Answer ALL TWENTY FOUR questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1   The table shows information about the lengths, in minutes, of 50 telephone calls.

Length of telephone call (m minutes)	Frequency	
$0 < m \leqslant 5$ **2.5**	$\times$ 8	= 20
$5 < m \leqslant 10$ **7.5**	$\times$ 2	= 15
$10 < m \leqslant 15$ **12.5**	$\times$ 6	= 75
$15 < m \leqslant 20$ **17.5**	$\times$ 4	= 70
$20 < m \leqslant 25$ **22.5**	$\times$ 12	= 270
$25 < m \leqslant 30$ **27.5**	$\times$ 18	= 495

50

(a) Write down the modal class.

$25 < m \leqslant 30$

(1)

(b) Work out an estimate for the total length, in minutes, of these telephone calls.

not mean

$20 + 15 + 75 + 70 + 270 + 495 = 945$

.

945 minutes

(3)

(Total for Question 1 is 4 marks)

Turn over ▶

2 The diagram shows triangle *ABC* and triangle *ECD*
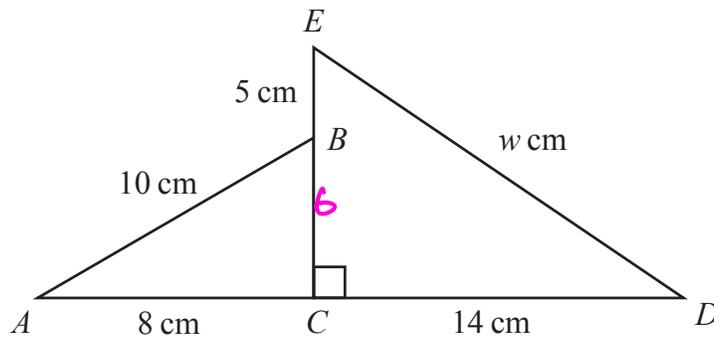

Diagram **NOT** accurately drawn

ACD and *EBC* are straight lines.

AB = 10 cm   *AC* = 8 cm   *EB* = 5 cm   *CD* = 14 cm   *ED* = *w* cm

Work out the value of *w*
Give your answer correct to one decimal place.

$\triangle ABC$

$BC = \sqrt{10^2 - 8^2}$

$= \sqrt{100 - 64} \quad = \sqrt{36} \quad = 6$

$\triangle CDE$

$ED = \sqrt{11^2 + 14^2}$

$= \sqrt{121 + 196}$

$= \sqrt{317}$

$= 17.804493$
1 dp

w = ...**17.8**........

(Total for Question 2 is 4 marks)

3
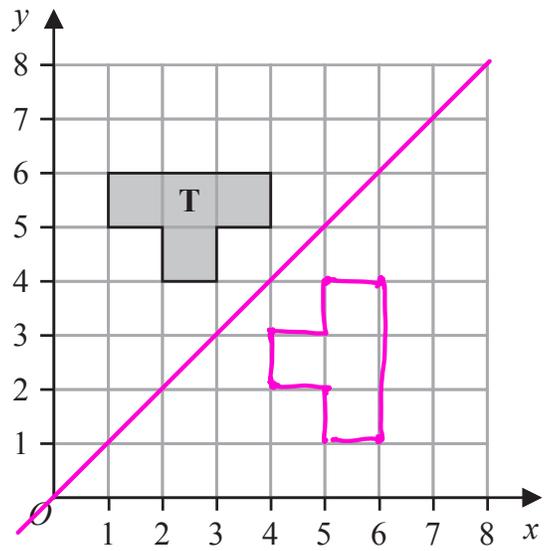
(a) Reflect shape **T** in the line $y = x$

(2)
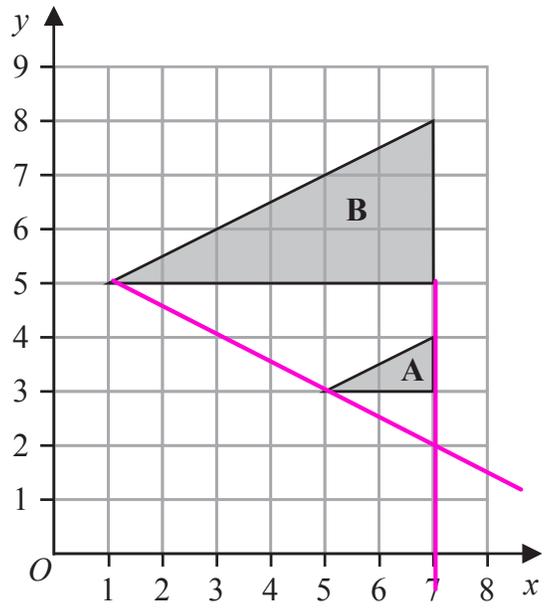
(b) Describe fully the single transformation that maps triangle **A** onto triangle **B**

