Cambridge Assessment International Education

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CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/51**

Paper 5 Investigation (Core) **May/June 2020**

1 hour 10 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 36.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **8** pages. Blank pages are indicated.

[Turn over

Answer **all** the questions.

COMBINING TRIANGLE NUMBERS

This investigation looks at results when adding or subtracting triangle numbers.

Here is a table of the first 21 triangle numbers, T_1 to T_{21}.

T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}	T_{15}	T_{16}	T_{17}	T_{18}	T_{19}	T_{20}	T_{21}
1	3	6	10	15	21	28	36	45	55	66	78	91	105	120	136	153	171	190	210	231

1 Find the next two triangle numbers.

$T_{22} =$ .....

$T_{23} =$ ..... [4]

2 **(a)** Complete the table.

T_1	1
$T_2 - T_1$	2
$T_3 - T_2$	
$T_4 - T_3$	
$T_5 - T_4$	
$T_6 - T_5$	6
$T_n - T_{n-1}$	

[2]

(b) **(i)** $T_n - T_{n-1} = 100$.

Write down the value of n.

..... [1]

(ii) Write down the difference between the 50th and the 49th triangle numbers.

..... [1]

3 Complete the table for adding two consecutive triangle numbers.

T_1	1
$T_2 + T_1$	4
$T_3 + T_2$	9
$T_4 + T_3$	
$T_5 + T_4$	
$T_6 + T_5$	
$T_n + T_{n-1}$	

[2]

4 **(a)** Use the last row of the table in **Question 2(a)** to complete the equation $T_n - T_{n-1} =$ .....

Use the last row of the table in **Question 3** to complete the equation $T_n + T_{n-1} =$ .....

By adding these two equations together show that $T_n = \frac{n^2 + n}{2}$.

[1]

(b) Find T_{1000}.

..... [2]

5 **(a)** The table shows the difference of the squares of two consecutive triangle numbers. Complete the table.

$(T_1)^2$	1
$(T_2)^2 - (T_1)^2$	8
$(T_3)^2 - (T_2)^2$	
$(T_4)^2 - (T_3)^2$	
$(T_5)^2 - (T_4)^2$	125
$(T_6)^2 - (T_5)^2$	216
$(T_n)^2 - (T_{n-1})^2$	

[3]

(b) Calculate the difference between the squares of the 50th and the 49th triangle numbers.

..... [2]

6 The sum of two **different** triangle numbers sometimes equals another triangle number. When this happens, we have a *triangle triple*.

Example

- Start with the triangle number $T_3 = 6$.
- From the table in **question 2(a)** $T_6 - T_5 = 6$.

  So $T_6 - T_5 = T_3$.
- Rearrange the equation $T_3 + T_5 = T_6$.
- The *triangle triple* is then (3, 5, 6).

The three different numbers must be written in order of increasing size.

(a) Start with triangle number $T_5 = 15$ and complete the method of the Example to find another triangle triple.

T_{15} − ..... = ...

So ..... − ..... = T_5

T_5 + ... = ...

The triangle triple is (5, ..., ...) [4]

(b) In the table, each row is a triangle triple.
Use your answer to **part (a)** and any patterns you notice to complete the table.

Triangle triple		
3	5	6
4	9	10
5		
6		
7		

[5]

(c) Use the list of triangle numbers on page 2 to check the triangle triple beginning with 6.

[1]

7 **(a)** The triangle numbers T_1 and T_3 are not consecutive. They are two apart.
Complete the table for subtracting triangle numbers that are two apart.

$T_3 - T_1$	5
$T_4 - T_2$	
$T_5 - T_3$	
$T_6 - T_4$	
$T_7 - T_5$	13
$T_n - T_{n-2}$	

[4]

(b) Use the triangle number $T_9 = 45$ to find a triangle triple where

- the smallest number is 9
- the difference between the other two numbers is 2.

Hints: Use the last row of the table in **part (a)**.
Use a method similar to that in the Example in **Question 6**.

(9 , ... , ...) [4]

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