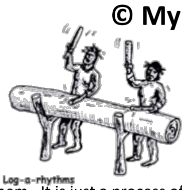
log (😅) = 💧log (😄)
log (😂) = 💧💧log (😄)
log(😴)=💤log(😲)
log(👫)=log(💁‍♂️)+log(💁‍♀️)

www.mymathscloud.com

Log Rules:



log (😅) = 💧log (😄)
log (😂) = 💧💧log (😄)
log(😴)=💤log(😲)
log(👫)=log(💁‍♂️)+log(💁‍♀️)

The key to success at logs is memorising the 5 rules and being good at algebra. Just know your rules and when to use them. It is just a process of elimination! If one rule doesn't work ask yourself, can I apply the other? Also remember that the rules work both ways, not only from left to right, hence the $\Leftrightarrow$ sign below. A log has the form $log_a b = c \ where = a > 0, b > 0$ (Note: when no base a is written it means base 10 by default)

Rule	In Words	Examples	Common Mistakes
Logs			
$c\, log_a\, b \Leftrightarrow log_a\, b^c$ **log x^n =** 	we can bring the power up and down	**Example 1:** $2\log_3 5$ This is the same as writing $\log_3 5^2 = \log_3 25$ **Example 2:** $\log_4 9$ This is the same as writing $\log_4 3^2 = 2\log_4 3$	$(\log_a b)^c \neq c\, log_a\, b$ This is because $(\log_a b)^c$ is not the same as $log_a\, b^c$ Instead, $log_a\, b^c = c\, log_a\, b$
$\log_a b = c \Leftrightarrow a^c = b$	writing a log is the same as writing an index form, so we can get rid of log completely and write it in index form. This rule is used all the time, especially for solving when we want to get rid of the log.	**Example 1:** simplify/evaluate $\log_2 8$ we set this equal to x. If we can find what x is then we know the value of $\log_2 8$ $\log_2 8 = x$ $2^x = 8$ $x = 3$	Failure to realise the following extra cancellation laws resulting from this rule: $\log_a a^x = x$ $\log_a a = 1$ $a^{\log_a x} = x$
$\log_a b + \log_a c \Leftrightarrow \log_a bc$ The coefficients must be 1 to use this rule: $1\log_a b + 1\log_a c \Leftrightarrow \log_a bc$ If they aren't use rule $c\, log_a\, b \Leftrightarrow log_a\, b^c$	we can make two logs added into one multiplied and vice versa	**Example 1:** Simplify $4\log_3 x^2 + 3\log_3 y^5$ $\log_3 (x^2)^4 + \log_3 (y^5)^3 = \log_3 x^8 + \log_3 y^{15} = \log_3 x^8 y^{15}$ **Example 2:** Given $\log_5 x = p, \log_5 q$, find $\log_{10} xy^3$ in terms of p & q $\log_5 xy^3 = log_5 x + log_5 y^3 = log_5 x + 3log_5 y = p + 3q$	$log(a \pm b) \neq loga \pm logb$ You can't expand/distribute a log, it is fixed with its argument **ln(1+2+3) = ln1 + ln2 + ln3**  Remember ONE log multiplied goes to TWO added, two logs do not go to two logs!
$\log_a b - \log_a c \Leftrightarrow \log_a \frac{b}{c}$ The coefficients must be 1 to use this rule: $1\log_a b - 1\log_a c \Leftrightarrow \log_a \frac{b}{c}$ If they aren't use rule $c\, log_a\, b \Leftrightarrow log_a\, b^c$	we can make two logs added into one divided and vice versa	**Example 1:** Simplify $5\log_3 x^3 - 4\log_3 y^2$ $\log_3 (x^3)^5 - \log_3 (y^2)^4 = \log_3 x^{15} - \log_3 y^8 = \log_3 \frac{x^5}{y^8}$ **Example 2:** Given $\log_5 x = p, \log_5 = q$, find $\log_{10} \frac{x^4}{y^2}$ in terms of p & **q** $\log_5 \frac{x^4}{y^2} = log_5\, x^4 - log_5\, y^2 = 4\, log_5 x - 2log_5 y = 4p - 2q$	$\frac{loga}{logb} \neq loga - logb$ $\frac{loga}{logb} \neq log\frac{a}{b}$ Remember ONE log divided goes to TWO subtracted, two logs do not go to two logs!
$\log_a b \Leftrightarrow \frac{\log_c b}{\log_c a}$	we can change the base of any log if it isn't the base that we need/want	**Example 1:** $\log_a 2 = 2$, $\log_a 5 = y$. Find $\log_2 25$ & $\log_2 20$ in terms of y $\log_2 25 = \frac{log_a 25}{log_a 2} = \frac{log_a 5^2}{log_a 2} = \frac{2log_a 5}{log_a 2} = \frac{2(y)}{2} = y$ $\log_2 20 = \frac{log_a 20}{log_a 2} = \frac{log_a (4\times5)}{log_a 2} = \frac{log_a (2^2\times5)}{log_a 2} = \frac{2\log_a 2 + log_a 5}{log_a 2} = \frac{2(2)+y}{2} = \frac{4+y}{2}$	We only use this when the bases don't match. © MyMathsCloud