Enlargement, scale factor 3, centre (7,2)

(3)

(Total for Question 3 is 5 marks)

4 (a) Solve $\dfrac{2x+5}{6} = 2x - 5$

Show clear algebraic working.

$$2x + 5 = 6(2x - 5)$$

$$2x + 5 = 12x - 30$$
$$-2x \qquad\qquad -2x$$

$$5 = 10x - 30$$
$$+30 \qquad\qquad +30$$

$$10x = 35$$
$$x = \dfrac{35}{10}$$

$x =$ .... 3.5 ....

(3)

(b) Simplify $h^{15} \div h^3$

$$h^{15-3}$$

.... h^{12} ....

(1)

(c) Simplify fully $\left(2g^3 k^5\right)^4$

$$2g^3 k^5 \times 2g^3 k^5 \times 2g^3 k^5 \times 2g^3 k^5$$

$$= 16g^{12} k^{20}$$

.... $16g^{12} k^{20}$ ....

(2)

(d) Given that $\dfrac{y^5 \times y^n}{y^7} = y^{12}$

work out the value of n

$$5 + n - 7 = 12$$

$$n = 12 + 7 - 5$$

$$= 14$$

$n =$ .... 14 ....

(2)

(Total for Question 4 is 8 marks)

5 Avril bakes a cake.

She uses flour, butter and sugar such that

weight of flour : weight of butter = 6 : 5
weight of butter : weight of sugar = 3 : 2

Avril uses 120 grams of sugar.

Work out the weight of flour Avril uses.

B : S
3 : 2
×60) ×60
= 180g   120

F : B
6 : 5
×36) ×36
= 216g   180g

216 .... grams

(Total for Question 5 is 3 marks)

6 Show that $3\dfrac{3}{7} \div 2\dfrac{2}{3} = 1\dfrac{2}{7}$

$3\dfrac{3}{7} = \dfrac{24}{7}$

$2\dfrac{2}{3} = \dfrac{8}{3}$

$\dfrac{24}{7} \div \dfrac{8}{3}$

$= \dfrac{\overset{3}{\cancel{24}}}{7} \times \dfrac{3}{\cancel{8}_1}$

$= \dfrac{9}{7}$

$\dfrac{9}{7} = 1\dfrac{2}{7}$ as required.

(Total for Question 6 is 3 marks)

7 Hermione buys a boat for $26 800
The value of the boat depreciates by 8% each year.

$100 - 8 = 92\%$

Work out the value of the boat at the end of 3 years.
Give your answer correct to the nearest dollar.

$26800 \times 0.92^3 = 20868.8384$

nearest dollar

$.................\underline{20869}........$

(Total for Question 7 is 3 marks)

8 The mean number of goals scored by a hockey team in 8 matches is 6

The team plays 2 more matches and scores k goals in each match.

The mean number of goals scored by the hockey team in the 10 matches is 7

Work out the value of k

matches 8 { 8+2
              10

mean 6 { 7

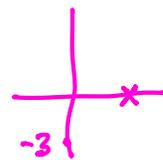total goals 8 × 6   { 10 × 7
             = 48   { = 70

so 70 - 48 goals scored in 2 games

= 22 in 2 games

22 ÷ 2 = 11

$k =$ __11__

(Total for Question 8 is 3 marks)

9 A straight line passes through the points with coordinates $(0, –3)$ and $(2, 0)$

Find an equation of the line.

gradient $= \dfrac{3}{2} = 1.5$

$y = 1.5x + c$ passes through $(2, 0)$
                                        x y

