



# Cambridge O Level

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**ADDITIONAL MATHEMATICS**

**4037/02**

Paper 2

**For examination from 2020**

MARK SCHEME

Maximum Mark: 80

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**Specimen**

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This document has **12** pages. Blank pages are indicated.

### Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

<p>GENERIC MARKING PRINCIPLE 1:</p> <p>Marks must be awarded in line with:</p> <ul style="list-style-type: none"> <li>• the specific content of the mark scheme or the generic level descriptors for the question</li> <li>• the specific skills defined in the mark scheme or in the generic level descriptors for the question</li> <li>• the standard of response required by a candidate as exemplified by the standardisation scripts.</li> </ul>
<p>GENERIC MARKING PRINCIPLE 2:</p> <p>Marks awarded are always <b>whole marks</b> (not half marks, or other fractions).</p>
<p>GENERIC MARKING PRINCIPLE 3:</p> <p>Marks must be awarded <b>positively</b>:</p> <ul style="list-style-type: none"> <li>• marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate</li> <li>• marks are awarded when candidates clearly demonstrate what they know and can do</li> <li>• marks are not deducted for errors</li> <li>• marks are not deducted for omissions</li> <li>• answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.</li> </ul>
<p>GENERIC MARKING PRINCIPLE 4:</p> <p>Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.</p>
<p>GENERIC MARKING PRINCIPLE 5:</p> <p>Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).</p>

**GENERIC MARKING PRINCIPLE 6:**

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

**MARK SCHEME NOTES**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

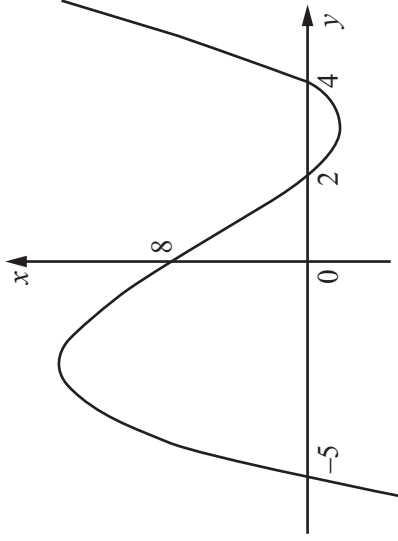
**Types of mark**

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, given for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the **M** marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several **B** marks allocated. The notation ‘dep’ is used to indicate that a particular **M** or **B** mark is dependent on an earlier mark in the scheme.

**Abbreviations**

<b>AG</b>	answer given
<b>awrt</b>	answer which rounds to
<b>cao</b>	correct answer only
<b>dep</b>	dependent
<b>FT</b>	follow through after error
<b>isw</b>	ignore subsequent working
<b>nfww</b>	not from wrong working
<b>oe</b>	or equivalent
<b>rot</b>	rounded or truncated
<b>SC</b>	special case
<b>soi</b>	seen or implied

Question	Answer	Marks	Partial Marks
1	$(xy)^4 y = 486$ oe $81y = 486$ $y = 6, x = \frac{1}{2}$	<b>M1</b>  <b>A2</b>	For elimination of one variable and attempt to solve  <b>A1</b> for each
2(a)		<b>2</b>	<b>B1</b> for shape <b>B1</b> for intercepts on coordinate axes
2(b)	Valid explanation, e.g. multiplying throughout by 5 does not change $x$ values because the $x$ -axis is invariant.	<b>1</b>	
2(c)	$x \leq -5$ $2 \leq x \leq 4$	<b>1</b>	<b>FT</b> their graph

Question	Answer	Marks	Partial Marks		
3(a)	$y = 2 + 4 \ln x$	<b>M1</b>	Complete method to find the inverse		
	$\ln x = \frac{y-2}{4}$				
	$g^{-1}(x) = e^{\frac{x-2}{4}}$			<b>A1</b>	Must be in a correct form
	Domain $x \in \mathbb{R}$			<b>B1</b>	Must be in a correct form
3(b)	Range $y > 0$	<b>B1</b>	Must be in a correct form		
	$g(x^2 + 4) = 10$	<b>M1</b>	For correct order		
	$2 + 4 \ln(x^2 + 4) = 10$	<b>M1</b>	dep For attempt to solve		
	Leading to $x = 1.84$ or $\sqrt{e^2 - 4}$ only	<b>A1</b>	One solution only		
3(b) alternative	$h(x) = x^2 + 4 = g^{-1}(10)$	<b>M1</b>	For correct order		
	$g^{-1}(10) = e^2$ , so $x^2 + 4 = e^2$	<b>M1</b>	dep For attempt to solve		
	Leading to $x = 1.84$ only	<b>A1</b>	One solution only		
	$\frac{4}{x} = 2x$	<b>B1</b>	For given equation, allow in this form		
3(c)	$x^2 = 2$	<b>M1</b>	For attempt to solve, must be using derivatives		
	$x = \sqrt{2}$	<b>A1</b>	One solution only, allow 1.41 or better		

