Cambridge Assessment International Education

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0541118281

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/53**

Paper 5 Investigation (Core) **October/November 2022**

1 hour 10 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 36.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **8** pages.

Answer **all** the questions.

INVESTIGATION **TWO-STEP SEQUENCES**

This investigation looks at *two-step sequences*.
These are sequences which use two steps to get from one term to the next.

> The first term in every sequence is 1.
>
> The two steps are:
> - multiply by a given number
> - then add a given number.

1 In this question the two steps are:
- multiply by 2
- then add 1.

1st term = 1

2nd term = 1st term $\times$ 2 + 1 = $1 \times 2 + 1$ = 3

3rd term = 2nd term $\times$ 2 + 1 = $3 \times 2 + 1$ = 7

4th term = 3rd term $\times$ 2 + 1 = $7 \times 2 + 1$ = 15

(a) Work out the 5th term of this sequence.

..... [2]

(b) The 3rd term of this sequence is 7.
You can write 7 as $2^3 - 1$.

Complete the table.

1st term	1	$2^1 - 1$
2nd term	3	$2^2 - 1$
3rd term	7	$2^3 - 1$
4th term	15	
5th term		

[1]

(c) Calculate the 20th term of this sequence.
Write down all the digits shown on your calculator.

..... [2]

(d) (i) Use the last column in the table to write down an expression for the nth term of this sequence.

..... [1]

(ii) Show that your expression gives the correct value for the 6th term of this sequence.

[2]

2 In this question the two steps are:
- multiply by 3
- then add 4.

The first term is 1.

(a) Calculate the 2nd, 3rd and 4th terms of this sequence.

1, ... , ... , ... [3]

(b) Complete the table.

1st term	1	$3^1 - 2$
2nd term		$3^2 - 2$
3rd term		
4th term		
5th term	241	$3^5 -$

[2]

(c) Write down an expression for the nth term of this sequence.

..... [1]

3 In this question the two steps are:
- multiply by 4
- then add 9.

The first term is 1.

Show that the expression for the nth term, $4^n - 3$, gives the correct value for the 3rd term of this sequence.

[4]

4 **(a)** Copy your results from **Question 1(d)(i)** and **Question 2(c)** into the table.

Use any patterns you notice to complete the table.

	Steps to get the next term	Expression for the nth term
Question 1(d)(i)	Multiply by 2, then add 1	.....
Question 2(c)	Multiply by 3, then add 4	.....
	Multiply by 4, then add 9	$4^n - 3$
	Multiply by ..... , then add 16	$5^n -$ .....
	Multiply by 6, then add	..... $- 5$
	Multiply by 7, then add 36	.....
	Multiply by ..... , then add	$8^n - 7$

[4]

(b) A sequence has the two steps that you found in the last row of the table.

Show that the expression for the nth term gives the correct value for the 3rd term of this sequence.

[3]

(c) The nth term of a two-step sequence is $22^n - 21$.

Find the two steps.

- .....
- ..... [2]

(d) In a two-step sequence the steps are:
- multiply by 11
- then add 100.

The first term is 1.

(i) Find the value of the term nearest to 20 000 000.
Write down all the digits shown on your calculator.

..... [2]

(ii) Which term in the sequence is your answer to **part (i)**?

..... [1]

Question 5 is printed on the next page.

5 In this question the steps in **Question 1** are in the reverse order.

The two steps are now:
- add 1
- then multiply by 2.

(a) The first term is 1.
The second term is 4.

Calculate the 3rd, 4th and 5th terms.

1, 4, ... , ... , ... [2]

(b) This two-step sequence has nth term equal to $a \times 2^n - 2$.

(i) The first term is 1.

Use this to find the value of a.

..... [2]

(ii) Use **part (i)** to show that the expression for the nth term gives the correct value for the 3rd term of this sequence.

[2]