$0 = 1.5 × 2 + c$

$c = -3$

so $y = 1.5x - 3$

$y = 1.5x - 3$

(Total for Question 9 is 2 marks)
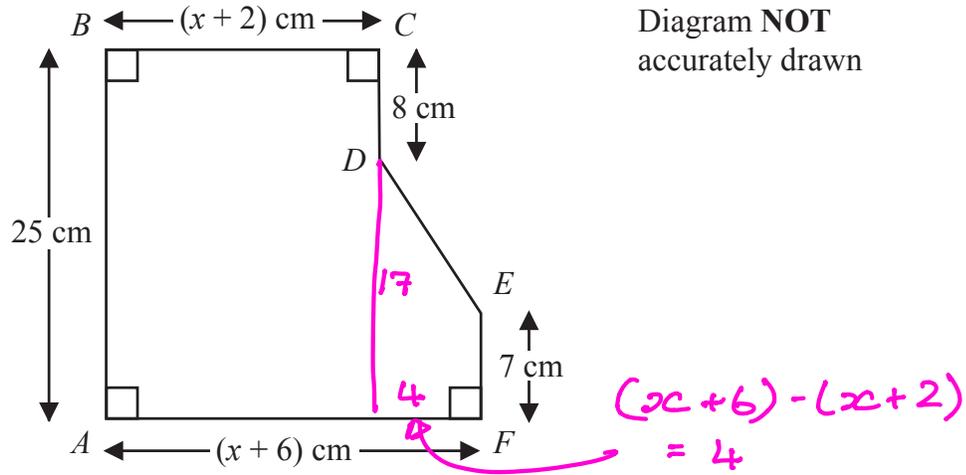
10 The diagram shows a hexagon *ABCDEF*

Diagram **NOT**
accurately drawn

B ← $(x + 2)$ cm → C

8 cm

D

25 cm

17

E

7 cm

4

A ← $(x + 6)$ cm → F

$(x + 6) - (x + 2)$
$= 4$

$AB = 25$ cm $BC = (x + 2)$ cm $CD = 8$ cm $EF = 7$ cm $AF = (x + 6)$ cm

The area of hexagon *ABCDEF* is 258 cm²

Work out the value of *x*

$$258 = 25(x + 2) + \frac{1}{2}(17 + 7) \times 4$$

$$= 25x + 50 + 48$$

$$258 = 25x + 98$$

$$258 - 98 = 25x$$

$$25x = 160$$

$$x = \frac{160}{25}$$

$$= 6.4$$

$x =$ ……………… 6.4 ………………

(Total for Question 10 is 5 marks)

11 Sid has 2 boxes of crayons, box **X** and box **Y**

5 of the 16 crayons in box **X** are red.
7 of the 20 crayons in box **Y** are red.
Sid takes at random one crayon from box **X** and one crayon from box **Y**

(a) Complete the probability tree diagram.

box **X**   box **Y**

$\frac{5}{16}$ — red

$\frac{7}{20}$ — red

$\frac{13}{20}$ — not red

$\frac{11}{16}$ — not red

$\frac{7}{20}$ — red

$\frac{13}{20}$ — not red

(2)

(b) Work out the probability that Sid takes two crayons that are red or two crayons that are not red.

$P(R, R) + P(\text{not }R, \text{not }R)$

$= \frac{5}{16} \times \frac{7}{20} + \frac{11}{16} \times \frac{13}{20}$

$= \frac{35}{320} + \frac{143}{320}$

$= \frac{178}{320}$

$\frac{178}{320}$ oe.

....

(3)

(Total for Question 11 is 5 marks)

12 $2^7 \times 4^5 = 4^x$

(a) Calculate the value of x

$4^5 = (2^2)^5 = 2^{10}$

$4^x = (2^2)^x = 2^{2x}$

so $2^7 \times 2^{10} = 2^{2x}$ $2x = 17$ $x = 8.5$

$x =$ .... 8.5 ....

(2)

(b) Simplify fully $\left(125p^6y^{24}\right)^{\frac{2}{3}}$

$\left(\sqrt[3]{125}\right)^2 = 5^2 = 25$

$p^{\overset{2}{6} \times \frac{2}{\cancel{3}}} = p^4$

$y^{\overset{8}{24} \times \frac{2}{\cancel{3}_1}} = y^{16}$

.... $25p^4y^{16}$ ....

(2)

(Total for Question 12 is 4 marks)

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13 Robert asked 11 people how many meetings they attended last week.

Here are the results in numerical order.

1   2   4   6   6   8   11   12   13   14   17

Find the interquartile range of the number of meetings.

$IQR = 13 - 4$

$= 9$

.... 9 ....

