Cambridge IGCSE

Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

0376227255

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/61**

Paper 6 (Extended) **October/November 2015**

1 hour 30 minutes

Candidates answer on the Question Paper

Additional Materials: Graphics Calculator

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, glue or correction fluid.
You may use an HB pencil for any diagrams or graphs.
DO **NOT** WRITE IN ANY BARCODES.

Answer both parts **A** and **B**.
You must show all relevant working to gain full marks for correct methods, including sketches.
In this paper you will also be assessed on your ability to provide full reasons and communicate your mathematics clearly and precisely.
At the end of the examination, fasten all your work securely together.
The total number of marks for this paper is 40.

This document consists of **12** printed pages.

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THE INVESTIGATION STARTS ON PAGE 3.

Answer **both** parts **A** and **B**.

A INVESTIGATION SUMS OF TWO SQUARES (20 marks)

You are advised to spend no more than 45 minutes on this part.

This investigation looks at the results when two square numbers are added together.

1 Here is a list of the first 11 prime numbers.

2 3 5 7 11 13 17 19 23 29 31

(a) In the list there are 4 numbers that are one more than a multiple of 4.
These are called *Pythagorean Primes*.
The smallest one is 5 and the largest one is 29.

Write down the other two.

5, ……….. , ……….. , 29

(b) The 17th century French mathematician Albert Girard proved that every Pythagorean Prime equals the sum of two square numbers.

Write your answers to **part (a)** as the sum of two square numbers.
Two have been written down for you.

$5 = 1^2 + 2^2$

……….. = ……….. + ………..

……….. = ……….. + ………..

$29 = 2^2 + 5^2$

(c) Another Pythagorean Prime is 101.
Write 101 as the sum of two square numbers.

101 = ……….. + ………..

2 The sum of two square numbers can equal a square number.
For example,

$$\mathbf{3}^2 + \mathbf{4}^2 = 9 + 16 = 25 = \mathbf{5}^2$$

We say that **3**, **4**, **5** is a *Pythagorean Triple*.

(a) Show, by calculation, that 7, 24, 25 is a Pythagorean Triple.

(b) Each row in this table is a Pythagorean Triple.

Complete the table.
Use patterns of numbers in the table to help you.

3	4	5
5	12	13
7	24	25
9	40	
11	60	
13		
		113

(c) What is the connection between the **square** of the smallest number and the other two numbers in each Pythagorean Triple in the table?

....

....

(d) Use your answer to **part (c)** and the patterns of numbers in the table to complete the following Pythagorean Triples.

(i)

………. , ………. , 421

(ii)

101 , ………. , ……….

3 Sometimes the sum of two square numbers can equal the sum of another pair of square numbers. For example,

$$5^2 \quad + \quad 5^2 \quad = \quad 1^2 \quad + \quad 7^2 \qquad \text{(Both sums equal 50.)}$$

(a) Show that $(x+y)^2 + (m-n)^2 = (x-y)^2 + (m+n)^2$ simplifies to $xy = mn$.

$$(x+y)^2 + (m-n)^2 = (x-y)^2 + (m+n)^2$$

$$xy = mn$$

(b) x, y, m and n are different positive integers with $x > y$ and $m > n$.

When $xy = mn = 6$ one solution is

$$x = 3,\ \ y = 2 \quad \text{and} \quad m = 6,\ \ n = 1.$$

The substitution of these values into

$(x + y)^2 + (m - n)^2 = (x - y)^2 + (m + n)^2$ gives these equal sums of square numbers.

$$5^2 \quad + \quad 5^2 \quad = \quad 1^2 \quad + \quad 7^2$$

Find all the possible solutions when $xy = mn = 12$.
For each solution, write the equal sums of square numbers.

(c) Complete the following equal sums of square numbers.

9^2 + ……… = 5^2 + ………

B MODELLING **POPULATION GROWTH (20 marks)**

You are advised to spend no more than 45 minutes on this part.

