Cambridge IGCSE®

Cambridge Assessment International Education

CANDIDATE NAME	

CENTRE NUMBER						CANDIDATE NUMBER			

MATHEMATICS **0580/04**

Paper 4 (Extended) **For examination from 2020**

SPECIMEN PAPER **2 hours 30 minutes**

You must answer on the question paper.

You will need: Geometrical instruments

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- For π, use either your calculator value or 3.142.

INFORMATION
- The total mark for this paper is 130.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

[Turn over

1 (a) Kristian and Stephanie share some money in the ratio 3 : 2.
Kristian receives $72.

   (i) Work out how much Stephanie receives.

$ .... [2]

  (ii) Kristian spends 45% of his $72 on a computer game.

Calculate the price of the computer game.

$ .... [1]

 (iii) Kristian also buys a meal for $8.40 .

Calculate the fraction of the $72 Kristian has left after buying the computer game and the meal.
Give your answer in its lowest terms.

.... [2]

 (iv) Stephanie buys a book in a sale for $19.20 .
This sale price is after a reduction of 20%.

Calculate the original price of the book.

$ .... [3]

(b) Boris invests \$550 at a rate of 2% per year simple interest.

Calculate the value of the investment at the end of 10 years.

$ .... [3]

(c) Marlene invests \$550 at a rate of 1.9% per year compound interest.

Calculate the value of the investment at the end of 10 years.

$ .... [2]

(d) Hans invests \$550 at a rate of x% per year compound interest.
At the end of 10 years, the value of the investment is \$638.30, correct to the nearest cent.

Find the value of x.

$x =$ .... [3]

2 **(a)** 200 students estimate the volume, $V\,\mathrm{m}^3$, of a classroom.
The cumulative frequency diagram shows their results.
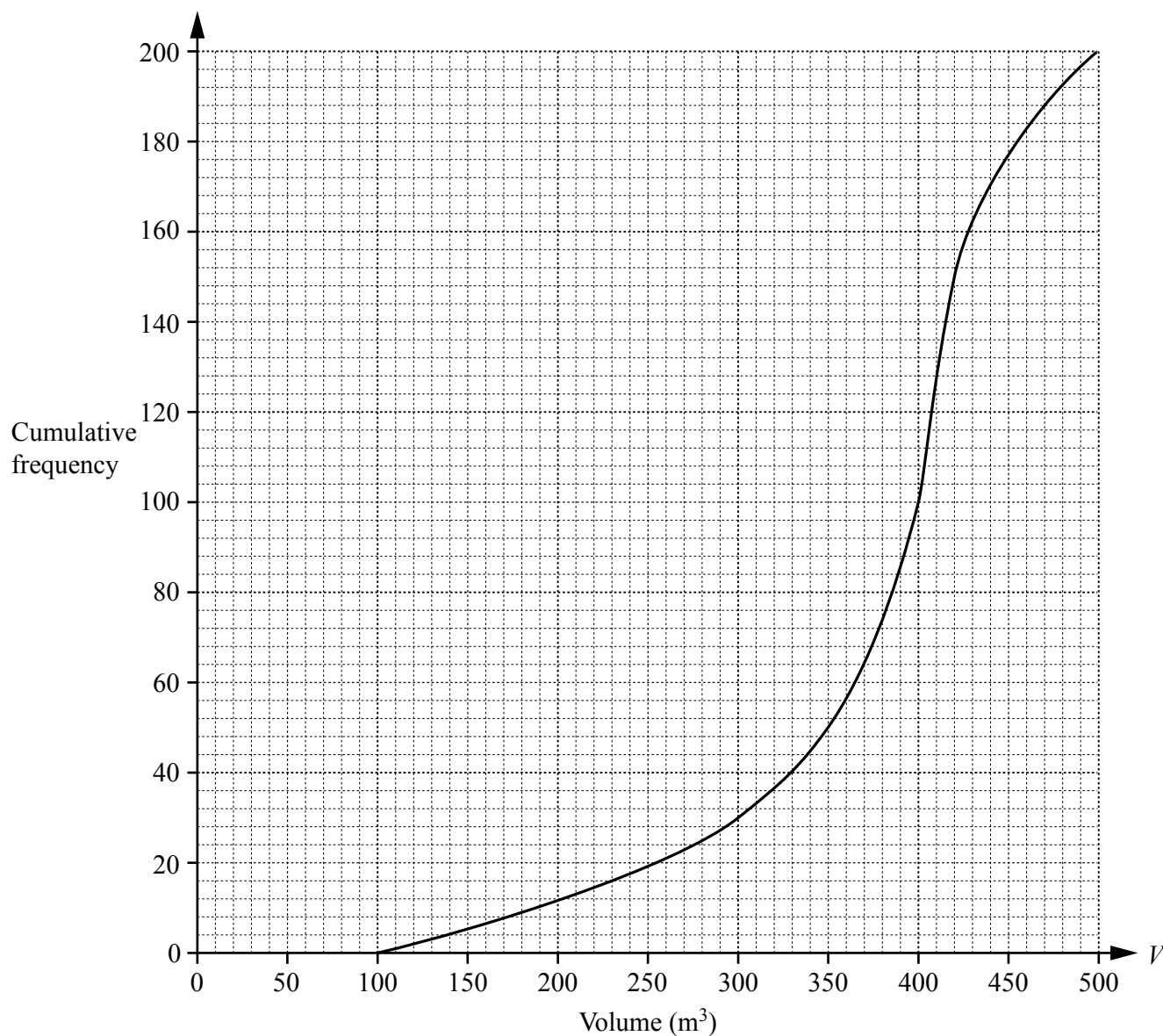
Use the graph to find an estimate of

