give your answer as an integer.

(i) $3\sqrt{2}+2\sqrt{8}$

.....

[1]

(ii) $\dfrac{21}{\sqrt{7}}$

.....

[2]

(iii) $\left(\sqrt{5}+2\sqrt{3}\right)\left(\sqrt{5}-2\sqrt{3}\right)$

23 Express $\sqrt{48}+\sqrt{108}$ in the form $k\sqrt{6}$ where k is a surd.

.....

(Total for Question 23 is 3 marks)

(*b*) Given that $\frac{8-\sqrt{18}}{\sqrt{2}} = a + b\sqrt{2}$, where a and b are integers,

find the value of a and the value of b.

$a =$ .....

$b =$ .....

(3)

19 (a) Show that $\left(5-\sqrt{8}\right)\left(7+\sqrt{2}\right)=31-9\sqrt{2}$

Show each stage of your working.

(3)

Given that c is a prime number,

(b) rationalise the denominator of $\dfrac{3c-\sqrt{c}}{\sqrt{c}}$

Simplify your answer.

C1 ALGEBRA — Worksheet A

1 Evaluate

a $\sqrt{49}$ **b** $\sqrt{121}$ **c** $\sqrt{\frac{1}{9}}$ **d** $\sqrt{\frac{4}{25}}$ **e** $\sqrt{0.01}$ **f** $\sqrt{0.09}$

g $\sqrt[3]{8}$ **h** $\sqrt[3]{1000}$ **i** $\sqrt[4]{81}$ **j** $\sqrt{1\frac{9}{16}}$ **k** $\sqrt[3]{0.125}$ **l** $\sqrt[3]{15\frac{5}{8}}$

2 Simplify

a $\sqrt{7} \times \sqrt{7}$ **b** $4\sqrt{5} \times \sqrt{5}$ **c** $(3\sqrt{3})^2$ **d** $(\sqrt{6})^4$

e $(\sqrt{2})^5$ **f** $(2\sqrt{3})^3$ **g** $\sqrt{2} \times \sqrt{8}$ **h** $2\sqrt{3} \times \sqrt{27}$

i $\dfrac{\sqrt{32}}{\sqrt{2}}$ **j** $\dfrac{\sqrt{3}}{\sqrt{12}}$ **k** $(\sqrt[3]{6})^3$ **l** $(3\sqrt[3]{2})^3$

3 Express in the form $k\sqrt{2}$

a $\sqrt{18}$ **b** $\sqrt{50}$ **c** $\sqrt{8}$ **d** $\sqrt{98}$ **e** $\sqrt{200}$ **f** $\sqrt{162}$

4 Simplify

a $\sqrt{12}$ **b** $\sqrt{28}$ **c** $\sqrt{80}$ **d** $\sqrt{27}$ **e** $\sqrt{24}$ **f** $\sqrt{128}$

g $\sqrt{45}$ **h** $\sqrt{40}$ **i** $\sqrt{75}$ **j** $\sqrt{112}$ **k** $\sqrt{99}$ **l** $\sqrt{147}$

m $\sqrt{216}$ **n** $\sqrt{800}$ **o** $\sqrt{180}$ **p** $\sqrt{60}$ **q** $\sqrt{363}$ **r** $\sqrt{208}$

5 Simplify

a $\sqrt{18} + \sqrt{50}$ **b** $\sqrt{48} - \sqrt{27}$ **c** $2\sqrt{8} + \sqrt{72}$

d $\sqrt{360} - 2\sqrt{40}$ **e** $2\sqrt{5} - \sqrt{45} + 3\sqrt{20}$ **f** $\sqrt{24} + \sqrt{150} - 2\sqrt{96}$

6 Express in the form $a + b\sqrt{3}$

a $\sqrt{3}(2 + \sqrt{3})$ **b** $4 - \sqrt{3} - 2(1 - \sqrt{3})$ **c** $(1 + \sqrt{3})(2 + \sqrt{3})$

d $(4 + \sqrt{3})(1 + 2\sqrt{3})$ **e** $(3\sqrt{3} - 4)^2$ **f** $(3\sqrt{3} + 1)(2 - 5\sqrt{3})$

7 Simplify

a $(\sqrt{5} + 1)(2\sqrt{5} + 3)$ **b** $(1 - \sqrt{2})(4\sqrt{2} - 3)$ **c** $(2\sqrt{7} + 3)^2$

d $(3\sqrt{2} - 1)(2\sqrt{2} + 5)$ **e** $(\sqrt{5} - \sqrt{2})(\sqrt{5} + 2\sqrt{2})$ **f** $(3 - \sqrt{8})(4 + \sqrt{2})$

