



# Cambridge O Level

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**ADDITIONAL MATHEMATICS**

**4037/01**

Paper 1

**For examination from 2020**

MARK SCHEME

Maximum Mark: 80

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**Specimen**

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This document has **12** pages. Blank pages are indicated.

### Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

<p>GENERIC MARKING PRINCIPLE 1:</p> <p>Marks must be awarded in line with:</p> <ul style="list-style-type: none"> <li>the specific content of the mark scheme or the generic level descriptors for the question</li> <li>the specific skills defined in the mark scheme or in the generic level descriptors for the question</li> <li>the standard of response required by a candidate as exemplified by the standardisation scripts.</li> </ul>
<p>GENERIC MARKING PRINCIPLE 2:</p> <p>Marks awarded are always <b>whole marks</b> (not half marks, or other fractions).</p>
<p>GENERIC MARKING PRINCIPLE 3:</p> <p>Marks must be awarded <b>positively</b>:</p> <ul style="list-style-type: none"> <li>marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate</li> <li>marks are awarded when candidates clearly demonstrate what they know and can do</li> <li>marks are not deducted for errors</li> <li>marks are not deducted for omissions</li> <li>answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.</li> </ul>
<p>GENERIC MARKING PRINCIPLE 4:</p> <p>Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.</p>
<p>GENERIC MARKING PRINCIPLE 5:</p> <p>Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).</p>

**GENERIC MARKING PRINCIPLE 6:**

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

**MARK SCHEME NOTES**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

**Types of mark**

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, given for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the **M** marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several **B** marks allocated. The notation ‘dep’ is used to indicate that a particular **M** or **B** mark is dependent on an earlier mark in the scheme.

**Abbreviations**

<b>AG</b>	answer given
<b>awrt</b>	answer which rounds to
<b>cao</b>	correct answer only
<b>dep</b>	dependent
<b>FT</b>	follow through after error
<b>isw</b>	ignore subsequent working
<b>nfww</b>	not from wrong working
<b>oe</b>	or equivalent
<b>rot</b>	rounded or truncated
<b>SC</b>	special case
<b>soi</b>	seen or implied



Question	Answer	Marks	Partial Marks
1(a)	$2(2)^3 - 3(2)^2 + 2q + 56 = 0$ with one correct interim step leading to $q = -30$	<b>1</b>	For convincingly showing $2(2)^3 - 3(2)^2 - 30(2) + 56 = 0$ or correct synthetic division at least as far as $\begin{array}{r rrrr} 2 & -3 & q & 56 & \\ & 4 & 2 & 2q & +4 \\ \hline & 2 & 1 & q+2 & 0 \end{array}$ then $q = -30$ or correct long division to, e.g. verify $-30$ , at least as far as: $\begin{array}{r} 2x^2 + x - 28 \\ x - 2 \overline{) 2x^3 - 3x^2 - 30x + 56} \\ \underline{2x^3 - 4x^2} \phantom{+ 56} \\ \phantom{2x^3 - 4x^2} x^2 - 30x \phantom{+ 56} \\ \phantom{2x^3 - 4x^2} \underline{x^2 - 2x} \phantom{+ 56} \\ \phantom{2x^3 - 4x^2} \phantom{x^2 - 30x} - 28x + 56 \\ \phantom{2x^3 - 4x^2} \phantom{x^2 - 30x} \underline{-28x + 56} \\ \phantom{2x^3 - 4x^2} \phantom{x^2 - 30x} \phantom{-28x + 56} 0 \end{array}$
1(b)	$2x^2 + x - 28$ oe $(x - 2)(2x - 7)(x + 4)$ oe $x = 2, x = -4, x = 3.5$ oe	<b>B2</b>  <b>M1</b>  <b>A1</b>	For any two terms correct For factorising the correct polynomial Answer only scores 0.

Question	Answer	Marks	Partial Marks
2(a)	$\left[\frac{dy}{dx} = \right] \frac{3}{2}\sqrt{x}$ oe	<b>B2</b>	Allow unsimplified, e.g. $\left[\frac{dy}{dx} = \right] x\left(\frac{1}{2}x^{-\frac{1}{2}}\right) + x^{\frac{1}{2}}$ from product rule <b>B1</b> for $y = x^{\frac{3}{2}}$ or for one correct term in the sum obtained using the product rule
2(b)	$[y = 8]$ $x = 4$	<b>B1</b>	
	$\frac{0.015}{\delta x} \approx \left(\textit{their} \frac{dy}{dx}\right)_{x=4}$ oe	<b>M1</b>	Condone $\frac{0.015}{\delta x} = \left(\textit{their} \frac{dy}{dx}\right)_{x=4}$
	0.005 oe nfvw	<b>A1</b>	

