Cambridge IGCSE

Cambridge Assessment International Education
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

6707905696

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/62**

Paper 6 (Extended) **May/June 2019**

1 hour 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Graphics calculator

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, glue or correction fluid.
You may use an HB pencil for any diagrams or graphs.
DO **NOT** WRITE IN ANY BARCODES.

Answer both parts **A** (Questions 1 to 6) and **B** (Questions 7 to 9).
You must show all relevant working to gain full marks for correct methods, including sketches.
In this paper you will also be assessed on your ability to provide full reasons and communicate your mathematics clearly and precisely.
At the end of the examination, fasten all your work securely together.
The total number of marks for this paper is 40.

This document consists of **12** printed pages.

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Answer **both** parts **A** and **B**.

A INVESTIGATION (QUESTIONS 1 to 6)

SQUARE ROOTS WITHIN SQUARE ROOTS (20 marks)

You are advised to spend no more than 45 minutes on this part.

This investigation looks at sequences of terms with square roots.

You can form a sequence by using square roots within square roots.

$\sqrt{6}$, $\sqrt{6+\sqrt{6}}$, $\sqrt{6+\sqrt{6+\sqrt{6}}}$, $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}$, ...

You can calculate the first four terms of this sequence as follows.

$\sqrt{6} = 2.4494\ldots$

$\sqrt{6+\sqrt{6}} = \sqrt{6+2.4494\ldots} = \sqrt{8.4494\ldots} = 2.9068\ldots$

$\sqrt{6+\sqrt{6+\sqrt{6}}} = \sqrt{6+2.9068\ldots} = \sqrt{8.9068\ldots} = 2.9844\ldots$

$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}} = \sqrt{6+2.9844\ldots} = \sqrt{8.9844\ldots} = 2.9974\ldots$

1 **(a)** Calculate the next term of the sequence, writing the decimal as far as the 4th decimal place.

.....

(b) As the sequence continues, the terms get closer and closer to an integer.
This integer is the *integer limit* of the sequence.

Write down the integer limit of this sequence.

.....

2 Here is a similar sequence of square roots.

$$\sqrt{30}\ ,\quad \sqrt{30+\sqrt{30}}\ ,\quad \sqrt{30+\sqrt{30+\sqrt{30}}}\ ,\quad \sqrt{30+\sqrt{30+\sqrt{30+\sqrt{30}}}}\ ,\quad ...$$

(a) Calculate the first three terms, writing each decimal as far as the 4th decimal place.

..... , ... , ...

(b) Write down the integer limit of this sequence.

.....

3 **(a)** Complete this table for similar sequences.

First term	Integer limit
$\sqrt{2}$	
$\sqrt{6}$	
$\sqrt{12}$	
$\sqrt{20}$	
$\sqrt{30}$	
$\sqrt{42}$	7

(b) **(i)** Use **part (a)** to find the first term of the sequence that has an integer limit of 8.

.....

(ii) Calculate the 4th term of the sequence in **part (b)(i)**.
Give your answer correct to 4 decimal places.

.....

4 The general sequence is $\sqrt{k}$, $\sqrt{k+\sqrt{k}}$, $\sqrt{k+\sqrt{k+\sqrt{k}}}$, $\sqrt{k+\sqrt{k+\sqrt{k+\sqrt{k}}}}$, ...

So, when x is one of the terms, the next term is $\sqrt{k+x}$.

The sequence reaches its limit, x, when one term equals the next term. That is, when $x = \sqrt{k+x}$.

(a) Make k the subject of the equation $x = \sqrt{k+x}$.

.....

(b) Use **part (a)** to check that the sequence below has a limit, x, equal to 7.

$$\sqrt{42}\ ,\quad \sqrt{42+\sqrt{42}}\ ,\quad \sqrt{42+\sqrt{42+\sqrt{42}}}\ ,\quad \sqrt{42+\sqrt{42+\sqrt{42+\sqrt{42}}}}\ ,\quad ...$$

5 You will find the following information useful in this question.

For the equation $ax^2+bx+c=0$, $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

The limit of $\sqrt{3}$, $\sqrt{3+\sqrt{3}}$, $\sqrt{3+\sqrt{3+\sqrt{3}}}$, $\sqrt{3+\sqrt{3+\sqrt{3+\sqrt{3}}}}$, ... is not an integer.

Find the **exact** value of this limit.

.....

6 Here is another type of sequence.

$\sqrt{5}$, $\sqrt{5+\sqrt{13}}$, $\sqrt{5+\sqrt{13+\sqrt{5}}}$, $\sqrt{5+\sqrt{13+\sqrt{5+\sqrt{13}}}}$, $\sqrt{5+\sqrt{13+\sqrt{5+\sqrt{13+\sqrt{5}}}}}$, ...

(a) The limit of this sequence is an integer, x.

By calculating some terms of this sequence, find x.

.....