(Total for Question 13 is 2 marks)

14 Here is the graph of the equation $2y + x = 1$ drawn on a grid.
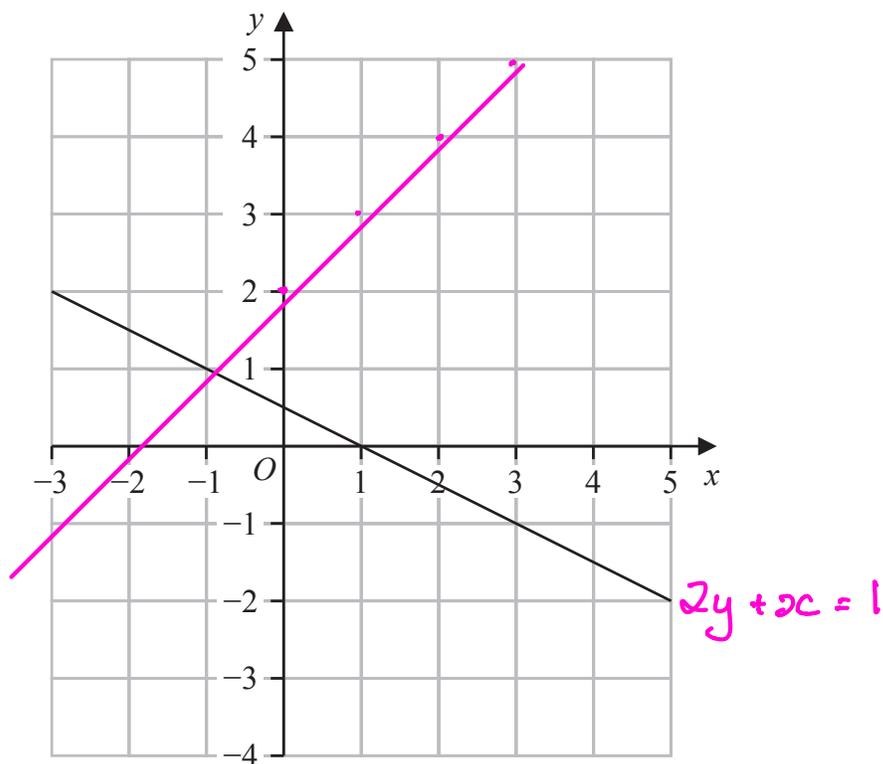
By drawing another straight line on the grid, solve the simultaneous equations

$$y - x - 2 = 0 \qquad y = x + 2$$
$$2y + x = 1$$

$x = \quad -1$

$y = \quad 1$

(Total for Question 14 is 3 marks)

15 (a) Use algebra to show that $0.3\dot{7}\dot{2} = \dfrac{41}{110}$

$$100x = 3\overset{6}{\cancel{7}}\overset{1}{.}2727272\ldots$$
$$x = 0.3727272\ldots$$
$$\overline{}$$
$$99x = 36.9$$

$$x = \frac{36.9}{99} \qquad \frac{36.9}{99} \times \frac{10}{10}$$

$$= \frac{369}{990}$$

$$\frac{369}{990} = \frac{41}{110}$$

(2)

(b) Express $\dfrac{\sqrt{125} + \sqrt{80}}{\sqrt{3}}$ in the form $\sqrt{n}$ where n is an integer.

Show your working clearly.

$$\sqrt{125} = \sqrt{5}\sqrt{25} = 5\sqrt{5}$$

$$\sqrt{80} = \sqrt{5}\sqrt{16} = 4\sqrt{5}$$

$$\frac{4\sqrt{5} + 5\sqrt{5}}{\sqrt{3}} = \frac{9\sqrt{5}}{\sqrt{3}}$$

$$\frac{9\sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{9\sqrt{15}}{3}$$

$$= 3\sqrt{15}$$

$$3\sqrt{15} = \sqrt{9}\sqrt{15} = \sqrt{135}$$

$\sqrt{135}$

(3)

(Total for Question 15 is 5 marks)

16 Expand and simplify $(2x + 3)(x - 5)(x + 4)$

$(2x + 3)(x^2 - x - 20)$

$= 2x^3 - 2x^2 - 40x + 3x^2 - 3x - 60$

$= 2x^3 + x^2 - 43x - 60$

$\underline{2x^3 + x^2 - 43x - 60}$

(Total for Question 16 is 3 marks)

17 $P = a(c + y)$

$a = $ 8.3 correct to 2 significant figures
$c = $ 2 correct to 1 significant figure
$y = 15$ correct to the nearest 5

Work out the upper bound for the value of P
Show your working clearly.