Question	Answer	Marks	Partial Marks
3(a)	$12\left(x - \frac{1}{4}\right)^2 + \frac{17}{4}$ isw	<b>3</b>	<b>B1</b> for each of $p, q, r$ correct; Allow correct equivalent values If 0 scored, <b>SC2</b> for $12\left(x - \frac{1}{4}\right) + \frac{17}{4}$ or <b>SC1</b> for correct 3 values but incorrect format
3(b)	$\frac{4}{17}$ is greatest value when $x = \frac{1}{4}$	<b>2</b>	Strict <b>FT</b> <i>their</i> $\frac{17}{4}$ and <i>their</i> $\frac{1}{4}$ <b>B1</b> for $\frac{4}{17}$ and <b>B1</b> for $x = \frac{1}{4}$ Each value must be correctly attributed; Condone $\left(\frac{1}{4}, \frac{4}{17}\right)$ for <b>B2</b> Condone $y = \frac{4}{17}$ for <b>B1</b>

Question	Answer	Marks	Partial Marks
4	$AX = \sqrt{45}$ soi	<b>B1</b>	May be implied by $3\sqrt{5}$
	$AX = 3\sqrt{5}$	<b>B1</b>	May be seen later
	$\frac{1}{2}(4 + \sqrt{5} + 2 + x) \times \text{their } \sqrt{45}$	<b>M1</b>	May be implied by, e.g. summation of rectangle and two triangles
	$15(\sqrt{5} + 2) =$ $\frac{1}{2}(4 + \sqrt{5} + 2 + x) \times \text{their } \sqrt{45}$ or better	<b>M1</b>	Must be correct apart from <i>their</i> $\sqrt{45}$
	Correctly divide <i>their</i> equation by <i>their</i> $\sqrt{5}$ or <i>their</i> $\sqrt{45}$ and rationalise denominator	<b>M1</b>	or correctly multiply both sides of <i>their</i> equation by <i>their</i> $\sqrt{5}$ or <i>their</i> $\sqrt{45}$ and obtain a rational coefficient of $x$ soi
	Completion to $4 + 3\sqrt{5}$ nfw	<b>A1</b>	

Question	Answer	Marks	Partial Marks
5(a)	Correct shape for both graphs	<b>B2</b>	<b>B1</b> for either Must touch the $x$ -axis in the correct quadrant
	Correct $y$ -intercept for both graphs and Correct $x$ -intercept for both graphs	<b>B2</b>	<b>B1</b> for either $y = 2$ , $y = 5$ or $x = 2$ , $x = -2.5$ or $y = 2$ , $x = -2.5$
5(b)	$2x + 5 = \pm(2 - x)$ oe	<b>M1</b>	For attempt to obtain 2 solutions; must be a complete method
	or $(2x + 5)^2 = (2 - x)^2$	<b>A1</b>	
	$x = -7$ , $x = -1$ $-7 \leq x \leq -1$	<b>A1</b>	<b>FT</b> <i>their</i> values of $x$

Question	Answer	Marks	Partial Marks
6	$\frac{1}{2}(2x)(x^2 + 5)^{-\frac{1}{2}}$ for quotient seen or $-\frac{1}{2}(2x)(x^2 + 5)^{-\frac{3}{2}}$ for product seen  Correct application of the quotient rule or product rule attempted  $\frac{dy}{dx} = \frac{2(x^2 + 5)^{\frac{1}{2}} - \frac{1}{2}(2x)(x^2 + 5)^{-\frac{1}{2}}(2x - 1)}{x^2 + 5}$ or $\frac{dy}{dx} = 2(x^2 + 5)^{-\frac{1}{2}} - \frac{1}{2}(2x)(x^2 + 5)^{-\frac{3}{2}}(2x - 1)$  $x = 2, y = 1$ $\frac{dy}{dx} = \frac{4}{9}$	<b>B1</b>	
		<b>M1</b>	
		<b>A1</b>	All correct; may be unsimplified
		<b>B2</b>	<b>B1</b> for each
	Gradient of normal = $-\frac{9}{4}$	<b>M1</b>	Valid attempt to obtain gradient of normal
	$y - 1 = \textit{their} \left(-\frac{9}{4}\right)(x - 2)$ oe	<b>M1</b>	
	$9x + 4y = 22$	<b>A1</b>	

Question	Answer	Marks	Partial Marks	
7(a)	$20 = \pi x^2 + xy$	<b>B1</b>		
	$y = \frac{20 - \pi x^2}{x}$	<b>B1</b>		
	$P = 2\pi x + 2x + 2y$ $= 2\pi x + 2x + 2\left(\frac{20}{x} - \pi x\right)$	<b>M1</b>	Attempt to use perimeter and obtain in terms of $x$ only	
	$= 2x + \frac{40}{x}$	<b>A1</b>	For all steps seen, nfwv AG	
	<b>Alternative</b> $20 = \pi x^2 + xy$ and $P = 2\pi x + 2x + 2y$	<b>B1</b>		
7(a)	$P = \frac{2}{x}(\pi x^2 + xy) + 2x$	<b>M1</b>	For attempt to use perimeter and write in $\frac{\pi x^2 + xy}{x}$	
	$= \frac{2}{x}(20) + 2x$	<b>B1</b>	For replacing $\pi x^2 + xy$ with 20	
	$= 2x + \frac{40}{x}$	<b>A1</b>	For all steps seen, nfwv AG	
	7(b)	$\frac{dP}{dx} = 2 - \frac{40}{x^2}$ When $\frac{dP}{dx} = 0$ $x = 2\sqrt{5}$ or 4.47 or $\sqrt{20}$ Leading to $P = 8\sqrt{5}$ or 17.9	<b>5</b>	<b>M1</b> for attempt to differentiate <b>DM1</b> for equating to zero and attempt to solve at least as far as $x^2 = \dots$ <b>A1</b> for $x$ <b>A1</b> for $P$ <b>A1</b> for this statement or use of gradient inspection either side of correct $x$
		$\frac{d^2P}{dx^2} = \frac{80}{x^3}$ Always positive so a minimum		
oe				