(i) the median,

.... m^3 [1]

(ii) the interquartile range,

.... m^3 [2]

(iii) the 60th percentile,

.... m^3 [1]

(iv) the number of students who estimate that the volume is greater than $300\,\mathrm{m}^3$.

.... [2]

**(b)** The 200 students also estimate the total area, $A\,\text{m}^2$, of the windows in the classroom.
The table shows their results.

Area ($A\,\text{m}^2$)	$20 < A \leqslant 60$	$60 < A \leqslant 100$	$100 < A \leqslant 150$	$150 < A \leqslant 250$
Frequency	32	64	80	24

(i) Calculate an estimate of the mean.
You must show all your working.

.... m^2 [4]

(ii) Complete the histogram to show the information in the table.

Area (m^2)
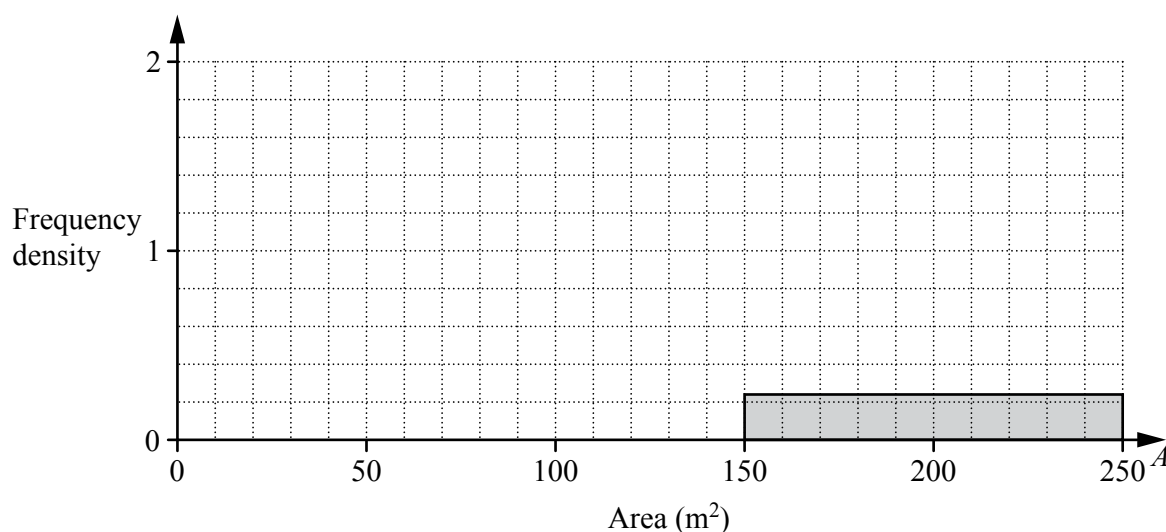
[4]

(iii) Two students are chosen at random from those students that estimated the area of the windows to be more than $100\,\text{m}^2$.

Find the probability that one of the two students estimates the area to be greater than $150\,\text{m}^2$ and the other student estimates the area to be $150\,\text{m}^2$ or less.