8 Express each of the following as simply as possible with a rational denominator.

a $\dfrac{1}{\sqrt{5}}$ **b** $\dfrac{2}{\sqrt{3}}$ **c** $\dfrac{1}{\sqrt{8}}$ **d** $\dfrac{14}{\sqrt{7}}$ **e** $\dfrac{3\sqrt{2}}{\sqrt{3}}$ **f** $\dfrac{\sqrt{5}}{\sqrt{15}}$

g $\dfrac{1}{3\sqrt{7}}$ **h** $\dfrac{12}{\sqrt{72}}$ **i** $\dfrac{1}{\sqrt{80}}$ **j** $\dfrac{3}{2\sqrt{54}}$ **k** $\dfrac{4\sqrt{20}}{3\sqrt{18}}$ **l** $\dfrac{3\sqrt{175}}{2\sqrt{27}}$

9 Simplify

a $\sqrt{8} + \frac{6}{\sqrt{2}}$ **b** $\sqrt{48} - \frac{10}{\sqrt{3}}$ **c** $\frac{6-\sqrt{8}}{\sqrt{2}}$

d $\frac{\sqrt{45}-5}{\sqrt{20}}$ **e** $\frac{1}{\sqrt{18}} + \frac{1}{\sqrt{32}}$ **f** $\frac{2}{\sqrt{3}} - \frac{\sqrt{6}}{\sqrt{72}}$

10 Solve each equation, giving your answers as simply as possible in terms of surds.

a $x(x + 4) = 4(x + 8)$ **b** $x - \sqrt{48} = 2\sqrt{3} - 2x$

c $x\sqrt{18} - 4 = \sqrt{8}$ **d** $x\sqrt{5} + 2 = \sqrt{20}\,(x - 1)$

11 **a** Simplify $(2 - \sqrt{3})(2 + \sqrt{3})$.

b Express $\frac{2}{2-\sqrt{3}}$ in the form $a + b\sqrt{3}$.

12 Express each of the following as simply as possible with a rational denominator.

a $\frac{1}{\sqrt{2}+1}$ **b** $\frac{4}{\sqrt{3}-1}$ **c** $\frac{1}{\sqrt{6}-2}$ **d** $\frac{3}{2+\sqrt{3}}$

e $\frac{1}{2+\sqrt{5}}$ **f** $\frac{\sqrt{2}}{\sqrt{2}-1}$ **g** $\frac{6}{\sqrt{7}+3}$ **h** $\frac{1}{3+2\sqrt{2}}$

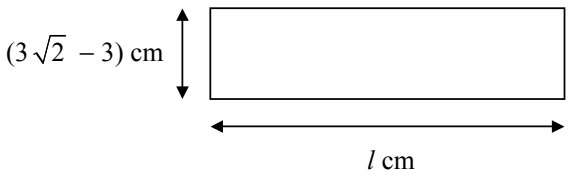
i $\frac{1}{4-2\sqrt{3}}$ **j** $\frac{3}{3\sqrt{2}+4}$ **k** $\frac{2\sqrt{3}}{7-4\sqrt{3}}$ **l** $\frac{6}{\sqrt{5}-\sqrt{3}}$

13 Solve the equation

$$3x = \sqrt{5}\,(x + 2),$$

giving your answer in the form $a + b\sqrt{5}$, where a and b are rational.

14



The diagram shows a rectangle measuring $(3\sqrt{2} - 3)$ cm by l cm.
Given that the area of the rectangle is 6 cm^2, find the exact value of l in its simplest form.

15 Express each of the following as simply as possible with a rational denominator.

a $\frac{\sqrt{2}}{\sqrt{2}+\sqrt{6}}$ **b** $\frac{1+\sqrt{3}}{2+\sqrt{3}}$ **c** $\frac{1+\sqrt{10}}{\sqrt{10}-3}$ **d** $\frac{3-\sqrt{2}}{4+3\sqrt{2}}$

e $\frac{1-\sqrt{2}}{3-\sqrt{8}}$ **f** $\frac{\sqrt{3}-5}{2\sqrt{3}-4}$ **g** $\frac{\sqrt{12}+3}{3-\sqrt{3}}$ **h** $\frac{3\sqrt{7}-2}{2\sqrt{7}-5}$