BioMedical Admissions Test (BMAT)

Section 2: Mathematics

Topic M7: Probability

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Probability Basics

- Probabilities lie between 0 and 1.
- The sum of all probabilities is 1.
- Mutually exclusive events have no impact on one another
- Combined probability:
    - P(A or B) = P(A) + P(B) if mutually exclusive
    - P(A or B) = P(A) + P(B)−P(A and B) if not mutually exclusive
    - P(A and B) = P(A) × P(B)

Example:
The probability of winning game A is 0.3. The probability of winning game B is 0.5. Winning on one machine does change the odds of winning on the other.
What is the probability that you lose on both games?

P(win at A and win at B) = P(win at A) x P (win at B) = 0.3 x 0.5 = 0.15

Experiments

The outcome of probability experiments can be recorded in a table.

Example:
There are 20 sweets in the bag. Calculate the probability of randomly picking each of the different types of sweets.

	Chocolate	Haribos	Licorice	Total
Frequency	10	6	4	20

Chocolate = $\frac{10}{20} = \frac{1}{2} = 0.5$
Haribos = $\frac{6}{20} = \frac{3}{10} = 0.33..$
Licorice = $\frac{4}{20} = \frac{1}{5} = 0.2$

Probabilities can either be written in decimal or fractions.

Sample space

A sample space is the set of all the possible outcomes that could happen in an experiment. For example, the probability of tossing a coin twice this could be written as {HH, TT, HT, TH}. When rolling a dice or spinning a spinner, if the die or spinner is **fair** then there is an equal chance of landing on each number.

Sample space diagram

We can also use a table to visualise the outcomes. For example, this sample space diagram shows the probabilities of rolling two die:

	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,1	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

Expected frequency $= probability \times number\ of\ trials$

Relative frequency $= \frac{frequency}{number\ of\ trials}$

We can see from the sample space diagram above that the probability of rolling the same number on 2 die = $\frac{6}{36} = \frac{1}{6}$

Therefore, if we rolled two die at the same time 10 times then the likelihood of rolling a double is:
Expected frequency = $\frac{1}{6} \times 10 = \frac{10}{6} = 1.67$

However, this is only an expected frequency, meaning that this might not be exactly the number of times the event actually happens. It might be more or less.

Theoretical probability

$$\text{Probability} = \frac{\textit{number of favourable outcomes}}{\textit{number of possible outcomes}}$$

Example:

P(lands of an even number) = $\frac{\textit{number of even numbers}}{\textit{number of possible outcomes}} = \frac{3}{6} = \frac{1}{2}$

Conditional probability

The probability of event B occurring once A has occurred is called the conditional probability. This is usually written as B given A.

Mutually exclusive and exhaustive events

Events are **mutually exclusive** if, when you flip a coin, it cannot land both heads and tails simultaneously. This means the probability of both occurring simultaneously is 0.

Exhaustive events are when at least one of the events must occur. For example, if a dice is rolled and one event is 'lands on an even number' and another event is 'lands on an odd number' then they are exhaustive, as at least one of these events must occur.

This can also be written as: P(A) + P(not A) = 1 as the two events are exhaustive and mutually exclusive.

Example: If the probability of it snowing next week is 0.25, what is the probability of it not snowing?

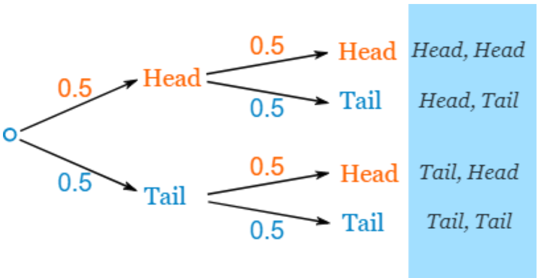
P(snowing) + P(not snowing) = 1

P(not snowing) = 1 - 0.25

P (not snowing) = 0.75

Tree diagrams

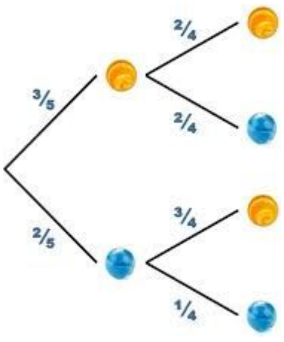
A tree diagram can also be used to list outcomes. First, we must draw out what each of the possible outcomes are and also what the probability is of each outcome occurring.



Probabilities independent of previous outcome:

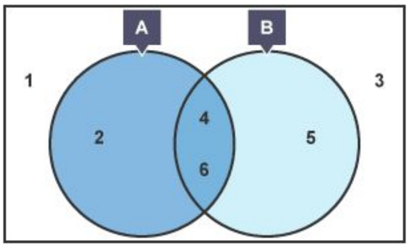
Here is an example of a tree diagram for flipping a coin twice. The chances of flipping heads are 50%, irrelevant of what the first toss was so it is independent of the previous outcome.
From the tree diagram we can see that the chances of getting, for example, 2 heads in a row is 0.5 x 0.5 = 0.25, as each time the chance is 50%.

Probabilities dependent of previous outcome:
There is a bag of marbles - 3 are yellow and 2 are blue. You pick one marble from the bag and without putting it back in, you pick another marble. Therefore, the probabilities change as there is one less marble in the bag after the first pick.
Therefore the probability of picking 2 yellow marbles is $\frac{3}{5} \times \frac{2}{4} = \frac{6}{20} = \frac{3}{10}$



Venn diagrams

Venn diagrams use circles to show which values fit into different sets. Values that fit into both sets are found within then the overlap. Numbers that are not members of either set are put outside the circle.



Example: A dice is rolled. Event A is rolling an even number and Event B is rolling a number greater than 3.
The numbers in set A are therefore {2,4,6} and set B is {4,5,6}.
A ∪ B is the **union** of the two sets, therefore {2,4,5,6}
A ∩ B is the **intersection** of the two sets, therefore {4,6}