.... [3]

3 $f(x) = \dfrac{20}{x} + x, \ x \neq 0$

(a) Complete the table.

x	-10	-8	-5	-2	-1.6		1.6	2	5	8	10
$f(x)$	-12	-10.5	-9	-12	-14.1		14.1	12			12

[2]

(b) On the grid, draw the graph of $y = f(x)$ for $-10 \leqslant x \leqslant -1.6$ and $1.6 \leqslant x \leqslant 10$.
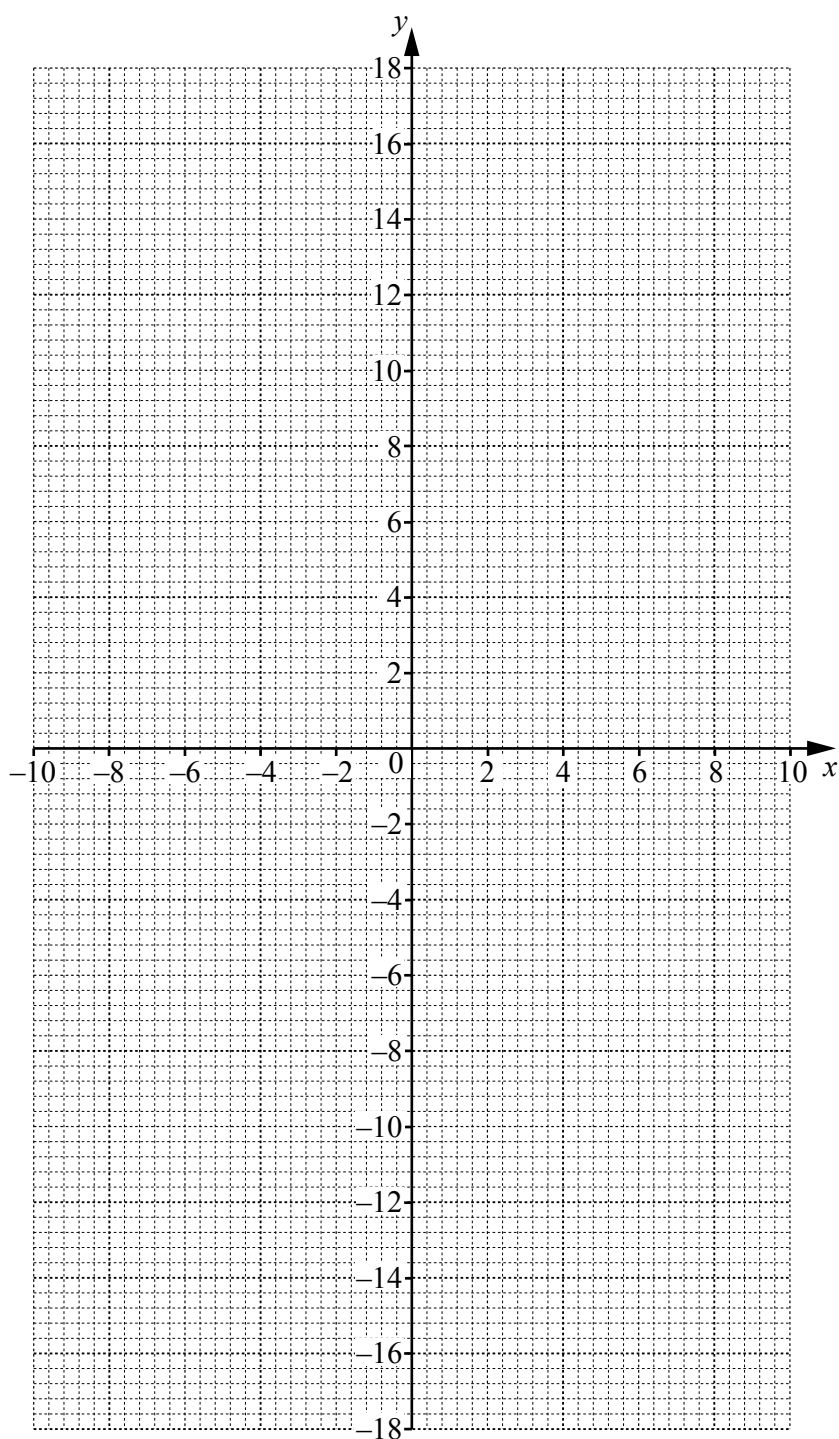
[5]

(c) Using your graph, solve the equation $f(x) = 11$.

$x =$ .... or $x =$ .... [2]

(d) k is a prime number and $f(x) = k$ has no solutions.

