## Cambridge IGCSE ${ }^{\text {TM }}$



CENTRE NUMBER


CANDIDATE NUMBER

## CAMBRIDGE INTERNATIONAL MATHEMATICS

Paper 5 Investigation (Core)
May/June 2023
1 hour 10 minutes
You must answer on the question paper.
No additional materials are needed.

## INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.


## INFORMATION

- The total mark for this paper is 36 .
- The number of marks for each question or part question is shown in brackets [ ].

Answer all the questions.

## INVESTIGATION

WINNING LINES
This investigation looks at the number of winning lines on a grid.
1 In a game you win by making a straight line of three $\mathbf{O}$ s on a 3 by 3 grid.
There are 8 winning lines of three $\mathbf{O}$ s on a 3 by 3 grid.
Show each winning line of three $\mathbf{O}$ s on the grids.
Three of the winning lines have been shown for you.


2 (a) Another grid is 4 by 4 .
You now need four $\mathbf{O s}_{\mathrm{s}}$ in a line to win.
Find the number of winning lines on a 4 by 4 grid.
You may use the grids to help you.

(b) Another grid is 5 by 5 .

You now need five $\mathbf{O}$ s in a line to win.
Find the number of winning lines on a 5 by 5 grid.
You may use the grids to help you.



(c) Another grid is 6 by 6 .

You now need six $\mathbf{O}$ s in a line to win.
Write down the number of winning lines on a 6 by 6 grid.
$\qquad$

3 (a) Complete this table using your answers to Question 1 and Question 2 and any patterns you notice.

| Size of grid | Number of winning lines |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Horizontal | Vertical | Diagonal | Total |
| 3 by 3 |  |  |  | 8 |
| 4 by 4 |  |  |  |  |
| 5 by 5 |  |  |  |  |
| 6 by 6 |  |  |  |  |
| 7 by 7 |  |  |  |  |
|  |  |  |  |  |
| 20 by 20 |  |  |  |  |

(b) A grid is $n$ by $n$.

You need $n \mathbf{O}$ s in a line to win.
Find an expression, in terms of $n$, for the number of winning lines.
$\qquad$
(c) Jibreel draws a very large square grid.

He thinks there will be 583 winning lines of $\mathbf{O}$ s on his grid.
Give a reason why he is wrong.
$\qquad$
(d) Harriet draws a square grid with 324 squares.

Find the number of winning lines of $\mathbf{O}$ s on this grid.

4 A grid is $n$ by $n$.
In a different game a winning line is one $\mathbf{O}$ less than $n$.
To make a line, the $\mathbf{O s}$ must be in squares that are next to each other.
(a) In a 3 by 3 grid you need two $\mathbf{O}$ in a line to win.

These diagrams show some of the diagonal winning lines.


Complete the table to find the number of winning lines with two $\mathbf{O}$ s.
You may use the grids below the table to help you.

| Size of grid | Number of winning lines |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Horizontal | Vertical | Diagonal | Total |
| 3 by 3 |  |  |  | 20 |


|  |  |  |
| :--- | :--- | :--- |
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(b) In a 4 by 4 grid you need three $\mathbf{O}$ s in a line to win.

Complete the table to find the number of winning lines with three $\mathbf{O}$ s. You may use the grids below the table to help you.

| Size of grid | Number of winning lines |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Horizontal | Vertical | Diagonal | Total |
| 4 by 4 |  |  |  |  |




(c) Copy your results from part (a) and part (b) into this table.

Complete the table.
You may use the grids below the table to help you.

| Size of grid | Number of winning lines |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Horizontal | Vertical | Diagonal | Total |
| 3 by 3 |  |  |  |  |
| 4 by 4 |  |  |  |  |
| 5 by 5 |  |  |  |  |
|  |  |  |  |  |
| $n$ by $n$ |  |  |  |  |




(d) In an $n$ by $n$ grid you need $(n-1) \mathbf{O}$ in a line to win. $n$ must be at least 3 .

In one grid the total number of winning lines is a square number less than 50 .
Find the grid size.

5 A rectangular grid has height 2 and width at least 2.
You need two $\mathbf{0}$ s in a line to win.
These diagrams show all the winning lines with two $\mathbf{O}$ s on a 2 by 3 grid.


There are 11 winning lines

(a) Complete the table for the number of winning lines with two $\mathbf{O}$ s. You may use the grid below the table to help you.

| Size of grid | Number of winning lines |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Horizontal | Vertical | Diagonal | Total |
| 2 by 2 | 2 | 2 | 2 | 6 |
| 2 by 3 | 4 | 3 | 4 | 11 |
| 2 by 4 |  |  |  |  |
| 2 by 5 |  |  |  |  |
|  |  |  |  |  |
| 2 by $w$ |  |  |  |  |


(b) A 2 by $w$ grid has 111 winning lines with two O .

Find the width of the grid.

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