(b) The general form of this sequence is

$\sqrt{a}$, $\sqrt{a+\sqrt{b}}$, $\sqrt{a+\sqrt{b+\sqrt{a}}}$, $\sqrt{a+\sqrt{b+\sqrt{a+\sqrt{b}}}}$, ...

where a and b are positive integers and $a \neq b$.

The sequence has a limit, x, when $x = \sqrt{a+\sqrt{b+x}}$.

Show that this equation can be written as $x^2 - a = \sqrt{b+x}$.

(c) (i) Use your answer to **part (a)** to show that the equation $x^2 - a = \sqrt{b+x}$ is correct for this sequence.

$\sqrt{5}$, $\sqrt{5+\sqrt{13}}$, $\sqrt{5+\sqrt{13+\sqrt{5}}}$, $\sqrt{5+\sqrt{13+\sqrt{5+\sqrt{13}}}}$, $\sqrt{5+\sqrt{13+\sqrt{5+\sqrt{13+\sqrt{5}}}}}$, ...

(ii) There are other pairs of positive integers, a and b, where $a \neq b$, such that the sequence

$\sqrt{a}$, $\sqrt{a+\sqrt{b}}$, $\sqrt{a+\sqrt{b+\sqrt{a}}}$, $\sqrt{a+\sqrt{b+\sqrt{a+\sqrt{b}}}}$, ... has the same limit as in **part (a)**.

Find all these pairs of positive integers.

B MODELLING (QUESTIONS 7 to 9)

MAKING CONES (20 marks)

You are advised to spend no more than 45 minutes on this part.

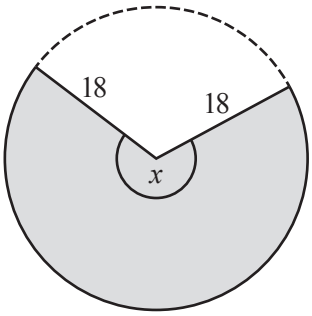
You may find some of these formulas useful.

Circumference, C, of a circle of radius r.	$C = 2\pi r$
Volume, V, of cone of radius r, height h.	$V = \frac{1}{3}\pi r^2 h$
Curved surface area, A, of cone of radius r, sloping edge l.	$A = \pi r l$

This task looks at modelling the volume of a cone, which is made from a circular disc.

In this part lengths are given in centimetres and angles in degrees.

Raj makes a cone from a circular disc of radius 18.
He cuts out part of the circular disc, leaving the shaded sector with sector angle x.
He joins the two radii of 18 together.
The cone that he makes has base radius r and height h.



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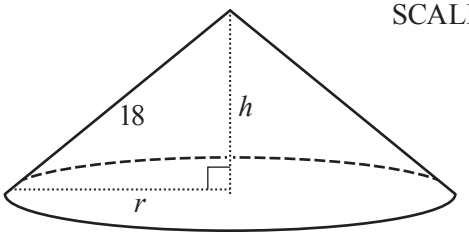


7 **(a)** Show that the arc length of the shaded sector is $\frac{x\pi}{10}$.

(b) This arc length forms the circumference of the base of the cone.

Find r in terms of x.

.....

(c) Show that $h = \sqrt{324 - \frac{x^2}{400}}$.

(d) **(i)** Show that a model for the volume, V, of this cone, is

$$V = 0.0026x^2\sqrt{324 - \frac{x^2}{400}} .$$

(ii) Use **part (i)** to find the volume of the cone if Raj uses a semicircle of radius 18.

.....

(e) Sketch the graph of the model for $0 < x < 360$.

(f) For the cone with maximum volume, find

(i) the angle, x,

.....

(ii) the maximum volume, V,

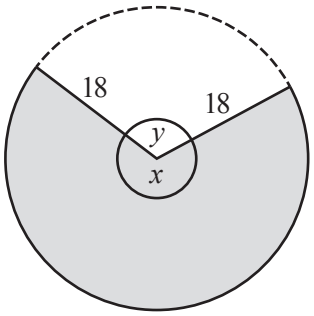
.....

(iii) the curved surface area.

.....

8 Raj decides that he can make two cones from his circular disc.

One comes from the sector with sector angle x.
The other comes from the remaining sector with sector angle y.



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(a) Write down a formula for y in terms of x.

.....

(b) The total volume of the two cones is

$$V = 0.0026x^2\sqrt{324 - \frac{x^2}{400}} + 0.0026y^2\sqrt{324 - \frac{y^2}{400}}\ .$$

Use your answer to **part (a)** to write down, in terms of x, a model for the total volume of the two cones.

.....

(c) Sketch the graph of this model.

(d) Find the **two** angles, x, that give the maximum total volume for the two cones.

..... and .....

Question 9 is printed on the next page.

9 Instead of the circular disc of radius 18, Raj now uses a disc of radius 36 to make **one** cone with th maximum volume.

Find

(a) the volume of this cone,

.....

(b) the angle that gives this volume.

.....