

1. ALGEBRA*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

- 1 Given that $y = \frac{\sin x}{\ln x^2}$, find an expression for $\frac{dy}{dx}$.

[4]

- 2 Find the values of k for which the equation $(k-1)x^2 + kx - k = 0$ has real and distinct roots.

[4]

3 (i) Given that $x-2$ is a factor of $ax^3 - 12x^2 + 5x + 6$, use the factor theorem to show that $a = 4$. [2]

(ii) Showing all your working, factorise $4x^3 - 12x^2 + 5x + 6$ and hence solve $4x^3 - 12x^2 + 5x + 6 = 0$. [4]

- 4 A circle has diameter x which is increasing at a constant rate of 0.01 cm s^{-1} . Find the exact rate of change of the area of the circle when $x = 6 \text{ cm}$. [5]

5 (i) Express $5x^2 - 15x + 1$ in the form $p(x+q)^2 + r$, where p , q and r are constants.

[3]

(ii) Hence state the least value of $x^2 - 3x + 0.2$ and the value of x at which this occurs.

[2]

- 6 (a) State the order of the matrix $\begin{pmatrix} 0 & 1 & 4 & 8 \\ 5 & 8 & 1 & 6 \end{pmatrix}$.

[1]

(b) $\mathbf{A} = \begin{pmatrix} 2 & -4 \\ -1 & 3 \end{pmatrix}$

- (i) Find \mathbf{A}^{-1} .

[2]

- (ii) Hence, given that $\mathbf{ABA} = \mathbf{I}$, find the matrix \mathbf{B} .

[3]

7 (a) Solve $\lg(x^2 - 3) = 0$.

[2]

(b) (i) Show that, for $a > 0$, $\frac{\ln a^{\sin(2x+5)} + \ln\left(\frac{1}{a}\right)}{\ln a}$ may be written as $\sin(2x+5) + k$, where k is an integer. [3]

(ii) Hence find $\int \frac{\ln a^{\sin(2x+5)} + \ln\left(\frac{1}{a}\right)}{\ln a} dx$. [3]

- 8 (a) In the binomial expansion of $\left(a - \frac{x}{2}\right)^6$, the coefficient of x^3 is 120 times the coefficient of x^5 . Find the possible values of the constant a . [4]

- (b) (i) Expand $(1 + 2x)^{20}$ in ascending powers of x , as far as the term in x^3 . Simplify each term. [2]

- (ii) Use your expansion to show that the value of 0.98^{20} is 0.67 to 2 decimal places. [2]

- 9 (a) Solve $6\sin^2x - 13\cos x = 1$ for $0^\circ \leq x \leq 360^\circ$.

[4]

- (b) (i) Show that, for $-\frac{\pi}{2} < y < \frac{\pi}{2}$, $\frac{4 \tan y}{\sqrt{1 + \tan^2 y}}$ can be written in the form $a \sin y$, where a is an integer. [3]

- (ii) Hence solve $\frac{4 \tan y}{\sqrt{1 + \tan^2 y}} + 3 = 0$ for $-\frac{\pi}{2} < y < \frac{\pi}{2}$ radians. [1]

10 (a) Find the unit vector in the direction of $5\mathbf{i} - 15\mathbf{j}$.

(b) The position vectors of points A and B relative to an origin O are $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 12 \\ 7 \end{pmatrix}$ respectively. The point C lies on AB such that $AC : CB$ is $2 : 1$.

(i) Find the position vector of C relative to O . [3]

The point D lies on OB such that $OD : OB$ is $1 : \lambda$ and $\overrightarrow{DC} = \begin{pmatrix} 6 \\ 1.25 \end{pmatrix}$.

(ii) Find the value of λ .

[3]

11 The velocity, $v \text{ ms}^{-1}$, of a particle travelling in a straight line, t seconds after passing through a fixed point O , is given by $v = \frac{4}{(t+1)^3}$.

(i) Explain why the direction of motion of the particle never changes. [1]

(ii) Showing all your working, find the acceleration of the particle when $t = 5$. [3]

(iii) Find an expression for the displacement of the particle from O after t seconds. [3]

(iv) Find the distance travelled by the particle in the fourth second. [2]

12 (a) The functions f and g are defined by

$$\begin{aligned} f(x) &= 5x - 2 \quad \text{for } x > 1, \\ g(x) &= 4x^2 - 9 \quad \text{for } x > 0. \end{aligned}$$

(i) State the range of g . [1]

(ii) Find the domain of gf . [1]

(iii) Showing all your working, find the exact solutions of $gf(x) = 4$. [3]

Question 12(b) is printed on the next page.

(b) The function h is defined by $h(x) = \sqrt{x^2 - 1}$ for $x \leq -1$.

(i) State the geometrical relationship between the graphs of $y = h(x)$ and $y = h^{-1}(x)$. [1]

(ii) Find an expression for $h^{-1}(x)$. [3]

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.