$8.3 \nearrow 8.35 \searrow 8.25$

a

$2 \nearrow 2.5 \searrow 1.5$

c

$15 \nearrow 17.5 \searrow 12.5$

y

$P_{UB} = 8.35(2.5 + 17.5)$

$= 8.35 \times 20$

$= 167$

$\underline{167}$

(Total for Question 17 is 3 marks)

18 A particle is moving along a straight line that passes through the fixed point O
The displacement, s metres, of the particle from O at time t seconds is given by

$$s = 2t^3 - 5t^2 + 6t - 5$$

Find the value of t when the acceleration of the particle is 5 m/s²

$$\frac{ds}{dt} = 6t^2 - 10t + 6$$

$$\frac{ds^2}{dt^2} = 12t - 10$$

$$12t - 10 = 5$$
$$12t = 15$$
$$t = \frac{15}{12}$$
$$= 1.25$$

$t = $ .... 1.25 ....

(Total for Question 18 is 4 marks)

19 The functions f and g are such that

$$f:x \mapsto 5x + 7$$

$$g:x \mapsto \frac{5}{2x - 9}$$

(a) State which value of x cannot be included in any domain of g

4.5

(1)

(b) Find fg(4)

$$g(4) = \frac{5}{8-9} = \frac{5}{-1} = -5$$

$$f(-5) = 5 \times (-5) + 7$$
$$= -25 + 7 = -18$$

-18

(2)

The function h is such that

$$h:x \mapsto 3x^2 - 12x + 8 \quad \text{where } x > 2$$

(c) Express the inverse function h^{-1} in the form $h^{-1}:x \mapsto ...$

$$y = 3(x^2 - 4x) + 8$$

$$= 3\left[(x-2)^2 - 4\right] + 8$$

$$= 3(x-2)^2 - 12 + 8$$

$$= 3(x-2)^2 + 4$$

$$\frac{y+4}{3} = (x-2)^2$$

$$x - 2 = \pm\sqrt{\frac{y+4}{3}}$$

$$x = 2 \pm \sqrt{\frac{y+4}{3}}$$

$$h^{-1}:x \mapsto 2 \pm \sqrt{\frac{x+4}{3}}$$

(4)

(Total for Question 19 is 7 marks)

Turn over ▶

20 The diagram shows equilateral triangle *ABC* with sides of length 10 cm.
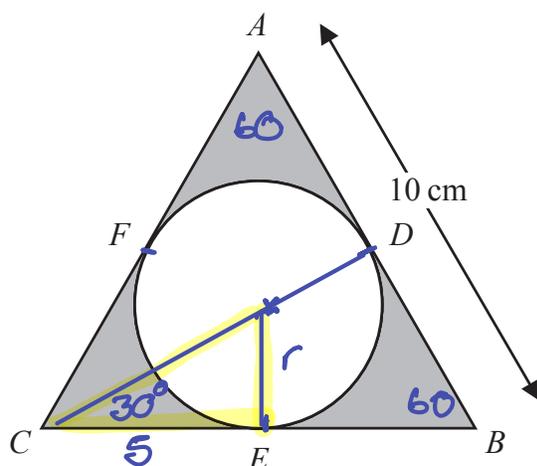A circle is drawn inside the triangle.

Diagram **NOT**
accurately drawn

D, *E* and *F* are points on the circle.

ADB, *BEC* and *CFA* are tangents to the circle.

Calculate the total area of the regions shown shaded in the diagram.
Give your answer correct to 3 significant figures.

area of triangle $= \frac{1}{2}$ $10 \times 10 \times \sin 60 = 25\sqrt{3}$

$\tan 30 = \frac{r}{5}$ $r = 5\tan 30$

$\qquad\qquad\qquad = \frac{5\sqrt{3}}{3}$

shaded area $= 25\sqrt{3} - \pi \times \left(\frac{5\sqrt{3}}{3}\right)^2$

$\qquad\qquad = 17.121331\ldots$

$\qquad\qquad\quad$ 3 s.f.