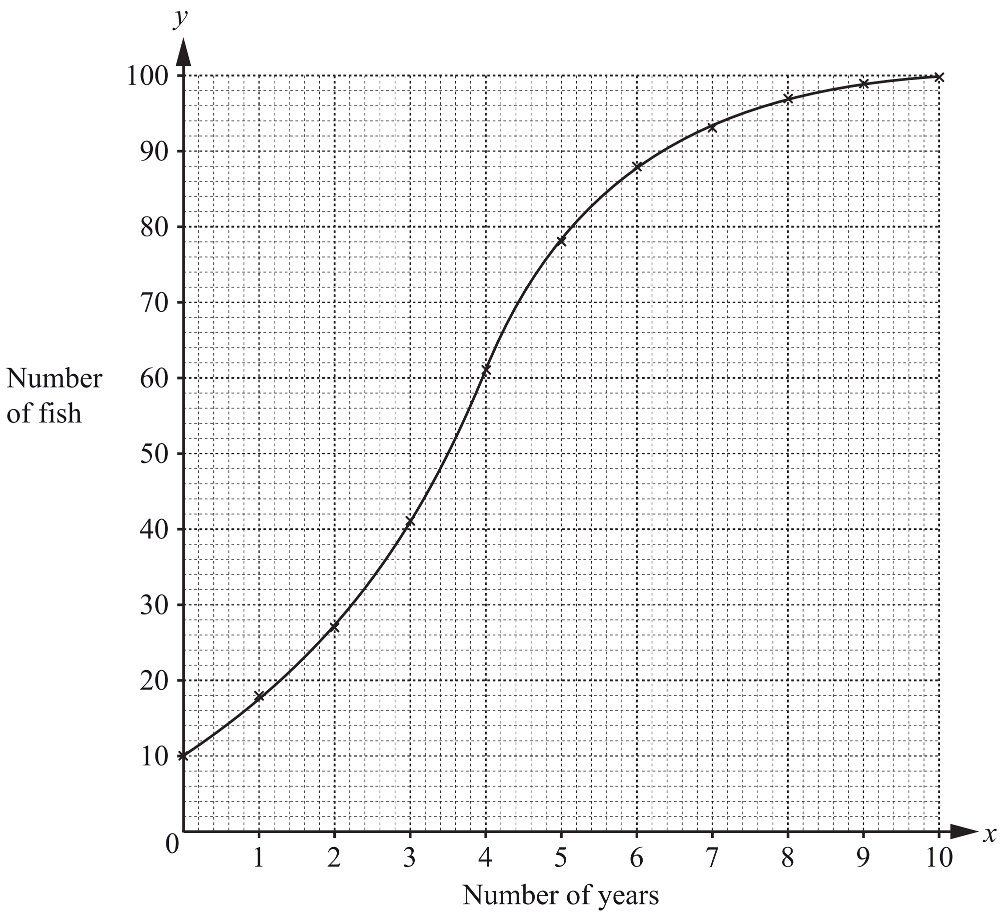
This modelling task compares three different models of population growth.

Ten fish are put into a lake to breed.
The maximum number of fish that can live in the lake is 100.
After ten years the number of fish stays approximately the same.

The table shows the number of fish, y, in the lake at the end of x years.

Number of years (x)	0	1	2	3	4	5	6	7	8	9	10
Number of fish (y)	10	18	27	41	61	78	88	93	97	99	100

This information is shown below.



1 **(a)** Why is it correct to join the points?

....

....

(b) Comment on the rate of increase in the number of fish when the number of fish approaches 100.

....

....

2 The data can be modelled using the cubic function $y = ax^3 + bx$.

(a) Find an equation in a and b so that the model gives the value of y in the table when

(i) $x = 1$,

....

(ii) $x = 5$.

....

(b) Solve the simultaneous equations from **part (a)** and write down the model.

....

3 The data can also be modelled by the trigonometric function $y = a + b\cos(18x)°$.

(a) Find an equation in a and b so that the model gives the value of y in the table when

(i) $x = 0$,

....

(ii) $x = 10$.

....

(b) Solve the equations from **part (a)** and write down your model.

....

4 The data can also be modelled by the logistic function $y = \dfrac{100 \times 2^x}{2^x + k}$.

(a) The model gives the value of y in the table when $x = 0$.

Find the value of k.

....

(b) Comment on the accuracy of the model when $x = 5$.

