Cambridge Assessment International Education

Cambridge IGCSE™

ADDITIONAL MATHEMATICS **0606/23**

Paper 2 **October/November 2021**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **10** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

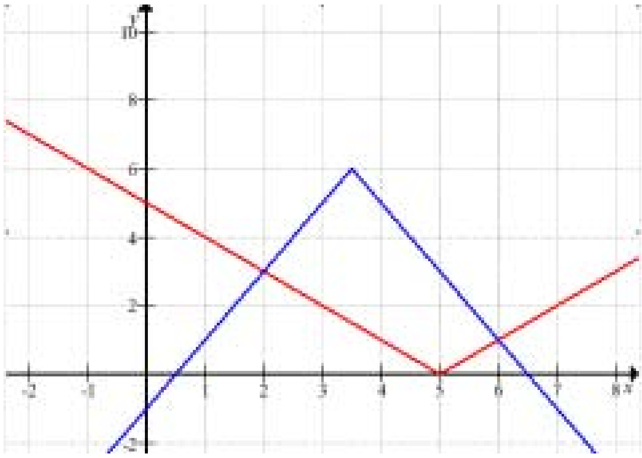
A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1(a)		4	**M1** for $y = \lvert x-5 \rvert$: $\vee$ shape with vertex at (5, 0) **A1** Correct graph with y-intercept at (0, 5) **M1** for $y = 6 - \lvert 2x-7 \rvert$: $\wedge$ shape with vertex at (3.5, 6) **A1** Correct graph with y-intercept at (0, −1)
1(b)	$x < 2$ or $x > 6$ final answer	**B2**	**B1** for exactly two correct critical values or **B1 FT** for exactly two correct FT critical values soi, FT dependent on at least M1 in **(a)** If the CVs are decimal allow BOD for reasonable values
2	Solves $2x+2y=6$ and $2x-\sqrt{3}y=5$ oe by elimination as far as $2y+\sqrt{3}y=1$ or substitutes $x = 3 - y$ into $2x-\sqrt{3}y=5$ oe OR solves $\sqrt{3}x+\sqrt{3}y=3\sqrt{3}$ and $2x-\sqrt{3}y=5$ oe by elimination as far as $2x+\sqrt{3}x=3\sqrt{3}+5$ or substitutes $y = 3 - x$ into $2x-\sqrt{3}y=5$ oe	**M1**	
	$y=\dfrac{1}{2+\sqrt{3}}$ or $x=\dfrac{3\sqrt{3}+5}{2+\sqrt{3}}$	**A1**	
	$y=\dfrac{1}{2+\sqrt{3}}\times\dfrac{2-\sqrt{3}}{2-\sqrt{3}}$ oe or $x=\dfrac{3\sqrt{3}+5}{2+\sqrt{3}}\times\dfrac{2-\sqrt{3}}{2-\sqrt{3}}$ oe	**M1**	**FT** *their* value of x or y providing of equivalent difficulty
	$y=2-\sqrt{3}$ and $x=1+\sqrt{3}$	**A2**	**A1** for either and no extra values
3(a)	$a=3$	**B1**	
	$b=2$	**B1**	
	$c=-1$	**B1**	
3(b)(i)	2	**B1**	

Question	Answer	Marks	Partial Marks
3(b)(ii)	$\frac{2\pi}{3}$ oe or 2.09 or 2.094[395…] rot to 4 or more sf	B1	
4(a)	$2x-3=6^{\frac{1}{2}}$ oe, soi	M1	
	$x=\frac{6^{\frac{1}{2}}+3}{2}$ or $x=\frac{\sqrt{6}+3}{2}$	A1	
4(b)	$\ln\frac{2u}{u-4}=\ln e$ soi or $\ln\frac{2u}{u-4}=1$ soi or $\ln 2u=\ln e(u-4)$ soi	M1	Condone one sign or bracketing error
	$\frac{2u}{u-4}=e$ or $2u=e(u-4)$ oe	M1	**FT** *their* logarithmic equation
	$u=\frac{4e}{e-2}$ or $u=\frac{-4e}{2-e}$ or equivalent exact form	A1	
4(c)	$\frac{3^v}{\left(3^3\right)^{2v-5}}=3^2$ oe soi or $\frac{9^{\frac{v}{2}}}{\left(9^{\frac{3}{2}}\right)^{2v-5}}=9$ oe soi or $\log 3^v-\log 27^{2v-5}=\log 9$ oe soi	B1	
	$15-5v=2$ oe or $v\log 3-(2v-5)\log 27=\log 9$	M1	**FT** *their* exponential equation in the same base or *their* logarithmic equation with any consistent base, providing *their* exponential or logarithmic equation has at most one sign or arithmetic error
	$v=\frac{13}{5}$ oe	A1	