Question	Answer	Marks	Partial Marks
4		4	B1 for shape B1 for 20° B1 for 100° B1 for (0, -1)

Question	Answer	Marks	Partial Marks
5(a)(i)	${}^9P_6 = 60480$	B1	Must be evaluated
5(a)(ii)	${}^4P_2 \times {}^3P_2 \times 2$ 144	M1 A1	For attempt at a product of 3 permutations Must be evaluated
5(a)(iii)	$840 \times 2$ 1680	B1	For either 840 or for realising that there are 2 possible positions for the symbols
5(b)(i)	${}^{10}C_6 \times {}^5C_3$ 2100	M1 A1	Unsimplified form
5(b)(ii)	${}^8C_4 \times {}^4C_2$ 420	M1 A1	Unsimplified form

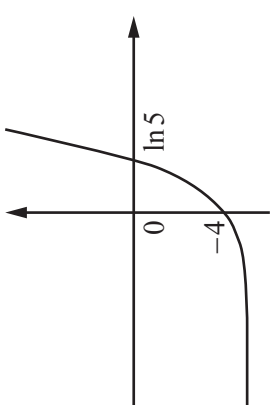
Question	Answer	Marks	Partial Marks
6(a)	27	1	
6(b)	$t^2 = 8 \ln 4$ $t = 3.33$ or better	M1 A1	Correct attempt to solve $e^{\frac{t^2}{8}} = 4$ , from correct working  M1 for a correct attempt to differentiate using the chain rule A1 all correct
6(c)	Acceleration = $3 \times \frac{2t}{8} e^{\frac{t^2}{8}} (e^{\frac{t^2}{8}} - 4)$ When $t = 1$ , $a = 6.98$	2 2	M1 for use of $t = 1$ in their acceleration

Question	Answer	Marks	Partial Marks
7(a)	$\lg y = x^2 \lg b + \lg A$ $\lg b = \pm 0.21$ $b = 0.617$ allow 0.62, $10^{-0.21}$ $\lg A = 0.94$ allow 0.93 to 0.95 $A = 8.71$ allow awrt 8.5 to 8.9	B1 B1 B1 B1	$\lg b = \pm 0.21$ may be implied
7(a) alternative	$5.37$ or $10^{0.73} = Ab$ $1.259$ or $10^{0.1} = Ab^4$ $b^3 = 10^{-0.63}$ $b = 0.617$ allow 0.62, $10^{-0.21}$ $A = 8.71$ allow awrt 8.5 to 8.9	B2 B1 B1	For both equations, allow correct to 2 SF
7(b)	$x = 1.5$ , $x^2 = 2.25$ $y = 2.93$ allow awrt 2.9 or 3.0	M1 A1	$y = \text{their } A \times \text{their } b^{1.5^2}$ or $\lg y = \lg \text{their } A + (1.5^2 \lg \text{their } b)$
7(c)	$\lg y = 0.301$ or $2 = 8.71(0.617)^{x^2}$ $x = 1.74$ Allow $\sqrt{3}$ or awrt 1.7, 1.8	M1 A1	$2 = \text{their } A(\text{their } b)^{x^2}$ or $\lg 2 = (\lg \text{their } b)x^2 + \lg(\text{their } A)$

Question	Answer	Marks	Partial Marks
8(a)	$2r \sin \theta$ <i>their</i> $(2r \sin \theta) + 2r\theta = 20$ $r = \frac{10}{\theta + \sin \theta}$	<b>B1</b> <b>M1</b> <b>A1</b>	
8(b)	$\frac{dr}{d\theta} = -\frac{10(1 + \cos \theta)}{(\theta + \sin \theta)^2}$ When $\theta = \frac{\pi}{6}$ , $\frac{dr}{d\theta} = -17.8$	<b>4</b>	<b>M1</b> for a correct attempt to differentiate <b>A2</b> , 1, 0, -1 each error <b>A1</b> , allow awrt - 17.8 <b>Answer only scores 0/4.</b>

Question	Answer	Marks	Partial Marks
9(a)	$\vec{DB} = 7\mathbf{a} - \mathbf{b}$	<b>1</b>	Mark final answer; allow unsimplified
9(b)	$\vec{DA} = 3\mathbf{a} - \mathbf{b}$	<b>1</b>	Mark final answer; allow unsimplified
9(c)	$\vec{AX} = \lambda(4\mathbf{a} + \mathbf{b})$	<b>1</b>	Mark final answer; allow unsimplified
9(d)	Valid method of finding $\vec{DX}$	<b>M1</b>	<b>FT</b> <i>their</i> $\vec{DA} + \vec{AX}$
	$\vec{DX} = 3\mathbf{a} - \mathbf{b} + \lambda(4\mathbf{a} + \mathbf{b})$	<b>A1</b>	Allow unsimplified
9(e)	$3\mathbf{a} - \mathbf{b} + \lambda(4\mathbf{a} + \mathbf{b}) = \mu(7\mathbf{a} - \mathbf{b})$	<b>M1</b>	Equating <i>their</i> $(3\mathbf{a} - \mathbf{b} + \lambda(4\mathbf{a} + \mathbf{b}))$ and $\mu \times$ <i>their</i> $(7\mathbf{a} - \mathbf{b})$
	Attempts to solve <i>their</i> $(3 + 4\lambda = 7\mu)$ and <i>their</i> $(-1 + \lambda = -\mu)$	<b>M1</b>	dep on first <b>M1</b>
	$\lambda = \frac{4}{11}$ , $\mu = \frac{7}{11}$	<b>A2</b>	<b>A1</b> for each



