Cambridge Assessment International Education

Cambridge IGCSE™

CANDIDATE NAME

CENTRE NUMBER

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CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/61**

Paper 6 Investigation and Modelling (Extended) **May/June 2020**

1 hour 40 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer both part **A** (Questions 1 to 8) and part **B** (Questions 9 to 12).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 60.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages. Blank pages are indicated.

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Answer both parts **A** and **B**.

A INVESTIGATION (QUESTIONS 1 to 8)

COMBINING TRIANGLE NUMBERS (30 marks)

You are advised to spend no more than 50 minutes on this part.

This investigation looks at results when adding, subtracting and multiplying triangle numbers.

Here is a table of the first 5 triangle numbers, T_1 to T_5.

T_1	T_2	T_3	T_4	T_5
1	3	6	10	15

1 Find the next triangle number.

.... [2]

2 Complete the table for subtracting consecutive triangle numbers.

T_1	1
$T_2 - T_1$	2
$T_3 - T_2$	
$T_4 - T_3$	
$T_5 - T_4$	5
$T_6 - T_5$	
$T_{n-2} - T_{n-3}$	
$T_{n-1} - T_{n-2}$	
$T_n - T_{n-1}$	

[2]

3 Complete the table for adding two consecutive triangle numbers.

T_1	1
$T_2 + T_1$	4
$T_3 + T_2$	9
$T_4 + T_3$	
$T_5 + T_4$	
$T_6 + T_5$	
$T_n + T_{n-1}$	

[2]

4 Use the last row of the table in **Question 2** to complete equation ①. $T_n - T_{n-1} =$ ....

Use the last row of the table in **Question 3** to complete equation ②. $T_n + T_{n-1} =$ ....

(a) By adding equations ① and ② together show that $T_n = \dfrac{n^2 + n}{2}$.

[1]

(b) **(i)** By multiplying equations ① and ② together, find a result about the squares of consecutive triangle numbers.

[2]

(ii) Give a numerical example of this result.

[2]

5 The sum of two **different** triangle numbers sometimes equals another triangle number. When this happens, we have a *triangle triple*.

<u>Example</u>

- Start with the triangle number $T_3 = 6$
- From the table in **Question 2** $T_6 - T_5 = 6$

  So $T_6 - T_5 = T_3$
- Rearrange the equation $T_3 + T_5 = T_6$
- The *triangle triple* is then (3, 5, 6)

The three different numbers must be written in order of increasing size.

(a) Start with triangle number $T_5 = 15$ and complete the method of the Example to find another triangle triple.

T_{15} − .... = ...

So .... − .... = T_5

T_5 + ... = ...

The triangle triple is (5, ... , ...) [3]

(b) In the table, each row is a triangle triple.

Use **part (a)** and any patterns you notice to complete the table.

Triangle triple		
3	5	6
4		
5		
6		
7		

[3]

6 **(a)** When you add the last two rows in the table in **Question 2**, you get an expression for $T_n - T_{n-2}$. This is the difference between triangle numbers that are two apart.

Give this expression in its simplest form.

.... [1]

(b) **(i)** Find n when $T_n - T_{n-2} = 15$.

.... [1]

(ii) $T_5 = 15$

Use your answer to **part (i)** to find a triangle triple where

- the smallest number is 5
- the difference between the other two numbers is 2.

(5, ... , ...) [2]

7 **(a)** By adding rows in the table in **Question 2**, show that $T_n - T_{n-3} = 3n - 3$.
This is the difference between triangle numbers that are three apart.

[1]

(b) $T_{14} = 105$

Use **part (a)** to find a triangle triple where

- the smallest number is 14
- the difference between the other two numbers is 3.

(14, ... , ...) [3]

8 Find all the triangle triples where the smallest number is 14.

[5]

B MODELLING (QUESTIONS 9 to 12)

SPEED OF PLANETS (30 marks)

You are advised to spend no more than 50 minutes on this part.

This task looks at models for the distance of a planet from the Sun and the time it takes to travel once round the Sun. The task uses these models to find a model for the speed of a planet.

Astronomers use astronomical units, au, to measure distance in space.
1 au = the distance from the Sun to the Earth.

9 In the 18th century, the German astronomer Bode numbered the planets Venus to Neptune from 0 to 7.

The table shows his numbers and the distance from the Sun to each planet.

Bode's number, n	Planet	Distance from Sun, R au	Bode's estimates (au)
	Mercury	0.39	
0	Venus	0.72	0.7
1	Earth	1.00	
2	Mars	1.52	
3	unknown		
4	Jupiter	5.20	
5	Saturn	9.55	
6	Uranus	19.22	19.6
7	Neptune	30.11	

Bode estimated the distance from the Sun to Venus as 0.7 au.
After that, for each planet, he used the following rule to estimate the distance of the **next** planet in the table.

Double the estimate for the distance and subtract 0.4

(a) Complete the table for Bode's estimates. [3]

(b) Bode's rule gives a good model for the planets Venus to Uranus.

Work out Bode's estimate for Neptune. Is it a good estimate?

.... [2]

(c) Find an estimate for Mercury using Bode's rule.

.... [2]

10 Bode's rule requires doubling each time.
So a possible model for the distance of a planet from the Sun, R au, is

$$R = a \times 2^n + b$$

where n is Bode's number and a and b are constants that transform the graph of $R = 2^n$.

(a) Write down the two types of transformation used.

.... , ... [2]

(b) Using the information for Earth and Jupiter on page 8, write two equations in a and b.

....

.... [2]

(c) Show how solving these simultaneous equations gives $R = 0.3 \times 2^n + 0.4$.

[2]

11 The table shows the time, T years, it takes a planet to go once round the Sun.
The table includes the 'dwarf planet' *Ceres* which Bode's rule predicts.

Planet	R (au)	T (years)
Mercury	0.39	0.24
Venus	0.72	0.62
Earth	1.00	1.00
Mars	1.52	1.88
Ceres	2.77	4.60
Jupiter	5.20	11.86
Saturn	9.55	29.46
Uranus	19.22	84.01
Neptune	30.11	164.80

A possible model for T is $T = k \times R^p$ where k and p are constants.

(a) **(i)** Use the information in the table for Earth to find k.

.... [2]

(ii) Using the result from **part (i)** and the information in the table for Jupiter, find p correct to 1 decimal place.

.... [3]

(b) Write down the model for T in terms of R.
Is this a good model for the time that it takes Mercury to go once round the Sun?
Show how you decide.

[2]

12 (a) Assume that planets travel in a circle with the Sun at the centre.

Use your model in **Question 11(b)** and $R = 0.3 \times 2^n + 0.4$ to show that a model for the average speed of a planet is

$$S = \frac{2\pi}{\sqrt{0.3 \times 2^n + 0.4}},$$

where n is Bode's number and S is measured in au/year.

[3]

(b) Sketch the graph of S for $0 \leqslant n \leqslant 5$.

[2]

Questions 12(c) and 12(d) are printed on the next page.

(c) The graph in **part (b)** is approximately a straight line.

Find a linear model for S, in terms of n, by finding the equation of this straight line.
Write the numbers in your model correct to 1 decimal place.

.... [3]

(d) Bode's number for Neptune is 7.

Show that your model does not give a sensible answer for the speed of Neptune.

[2]