Cambridge Assessment International Education

# Cambridge IGCSE™

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

*1364711255*

## CAMBRIDGE INTERNATIONAL MATHEMATICS 0607/51

Paper 5 Investigation (Core)

**October/November 2021**

**1 hour 10 minutes**

You must answer on the question paper.

No additional materials are needed.

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

### INFORMATION

- The total mark for this paper is 36.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **8** pages. Any blank pages are indicated.

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Answer **all** the questions.

## ADDING SQUARE NUMBERS

This investigation looks at adding two or more square numbers to make another square number.
In this investigation all numbers are positive integers.

**1** Complete the list of the first six square numbers.

$1^2 = 1$ $2^2 =$ ....... $3^2 = 9$ $4^2 =$ ....... $5^2 =$ ....... $6^2 = 36$ [1]

**2** **(a)** Work out

**(i)** $9^2$,

................................................ [1]

**(ii)** $40^2$.

................................................ [1]

**(b)** Show that $9^2 + 40^2 = 41^2$.

[2]

**3**

When $a^2 + b^2 = c^2$ then $(a, b, c)$ is a *3-square set*.
$a$, $b$ and $c$ are positive integers.

<u>Example</u>
In **Question 2(b)**, $a = 9$, $b = 40$ and $c = 41$.
$9^2 + 40^2 = 41^2$, so (9, 40, 41) is a 3-square set.

When $a^2 + b^2 = c^2$ then $c = \sqrt{a^2 + b^2}$.

Use this formula and any patterns you notice to complete the table on the next page for 3-square sets.

 

| $a$ | $b$ | $c$ |
|---|---|---|
| 3 | 4 | 5 |
| 5 | 12 | 13 |
| 7 | 24 | 25 |
| 9 | 40 | 41 |
| 11 | 60 | |
| 13 | 84 | 85 |
| | 112 | 113 |
| | 144 | |
| 19 | | 181 |
| 21 | | 221 |
| | | |
| 25 | 312 | 313 |

[6]

**4** When $a^2 + b^2 + c^2 = d^2$ then ($a$, $b$, $c$, $d$) is a *4-square set*.
It is possible to make a 4-square set using two rows in the table.

<u>Example</u> From the table row two $5^2 + 12^2 = 13^2$
row six $13^2 + 84^2 = 85^2$

Replace $13^2$ in the second equation with $5^2 + 12^2$ from the first equation: $5^2 + 12^2 + 84^2 = 85^2$.

So (5, 12, 84, 85) is a 4-square set.

Use the same method with rows from the table to find two more 4-square sets.

(......... , ......... , ......... , .........) and (......... , ......... , ......... , .........) [3]

**5** **(a)** Show that (6, 12, 12, 18) is a 4-square set.

[2]

**(b)** $k$ is any positive integer greater than 1.
If ($ka$, $kb$, $kc$, $kd$) is a 4-square set, then $(ka)^2 + (kb)^2 + (kc)^2 = (kd)^2$.

Show that ($a$, $b$, $c$, $d$) must also be a 4-square set.

[2]

**(c)** The numbers in the 4-square set (6, 12, 12, 18) have common factors.

**(i)** Find a common factor of 6, 12, 12 and 18 that is greater than 1.

................................................. [1]

**(ii)** Use (6, 12, 12, 18) and **part (b)** to find a 4-square set where *a*, *b*, *c* and *d* do not have a common factor greater than 1.

(......... , ......... , ......... , .........) [2]

**6** Here is another method for finding a 4-square set ($a$, $b$, $c$, $d$).

Choose two positive integers $a$ and $b$ with $a$ less than $b$.

Then $c = \dfrac{a^2 + b^2 - 1}{2}$ and $d = \dfrac{a^2 + b^2 + 1}{2}$ make the 4-square set ($a$, $b$, $c$, $d$).

**(a)** Use this to find a 4-square set when

**(i)** $a = 2$ and $b = 3$,

(2, 3, ......... , .........) [3]

**(ii)** $a = 2$ and $d = 43$.

(2, ......... , ......... , 43) [3]

**(b)** **(i)** Use your answers to **part (a)** and any patterns you notice to complete the table for 4-square sets that start with 2.

| $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|
| 2 | 3 | | |
| 2 | 5 | 14 | 15 |
| 2 | 7 | 26 | 27 |
| 2 | | | 43 |
| 2 | | | |

[3]

**(ii)** Write down an equation connecting $c$ and $d$.

................................................. [1]

 

**(c)** When $a$ and $b$ are both even then $c = \frac{a^2 + b^2 - 1}{2}$ and $d = \frac{a^2 + b^2 + 1}{2}$ do not give a 4-square set.

Give an example to show this.

[2]

**(d)** When $a$ and $b$ are both odd there are no 4-square sets.

In a 4-square set, $d = 23$.

**(i)** Show that $a^2 + b^2 = 45$.

[1]

**(ii)** Find a 4-square set when $d = 23$.

(......... , ......... , ......... , 23) [2]

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