........................17.1........................ cm²

(Total for Question 20 is 4 marks)

21 The line with equation $x + 2y = 5$ intersects the curve with equation $x^2 + 3y^2 = 13$ at the points A and B

Find the coordinates of A and the coordinates of B
Show clear algebraic working.

$x = 5 - 2y$ $x^2 = (5-2y)(5-2y)$
$= 25 - 20y + 4y^2$

sub into $x^2 + 3y^2 = 13$

$25 - 20y + 4y^2 + 3y^2 = 13$

$7y^2 - 20y + 12 = 0$

$7 \times 12 = 84$
$3, 28$
$4, 21$
$6, 14$

$7y^2 - 14y - 6y + 12 = 0$

$7y(y-2) - 6(y-2) = 0$

$(7y-6)(y-2) = 0$

$y = \frac{6}{7}$ OR $y = 2$

$x = 5 - 2 \times \frac{6}{7}$ $x = 5 - 2 \times 2$
$= 1$

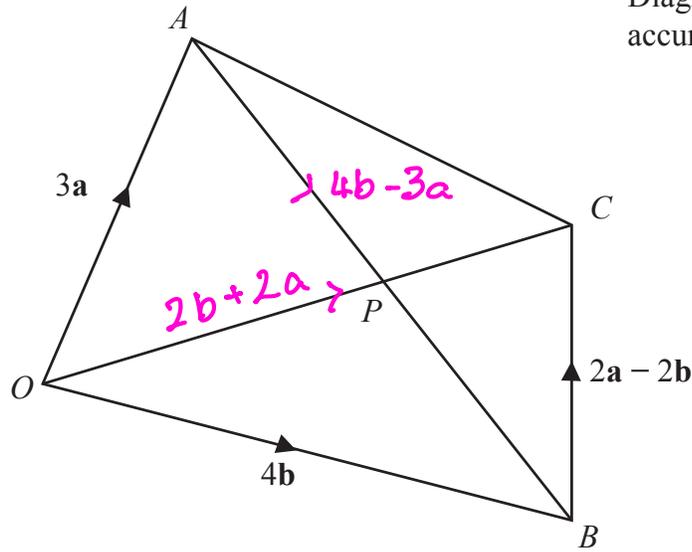$= \frac{35}{7} - \frac{12}{7}$

$= \frac{23}{7}$

$= 3\frac{2}{7}$

(........1........ , ...2........)

(......$3\frac{2}{7}$...... , ...$\frac{6}{7}$......)

(Total for Question 21 is 5 marks)

Diagram **NOT**
accurately drawn

A

3**a**

↗ 4**b** − 3**a**

2**b** + 2**a** ↗ P

C

2**a** − 2**b**

O

4**b**

B

OACB is a quadrilateral.

$\overrightarrow{OA} = 3\mathbf{a}$ $\overrightarrow{OB} = 4\mathbf{b}$ $\overrightarrow{BC} = 2\mathbf{a} - 2\mathbf{b}$

(a) (i) Find the vector $\overrightarrow{OC}$ in terms of **a** and **b**
Simplify your answer.

$4b + 2a - 2b$
$= 2b + 2a$

$\overrightarrow{OC} =$ .... $2b + 2a$ ....
(1)

(ii) Find the vector $\overrightarrow{AB}$ in terms of **a** and **b**

$\overrightarrow{AB} =$ .... $4b - 3a$ ....
(1)

P 7 3 4 6 9 A 0 2 0 2 4

The point P lies on AB and on OC

(b) Using a vector method, find the ratio $AP : PB$
Show your working clearly.

$$\vec{AP} = \lambda(4b - 3a)$$

$$\vec{AP} = -3a + k(2b + 2a)$$

$$4\lambda b - 3\lambda a = -3a + 2kb + 2ka$$

$$4\lambda b - 3\lambda a = (-3 + 2k)a + 2kb$$

comparing
coefficients

$$-3\lambda = -3 + 2k \quad ①$$
$$4\lambda = 2k \quad ②$$

sub ② in ①

$$-3\lambda = -3 + 4\lambda$$
$$7\lambda = 3$$
$$\lambda = \frac{3}{7}$$

$$4 \times \frac{3}{7} = 2k$$
$$k = \frac{6}{7}$$

so:

$$AP = \frac{3}{7}(4b - 3a) \text{ and}$$

$$PB = \frac{4}{7}(4b - 3a)$$

$$AP : PB = 3 : 4$$

............3 : 4............