Find the possible values of k.

.... [2]

(e) The gradient of the graph of $y = f(x)$ at the point (2, 12) is -4.

Write down the coordinates of the other point on the graph of $y = f(x)$ where the gradient is -4.

(........................ , ...) [1]

(f) **(i)** The equation $f(x) = x^2$ can be written as $x^3 + px^2 + q = 0$.

Show that $p = -1$ and $q = -20$.

[2]

(ii) On the grid opposite, draw the graph of $y = x^2$ for $-4 \leqslant x \leqslant 4$. [2]

(iii) Using your graphs, solve the equation $x^3 - x^2 - 20 = 0$.

$x =$ .... [1]

(iv)
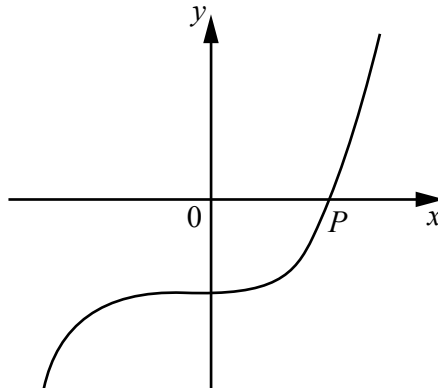
NOT TO
SCALE

The diagram shows a **sketch** of the graph of $y = x^3 - x^2 - 20$.
P is the point $(n, 0)$.

Write down the value of n.

$n =$ .... [1]

4
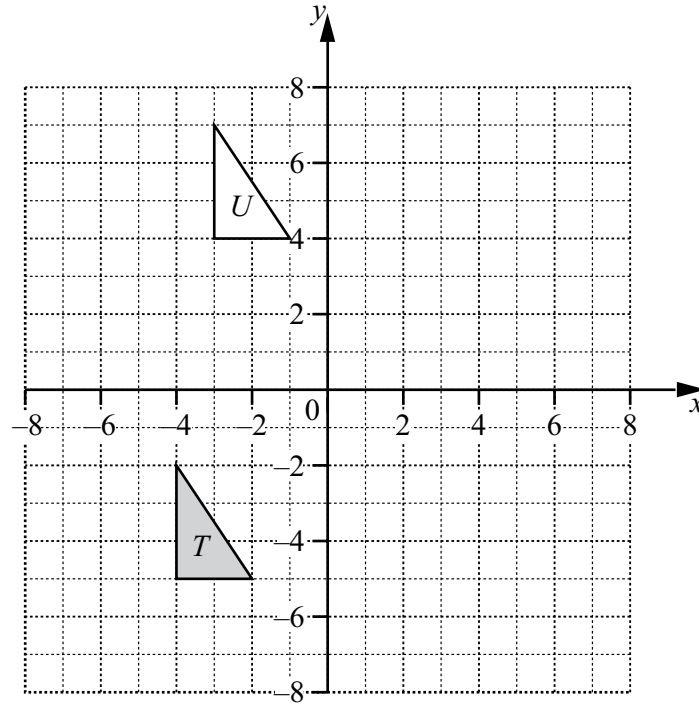
(a) (i) Draw the reflection of triangle *T* in the line *x* = 0. [2]

(ii) Draw the rotation of triangle *T* about (–2, –1) through 90° clockwise. [2]

(b) Describe fully the **single** transformation that maps triangle *T* onto triangle *U*.

....

.... [2]

5 (a)

x cm
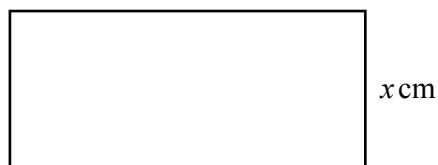
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The perimeter of the rectangle is 80 cm.
The area of the rectangle is A cm^2.

(i) Show that $x^2 - 40x + A = 0$.

[3]

(ii) When $A = 300$, solve the equation $x^2 - 40x + A = 0$ by factorising.

$x =$ .... or $x =$ .... [3]

(iii) When $A = 200$, solve the equation $x^2 - 40x + A = 0$ using the quadratic formula.
Show all your working and give your answers correct to 2 decimal places.

$x =$ .... or $x =$ .... [4]

(b) A car completes a 200 km journey at an average speed of x km/h.
The car completes **the return journey** of 200 km at an average speed of $(x + 10)$ km/h.

(i) Show that the difference between the time taken for each of the two journeys is $\dfrac{2000}{x(x + 10)}$ hours.