....

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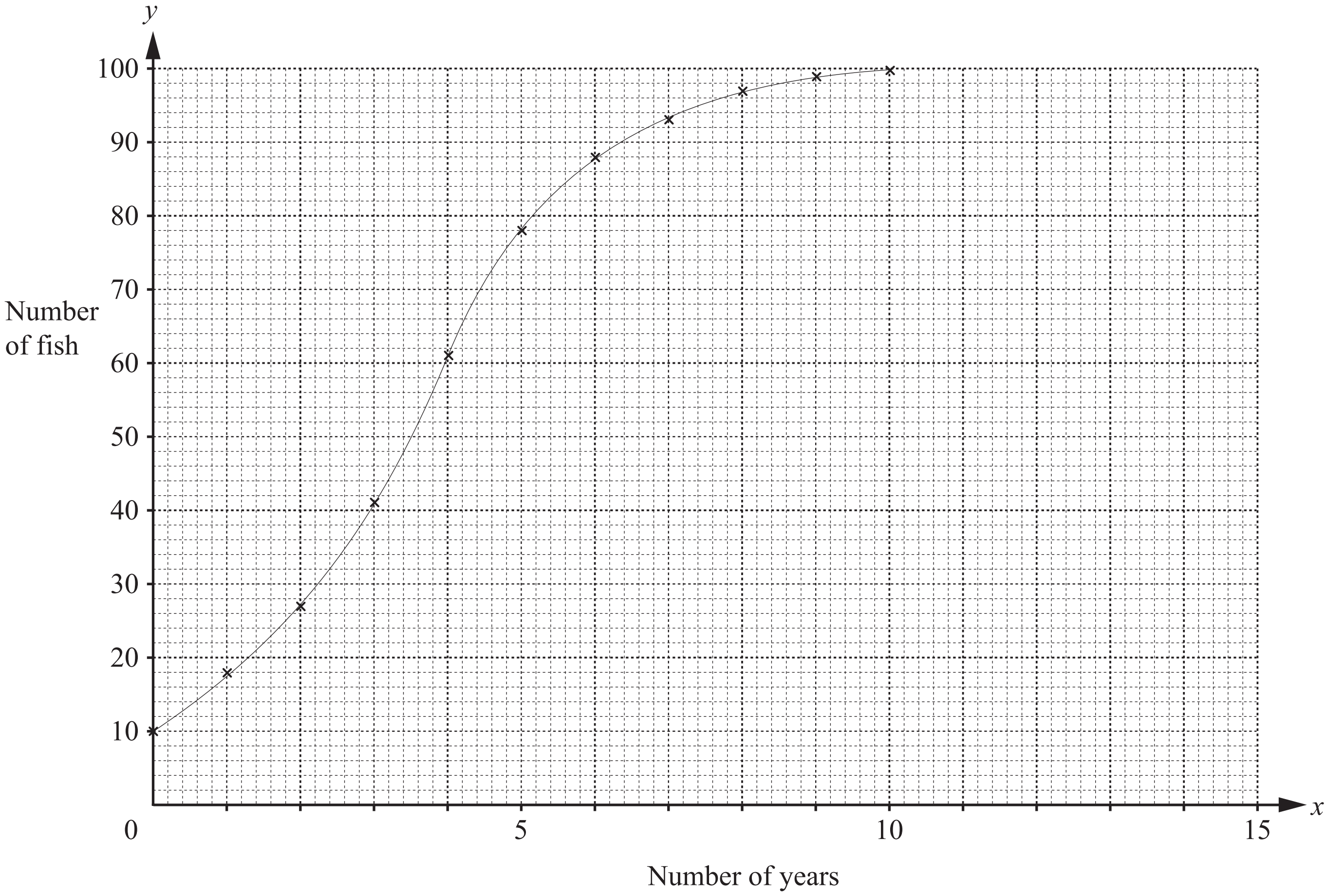
Question 5 is printed on the next page.

5 **(a)** Sketch the graphs of your models in **questions 2**, **3** and **4** for $0 \leqslant x \leqslant 15$.
The original data has been shown again.



(b) Give two reasons why the logistic model is the best.

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