Question	Answer	Marks	Partial Marks
5(a)	$\frac{\sin x}{1-\sin x}+\frac{\sin x}{1+\sin x}$ or $\frac{\operatorname{cosec} x+1+\operatorname{cosec} x-1}{\operatorname{cosec}^2 x-1}$ oe	**M1**	
	$\frac{\sin x+\sin^2 x+\sin x-\sin^2 x}{1-\sin^2 x}$ or $\frac{2\operatorname{cosec} x}{\cot^2 x}$ oe	**A1**	
	$\frac{2\sin x}{\cos^2 x}$ or $\frac{2\sin^2 x}{\sin x\cos^2 x}$ oe	**A1**	
	Fully correct justification of given answer: $\frac{2\sin x}{\cos x}\times\frac{1}{\cos x}=2\tan x\sec x$ or $2\tan x\times\frac{1}{\cos x}=2\tan x\sec x$ or $\frac{2\sin x}{\cos x}\times\sec x=2\tan x\sec x$ or equivalent	**A1**	
5(b)	$2\tan^2 x=5$ or better, soi or $7\cos^2 x=2$ or better, soi or $7\sin^2 x=5$ or better, soi	**B1**	
	$\tan x=[\pm]\sqrt{\frac{5}{2}}$ oe or [±] 1.58[1…] or $\cos x=[\pm]\sqrt{\frac{2}{7}}$ oe or [±] 0.534[5…] or $\sin x=[\pm]\sqrt{\frac{5}{7}}$ oe or [±] 0.845[1…]	**M1**	**FT** an equation of the form $a\tan^2 x=b$ $a>0, b>0$ or $p\sin^2 x=q$ or $p\cos^2 x=q$ where $p>0$, $q>0$ and $p>q$
	57.7 or 57.6884… rot to 2 or more dp 237.7 or 237.6884… rot to 2 or more dp 122.3 or 122.3115… rot to 2 or more dp 302.3 or 302.3115… rot to 2 or more dp	**A2**	no extras in range **A1** for any two correct answers
6(a)	$y=(x-2)^2+4$ oe, isw	**B2**	**B1** for a correct expression in x and y only, that is not of the form $y=\mathrm{f}(x)$
6(b)	$\left[\frac{\mathrm{d}y}{\mathrm{d}x}=\right]2(x-2)$ oe	**B1**	**dep** on **B2** in **(a)**

Question	Answer	Marks	Partial Marks
6(c)	[When $\theta = \frac{\pi}{3}$] $x = 4$ soi	B1	
	[When $\theta = \frac{\pi}{3}$] $y = 8$ soi	B1	
	[When $x = 4$ or $\theta = \frac{\pi}{3}$] $\frac{dy}{dx} = 4$	M1	**FT** *their* $\left.\frac{dy}{dx}\right\|_{x=4}$ providing non-zero
	$y - 8 = 4(x - 4)$ oe isw	A1	**FT** *their* $\left.\frac{dy}{dx}\right\|_{x=4}$ providing non-zero
7(a)	[**p** =] $-15\mathbf{i} + 36\mathbf{j}$ isw	B2	**B1** for multiplier $\frac{39}{\sqrt{5^2 + 12^2}}$ soi or unit vector $\frac{-5\mathbf{i} + 12\mathbf{j}}{\sqrt{5^2 + 12^2}}$
	[**q** =] $30\mathbf{i} - 16\mathbf{j}$ isw	B2	**B1** for multiplier $\frac{34}{\sqrt{15^2 + 8^2}}$ soi or unit vector $\frac{15\mathbf{i} - 8\mathbf{j}}{\sqrt{15^2 + 8^2}}$ soi
7(b)	[**p** + **q** =] $15\mathbf{i} + 20\mathbf{j}$ or $\begin{pmatrix} 15 \\ 20 \end{pmatrix}$ soi	B1	
	$\left[\|\mathbf{p} + \mathbf{q}\| = \sqrt{15^2 + 20^2} = \right] 25$	B1	**FT** *their*(**p** + **q**) of the form $\begin{pmatrix} x \\ y \end{pmatrix}$ or $x\mathbf{i} + y\mathbf{j}$ where $x \neq 0$, $y \neq 0$
	53.1[°] or 53.13[01…] rot to 2 or more dp OR 0.927 [rads] or 0.9272[95…] rot to 4 or more sf	B2	**M1 FT** *their*(**p** + **q**) of the form $\begin{pmatrix} x \\ y \end{pmatrix}$ or $x\mathbf{i} + y\mathbf{j}$ where $x \neq 0$, $y \neq 0$ and $x \neq y$ for $\tan(...) = \frac{their 20}{their 15}$ oe or $\cos(...) = \frac{their 15}{their 25}$ oe or $\sin(...) = \frac{their 20}{their 25}$ oe