[3]

(ii) Find the difference between the time taken for each of the two journeys when $x = 80$.
Give your answer in **minutes** and **seconds**.

.... min .... s [3]

6
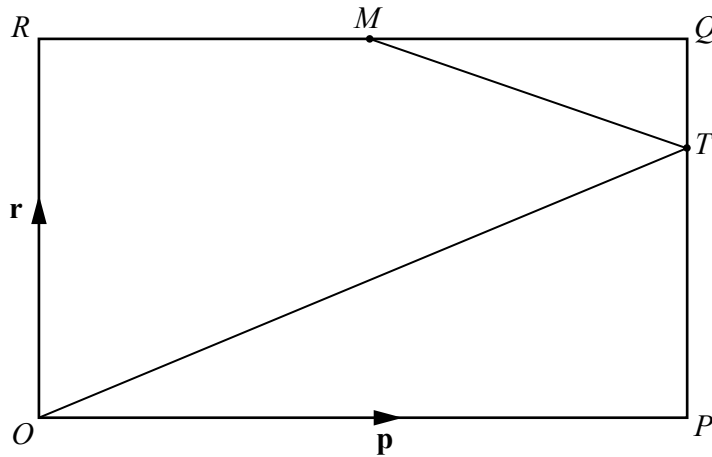
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OPQR is a rectangle and *O* is the origin.
M is the midpoint of *RQ* and *PT* : *TQ* = 2 : 1.
$\overrightarrow{OP}$ = **p** and $\overrightarrow{OR}$ = **r**.

(a) Find, in terms of **p** and/or **r**, in its simplest form

   (i) $\overrightarrow{MQ}$,

$\overrightarrow{MQ}$ = ... [1]

   (ii) $\overrightarrow{MT}$,

$\overrightarrow{MT}$ = ... [1]

   (iii) $\overrightarrow{OT}$.

$\overrightarrow{OT}$ = ... [1]

(b) *RQ* and *OT* are extended and meet at *U*.

Find the position vector of *U* in terms of **p** and **r**.
Give your answer in its simplest form.

.... [2]

(c) $\overrightarrow{MT} = \begin{pmatrix} 2k \\ -k \end{pmatrix}$ and $|\overrightarrow{MT}| = \sqrt{180}$.

     Find the positive value of k.

$k =$ .... [3]

7 $\qquad$ $f(x) = 2x + 1$ $\qquad$ $g(x) = x^2 + 4$ $\qquad$ $h(x) = 2^x$

(a) Solve the equation $f(x) = g(1)$.

$x =$ .... [2]

(b) Find $f^{-1}(x)$.

$f^{-1}(x) =$ .... [2]

(c) Find $gf(x)$ in its simplest form.

.... [3]

(d) Solve the equation $h^{-1}(x) = 0.5$.

$x =$ .... [1]

(e) $\dfrac{1}{h(x)} = 2^{kx}$

Write down the value of k.

$k =$ .... [1]

8 The grid shows the graph of $y = \cos x$ for $0° \leqslant x \leqslant 360°$.
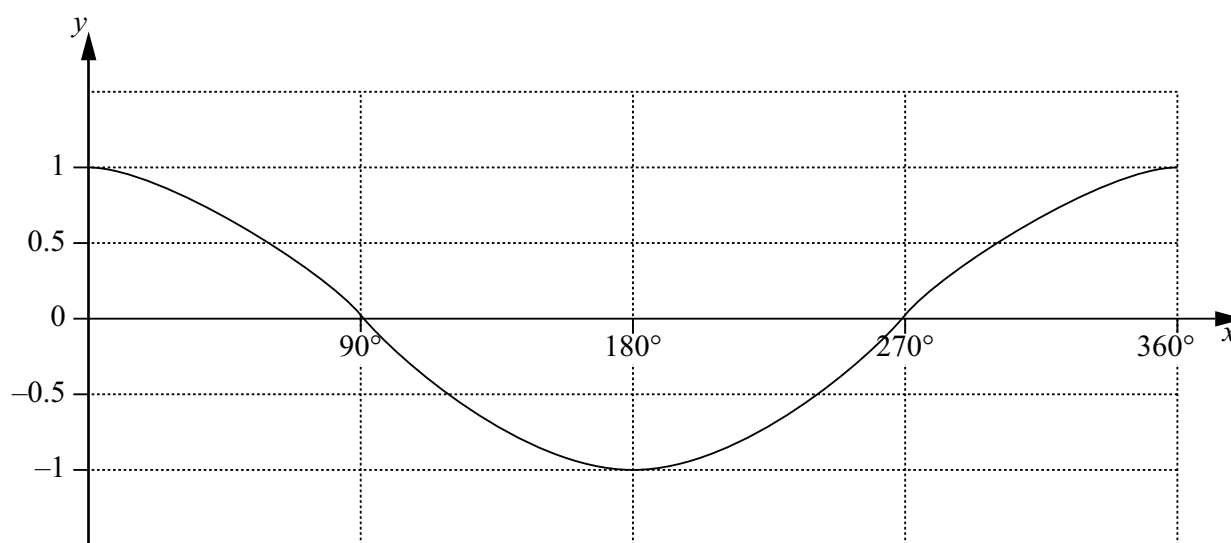
(a) Solve the equation $3\cos x = 1$ for $0° \leqslant x \leqslant 360°$.
Give your answers correct to 1 decimal place.

