Cambridge IGCSE

Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

7555901685

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/53**

Paper 5 (Core) **October/November 2015**

1 hour

Candidates answer on the Question Paper.

Additional Materials: Graphics Calculator

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, glue or correction fluid.
You may use an HB pencil for any diagrams or graphs.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
You must show all relevant working to gain full marks for correct methods, including sketches.
In this paper you will also be assessed on your ability to provide full reasons and to communicate your mathematics clearly and precisely.
At the end of the examination, fasten all your work securely together.
The total number of marks for this paper is 24.

This document consists of **7** printed pages and **1** blank page.

DC (ST) 118176

CAMBRIDGE International Examinations

THE INVESTIGATION STARTS ON PAGE 3.

Answer **all** the questions.

INVESTIGATION **SUMS OF TWO SQUARES**

This investigation looks at the results when two square numbers are added together.

1 Here is a list of the first 11 prime numbers.

2 3 5 7 11 13 17 19 23 29 31

(a) In the list there are 4 numbers that are one more than a multiple of 4.
These are called *Pythagorean Primes*.
The smallest one is 5 and the largest one is 29.

Write down the other two.

5, ... , ... , 29

(b) The 17th century French mathematician Albert Girard proved that every Pythagorean Prime equals the sum of two square numbers.

Write your answers to **part (a)** as the sum of two square numbers.
Two have been written down for you.

$5 = 1^2 + 2^2$

.... = ... + ...

.... = ... + ...

$29 = 2^2 + 5^2$

(c) Another Pythagorean Prime is 101.
Write 101 as the sum of two square numbers.

101 = ... + ...

2 The sum of two square numbers can equal another square number.
For example,

$$\begin{aligned} \mathbf{3}^2 + \mathbf{4}^2 &= 9 + 16 \\ &= 25 \\ &= \mathbf{5}^2 \end{aligned}$$

We say that **3**, **4**, **5** is a *Pythagorean Triple*.

(a) Show, by calculation, that 7, 24, 25 is a Pythagorean Triple.

(b) Each row in this table is a Pythagorean Triple.

Complete the table.
Use patterns of numbers in the table to help you.

3	4	5
5	12	13
7	24	25
9	40	
11	60	
13		
		113

(c) What is the connection between the **square** of the smallest number and the other two numbers in each Pythagorean Triple in the table?

....

....

(d) Use your answer to **part (c)** and the patterns of numbers in the table to complete the following Pythagorean Triple.

.... , ... , 421

3 $2\sqrt{x},\ x-1,\ x+1$ is a Pythagorean Triple when x is a square number.

(a) **(i)** Find the Pythagorean Triple when $x = 16$.

.... , ... , ...

(ii) Check that your answer to **part (a)(i)** is a Pythagorean Triple.
Use the method of the example in **question 2**.

(b) In the table, x is the square of an even number.
Each row is a Pythagorean Triple.

	$2\sqrt{x}$	$x-1$	$x+1$
($x = 16$)			
($x = 36$)	12		37
	16	63	65
		99	
	24		145

Write your answer to **part (a)(i)** in the first row of this table.

Complete the three columns of the table.
You may use patterns or the fact that $2\sqrt{x},\ x-1,\ x+1$ is a Pythagorean Triple to help you.

(c) What is the connection between the **square** of the smallest number and the sum of the other two numbers in each of the Pythagorean Triples in the table?

....

....

(d) Show algebraically that $2\sqrt{x},\ x-1,\ x+1$ satisfies your connection in **part (c)**.

BLANK PAGE