(3)

(Total for Question 22 is 5 marks)

23 Here is a frustum of a cone.
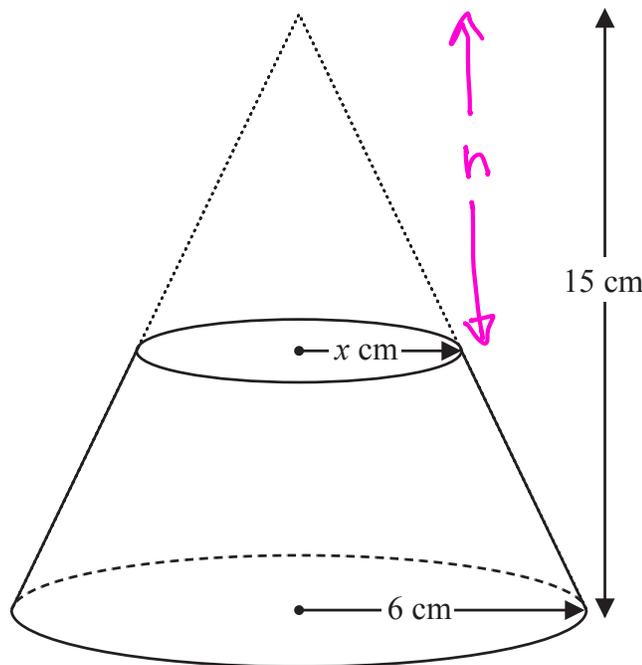The frustum is made by removing a small cone from a similar large cone.

Diagram **NOT** accurately drawn

15 cm

x cm

6 cm

$SF = \dfrac{15}{h}$

$x = 6 \div \dfrac{15}{h}$

$= \dfrac{6h}{15} = \dfrac{2}{5}h$

The height of the large cone is 15 cm.

The radius of the base of the large cone is 6 cm.
The radius of the base of the small cone is *x* cm.

Given that the volume of the frustum is $\dfrac{4212}{25}\pi$ cm^3

work out the value of *x*
Show clear algebraic working.

$$\dfrac{4212}{25}\pi = \dfrac{1}{3}\pi \times 6^2 \times 15 - \dfrac{1}{3}\pi \times \left(\dfrac{2}{5}h\right)^2 \times h$$

$$\dfrac{3 \times 4212}{25} = 36 \times 15 - \dfrac{4h^3}{25}$$

$$\dfrac{4h^3}{25} = 36 \times 15 - \dfrac{3 \times 4212}{25}$$

$$h^3 = 25 \times \dfrac{36^9}{4} \times 15 - \dfrac{3 \times 4212 \times 25}{25 \times 4}$$

$$= 3378 - 3159$$

$$h^3 = 216$$

$$h = \sqrt[3]{216}$$

$$= 6$$

$$x = \frac{2}{5}h = \underline{\underline{2.4}}$$

$x =$ ....**2.4**...........

(Total for Question 23 is 5 marks)

Turn over for Question 24

24 Solve $\dfrac{45x^3 - 80x}{3x^2 + x - 4} \times \left(\dfrac{1}{3x-4} + \dfrac{1}{x}\right) = \dfrac{4(x+2)}{5x-8}$.

Show clear algebraic working.

$$\frac{1}{3x-4} + \frac{1}{x} = \frac{x + 3x - 4}{x(3x-4)} = \frac{4x-4}{x(3x-4)} = \frac{4(x-1)}{x(3x-4)}$$

$$45x^3 - 80x = 5x(9x^2 - 16) = 5x(3x+4)(3x-4)$$

$$3x^2 + x - 4 = (3x+4)(x-1)$$

$$\frac{5x(3x+4)(3x-4)}{(3x+4)(x-1)} \times \frac{4(x-1)}{x(3x-4)} = \frac{4(x+2)}{5x-8}$$

$$\frac{20x}{x} = \frac{4(x+2)}{5x-8}$$

$$20(5x-8) = 4x+8$$

$$100x - 160 = 4x + 8$$

$$96x = 168$$

$$x = \frac{168}{96}$$

$$= \frac{7}{4}$$

$x =$ ......$\frac{7}{4}$......

(Total for Question 24 is 5 marks)

TOTAL FOR PAPER IS 100 MARKS