.... and .... [4]

(b) On the same grid, sketch the graph of $y = \sin x$ for $0° \leqslant x \leqslant 360°$. [2]

9

The diagram shows a trapezium *ABCD*.
AB is parallel to *DC*.
AB = 55 m, *BD* = 70 m, angle *ABD* = 40° and angle *BCD* = 32°.

(a) Calculate *AD*.

AD = ... m [4]

(b) Calculate *BC*.

BC = ... m [4]

(c) Calculate the area of *ABCD*.

.... m² [3]

(d) Calculate the shortest distance from *A* to *BD*.

.... m [2]

10 (a) Show that the volume of a metal sphere of radius 15 cm is 14 140 cm³, correct to 4 significal.
figures.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

[2]

(b) (i) The sphere is placed inside an empty cylindrical tank of radius 25 cm and height 60 cm.
The tank is filled with water.
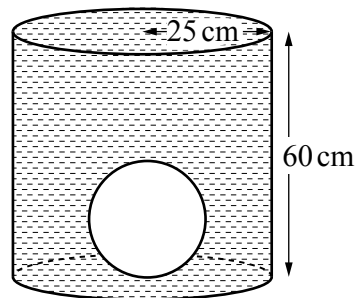
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Calculate the volume of water needed to fill the tank.

....cm³ [3]

(ii) The sphere is removed from the tank.
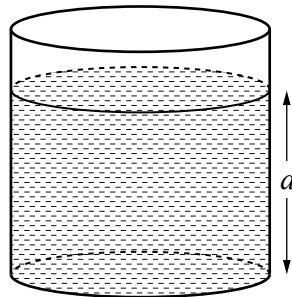
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Calculate the depth, d, of water in the tank.

$d = $ .... cm [2]

(c) The diagram below shows a solid circular cone and a solid sphere.
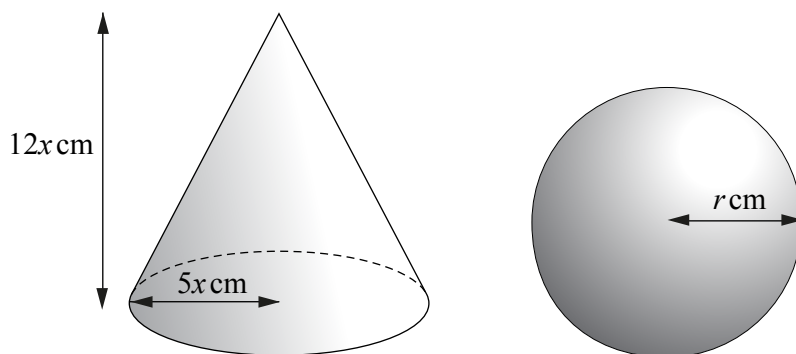
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The cone has radius $5x$ cm and height $12x$ cm.
The sphere has radius r cm.
The cone has the same **total** surface area as the sphere.

Show that $r^2 = \dfrac{45}{2}x^2$.

[The curved surface area, A, of a cone with radius r and slant height l is $A = \pi r l$.]
[The surface area, A, of a sphere with radius r is $A = 4\pi r^2$.]

[5]

11 A curve has equation $y = x^3 - 6x^2 + 16$.

(a) Find the coordinates of the two turning points.

(............ , ...) and (............ , ...) [6]

(b) Determine whether each of the turning points is a maximum or a minimum.
Give reasons for your answers.

[3]