Question	Answer	Marks	Partial Marks
10(a)(i)		3	<b>B1</b> correct shape <b>B1</b> through (0, -4) <b>B1</b> through (ln 5, 0)
10(a)(ii)	$k \leq -5$	1	
10(b)	$\frac{1}{2} \log_a 2 + 3 \log_a 2 - \log_a 2$ or $\log_a (2^{\frac{1}{2}} \times 2^3 \times 2^{-1})$ oe $2^{\frac{1}{2}} \log_a 2$ oe	<b>M1</b> <b>A1</b>	Condone one error <b>Answer only scores 0/2.</b>
10(c)	$\log_9 4x = \frac{\log_3 4x}{\log_3 9}$ or $\log_3 x = \frac{\log_9 x}{\log_9 3}$ $\log_3 x - \frac{\log_3 4x}{2} = 1$ or $\frac{\log_9 x}{(\frac{1}{2})} - \log_9 4x = 1$ $\log_3 \frac{x}{(4x)^{\frac{1}{2}}} = \log_3 3$ or $\log_9 \frac{x^2}{4x} = \log_9 9$ oe $x = 36$	<b>B1</b> <b>M1</b> <b>M1</b> <b>M1</b> <b>A1</b>	soi

Question	Answer	Marks	Partial Marks
11(a)(i)	$\frac{\operatorname{cosec} \theta}{\operatorname{cosec} \theta - \sin \theta} = \frac{1}{\frac{\sin \theta}{\sin \theta - \sin \theta}}$ $= \frac{1}{1 - \sin^2 \theta} \text{ or } \frac{1}{\frac{\sin \theta}{(1 - \sin^2 \theta)}} = \frac{1}{\sin \theta}$	M1	For using $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$ and either attempt to multiply top and bottom by $\sin \theta$ or an attempt to combine terms in the denominator
		M1	dep For correct use of $1 - \sin^2 \theta = \cos^2 \theta$
	$= \frac{1}{\cos^2 \theta}$ $= \sec^2 \theta$	A1	Completion of proof
11(a)(i) alternative	$\frac{\operatorname{cosec} \theta}{\operatorname{cosec} \theta - \sin \theta} = \frac{\operatorname{cosec} \theta}{\operatorname{cosec} \theta - \frac{1}{\operatorname{cosec} \theta}}$ $= \frac{\operatorname{cosec}^2 \theta}{\operatorname{cosec}^2 \theta - 1}$ $= \frac{1 + \cot^2 \theta}{\cot^2 \theta}$ $= \tan^2 \theta + 1$ $= \sec^2 \theta$	M1	For using $\sin \theta = \frac{1}{\operatorname{cosec} \theta}$ and an attempt to combine terms in the denominator
		M1	dep For use of $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$
		A1	Completion of proof
11(a)(ii)	$\cos^2 \phi = \frac{1}{4}, \cos \phi = \pm \frac{1}{2}$ $\text{or } \tan^2 \phi = 3, \tan \phi = \pm \sqrt{3}$ $\text{or } \sin^2 \phi = \frac{3}{4}, \sin \phi = \pm \frac{\sqrt{3}}{2}$ $\phi = 60^\circ, 120^\circ, 240^\circ, 300^\circ$	M1	For using (i) to obtain a value for either $\cos^2 \phi$ , $\tan^2 \phi$ or $\sin^2 \phi$ and then taking the square root
		A2	A1 for two correct values A1 for two further correct values and no extras in the range

Question	Answer	Marks	Partial Marks
11(b)	$\tan\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{3}}$ $x = \frac{\pi}{6} - \frac{\pi}{4}, \frac{7\pi}{6} - \frac{\pi}{4}, \frac{13\pi}{6} - \frac{\pi}{4}$ $x = \left(-\frac{\pi}{12}\right), \frac{11\pi}{12}, \frac{23\pi}{12}$	<p><b>M1</b></p> <p><b>A2</b></p>	<p>For correct order of operations, can be implied by <math>x = -\frac{\pi}{12}</math></p> <p><b>A1</b> for <math>x = \frac{11\pi}{12}</math> <b>A1</b> for <math>x = \frac{23\pi}{12}</math> If there are extra solutions in the range in addition to the two correct ones then <b>A1A0</b></p>



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