Question	Answer	Marks	Partial Marks
8(a)	$\frac{dy}{dx} = -5(x-1)^{-2} + 2$ oe	B2	**B1** for $\frac{d}{dx}(-5(x-1)^{-1}) = k(x-1)^{-2}$ soi
	$(x-1)^2 = \frac{5}{2}$ or $2x^2 - 4x - 3 = 0$	M1	**dep** on at least **B1**
	$x = 1 + \frac{\sqrt{10}}{2}$ oe, isw or 2.58[11…]	A1	implies **M1**
	$y = 2 + 2\sqrt{10}$ oe, isw or 8.32 to 8.325	A1	
8(b)	[Area of triangle =] 9 soi	B1	
	[Area under curve = F(x) =] $\left[5\ln(x-1) + \frac{2x^2}{2}\right]_2^4$ oe	M2	**M1** for $\int \frac{5}{x-1}\,dx = k\ln(x-1)$ $k \neq 0$ soi or for $5\ln x - 1$
	their 9 + F(4) – F(2)	M1	**dep** on at least **M1**
	21 + 5ln3 isw or 26.49 to 26.5	A1	
9(a)	Attempts to solve $a + 2d = 13$ and $a + 9d = 41$ oe	M2	**M1** for $a + 2d = 13$ and $a + 9d = 41$ soi
	$d = 4$ and $a = 5$	A2	**A1** for $d = 4$ or $a = 5$
9(b)	$\frac{n}{2}\{2(5) + (n-1)4\}$ soi	M1	**FT** *their a* and *their d*
	$2n^2 + 3n - 2555\,[*0\,]$	A1	where * could be = or any inequality sign
	Solves *their* 3-term quadratic of the form $ax^2 + bx + c$ [*0] by factorising or formula or *their* 3-term quadratic of the form $ax^2 + bx * c$ or better if completing the square	M1	
	35	A1	

Question	Answer	Marks	Partial Marks
9(c)	May work ***consistently*** in n throughout but must conclude in k to earn the final mark		
	$S_{2k}=\frac{2k}{2}\{10+(2k-1)4\}$ soi	**B1**	**FT** *their a* and *their d*
	$\frac{2k}{2}\{10+(2k-1)4\}-\frac{k}{2}\{10+(k-1)4\}$ soi	**M1**	**FT** *their a* and *their d* ; condone at most one error
	Simplifies as far as e.g. $8k^2+6k-(3k+2k^2)$ or $8k^2+6k-3k-2k^2$	**A1**	
	Correct completion to given answer: $6k^2+3k=3k(1+2k)$	**A1**	
	Alternative method		
	$\frac{2k}{2}\{2a+(2k-1)d\}$ **and** $a=$ *their* 5 and $d=$ *their* 4 substituted at some point	**(B1)**	
	$ak-\frac{d}{2}k+\frac{3}{2}dk^2$ oe	**(M1)**	condone at most one error
	$5k-\frac{4}{2}k+\frac{3}{2}\times4\times k^2$	**(A1)**	
	Correct completion to given answer: $6k^2+3k=3k(1+2k)$	**(A1)**	
10(a)	$[f'(x)=]12x^2-8x-15$	**M2**	**M1** for any two terms correct or $12x^2-8x-15+c$
	$y=3$ and $f'(1)=-11$	**A1**	
	$[m_{\perp}=]\frac{1}{11}$ soi	**M1**	**FT** $\frac{-1}{their\ f'(1)}$
	$y-3=\frac{1}{11}(x-1)$ oe, isw	**A1**	**FT** *their* $m_{\perp}$ and *their* 3, provided *their* $3\neq1$ or 0 or -11

Question	Answer	Marks	Partial Marks
10(b)	[f(–2) =] –32 – 16 + 30 + 18 = 0 or [f(–*a*) =] $-4a^3 - 4a^2 + 15a + 18$ and shows this to be 0 when $a = 2$ or uses algebraic long division or synthetic division to show that $x + 2$ is a factor of f(x) or that $a - 2$ is a factor of f($-a$)	**M1**	Method must be seen and be fully correct with no clear evidence of calculator use
	$a = 2$	**A1**	as the only value of a
	Uses $(x + 2)$ is a factor to find the correct quadratic factor $4x^2 - 12x + 9$	**B2**	**B1** for any two out of three terms correct
	Correctly solves *their*$(4x^2 - 12x + 9)(x + 2) = 0$ or correctly factorises *their*$(4x^2 - 12x + 9)(x + 2)$	**M1**	**dep** on using a quadratic factor that has earned at least **B1;** method must be seen; **M0** if *their* quadratic factor does not have real roots
	$x = -2$ or 1.5	**A1**	**dep** on **M1 B2 M1**