Ln and Exponentials

The rules for $ln\ x$ work the exact same way for $log\ x$. $\ln x$ is the same as $\log_e x$
$e^{lnx} = x$ (e and ln are inverses so cancel each other out - remember e takes a power)
$lne^x = x$ (e and ln are inverses so cancel each other out-remember ln takes an argument)

4 Solving Types

Solving with a power x and base other than e

1) Take the log of both sides if you can't make the bases the same (if have 2 terms, one on each side)
2) Use indices rules and then becomes a hidden quadratic if have 3 terms)

Example 1: $5^{x-2} = 7$
Log both sides
$log\ 5^{x-2} = log\ 7$
$(x-2)log\ 5 = log\ 7$
$(log\ 5)x - 2\ log\ 5 = log\ 7$
$(log\ 5)\ x = log\ 7 + 2\ log\ 5$
$x = \frac{log\ 7 + 2\ log\ 5}{log\ 5} = \frac{log\ 7 + log\ 5^2}{log\ 5} = \frac{log\ 175}{log\ 5}$

Example 2: $2^{2x+3} = 3^{2x+2}$. Give your answer in form $\frac{\log a}{\log b}$
$\log 2^{2x+3} = \log 3^{2x+2}$
$(2x+3)\log 2 = (2x+2)\log 3$
$(2\log 2)x + 3\log 2 = (2\log 3)x + 2\log 3$
$(2\log 2)x - (2\log 3)x = 2\log 3 - 3\log 2$
$x(2\log 2 - 2\log 3) = 2\log 3 - 3\log 2$
$x = \frac{2\log 3 - 3\log 2}{2\log 2 - 2\log 3} = \frac{\log 9 - \log 8}{\log 4 - \log 9} = \frac{\log\frac{9}{8}}{\log\frac{4}{9}}$

Watch out for hidden quadratic questions where we have to use a substitution first before taking logs

Solving with a power x and base e

Take the natural log of both sides

Example: $2e^{x-3} = 5$
$e^{x-3} = \frac{5}{2}$
$ln(e^{x-3}) = ln\left(\frac{5}{2}\right)$
$x - 3 = ln\left(\frac{5}{2}\right)$
$x = ln\left(\frac{5}{2}\right) + 3$

Solving with the form $\log_a b = c$

Use Rule $\log_a b = c \Leftrightarrow a^c = b$
If you have more than one log you will need to condense them into one log using the rules:
$\log_a b + \log_a c \Leftrightarrow \log_a bc$
$\log_a b - \log_a c \Leftrightarrow \log_a \frac{b}{c}$

Example 1: $\log_2 x + \log_2(x-2)$=3
$\log_2 x(x-2) = 3$
$\log_2(x^2 - 2x) = 3$
$2^3 = x^2 - 2x$
$x^2 - 2x - 8 = 0$
$x = 4, x = -2$

Example 2: $\log_2(11y - 5) - \log_2 3 = 2\log_2 y + 1$
$\log_2(11y - 5) - \log_2 3 - 2\log_2 y = 1$
$\log_2 \frac{11y - 5}{3y^2} = 1$
$\frac{11y - 5}{3y^2} = 2^1 = 2$
$11y - 5 = 6y^2$
$6y^2 - 11y + 5 = 0 \Longrightarrow y = 1, y = \frac{5}{6}$

Solving with the form $\ln x$

Either replace as $\log_e x$ and proceed as normal or raise both sides to the power e

Example: $\ln(x-2) = 5$

Way 1:	Way 2:
Raise e both sides	Replace with $\log_e x$
$e^{\ln(x-2)} = e^5$ $x - 2 = e^5$ $x = e^5 + 2$	$\log_e(x-2) = 5$ $e^5 = x - 2$ $x = e^5 + 2$