Question	Answer	Marks	Partial Marks
8(a)(i)	$ke^{5x-1}$	M1	
	$k = \frac{1}{5}$ oe	A1	
	$\frac{1}{5}(e^{5(1)-1} - e^{5(0.2)-1})$ or better	M1	
	$\frac{1}{5}(e^4 - 1)$ or $\frac{1}{5}e^4 - \frac{1}{5}$	A1	Answer only scores 0/4.
8(a)(ii)	Expands correctly	B1	May be unsimplified
	$\left[ \frac{1}{x} dx = \ln x \right]$ soi	B1	
	$\left[ \frac{x^3}{3} + 2 \text{ their } \ln x - \frac{x^{-3}}{3} \right]$	M1	Integration of their 3-term or, if unsimplified, 4-term expression
	$\left[ \frac{2^3}{3} + 2 \text{ their } \ln 2 - \frac{2^{-3}}{3} \right] - \left[ \frac{1^3}{3} + 2 \text{ their } \ln 1 - \frac{1^{-3}}{3} \right]$	M1	Condone omission of lower limit
	$2 \ln 2 + \frac{21}{8}$ oe	A1	Any equivalent answer that is simplified to two terms Answer only scores 0/5.
8(b)	$k \cos\left(\frac{x}{6}\right) (+c), k < 0$	M1	
	$-6 \cos\left(\frac{x}{6}\right) + c$	A1	

Question	Answer	Marks	Partial Marks
9	$\frac{n(n-1)(n-2)(n-3)(2^4)}{4 \times 3 \times 2 \times 1}$ $= 10 \frac{n(n-1)(2^2)}{2 \times 1} \text{ or better}$	<b>M3</b>	Condone omitting the factor of $n$ and/or $n-1$ ; must have dealt with factorials <b>M2</b> if one slip/omission or <b>M1</b> if two slips/omissions or <b>B1</b> for $\frac{n(n-1)}{2}(2)^2[x^2]$ seen and <b>B1</b> for $\frac{n(n-1)(n-2)(n-3)}{24}(2)^4[x^4]$ seen
	$n^2 - 5n - 24 [=0] \text{ oe}$ $(n+3)(n-8) [=0]$	<b>A1</b>	Equivalent must be 3 terms, e.g. $n^2 - 5n = 24$
	$n = 8$ only	<b>M1</b>	or any valid method of solution for <i>their</i> 3-term quadratic
		<b>A1</b>	<b>A0</b> if $-3$ also given as a final solution, i.e. not discarded

Question	Answer	Marks	Partial Marks
10(a)	$-200 > \frac{n}{2}(-10 + (n-1)(-3))$ leading to $3n^2 - 13n - 400 (> 0)$ $n = 13.9\dots$ so 14th term needed	<b>4</b>	<b>M1</b> for attempt to use sum to $n$ terms, allow use of $=$ or $\leq$ or $<$ <b>A1</b> for correct quadratic expression <b>DM1</b> for attempt to solve <b>A1</b> for correct conclusion
10(b)(i)	$ar^2 = \frac{81}{64}$ $ar^4 = \frac{729}{1024}$ $r^2 = \frac{9}{16}$ $r = \frac{3}{4}$ $a = \frac{9}{4}$	<b>5</b>	<b>B1</b> for 3rd term <b>B1</b> for 5th term <b>M1</b> for attempt to solve their equations to obtain either $r$ or $a$ <b>A1</b> for $r$ <b>A1</b> for $a$
10(b)(ii)	$S_\infty = 9$	<b>B1</b>	<b>FT</b> on <i>their</i> $a$ and $r$ , provided $ r  < 1$

Question	Answer	Marks	Partial Marks (Separate areas subtracted)
11	<b>Method 1</b>		
	$[x_B = x_C =]5 \text{ soi}$	<b>B1</b>	
	$\left[ \int (x^2 - 4x + 10) dx = \right]$ $\frac{x^3}{3} - \frac{4x^2}{2} + 10x$	<b>M2</b>	or <b>M1</b> for at least one term correct
	Correct or correct <b>FT</b> substitution of limits 0 and <i>their</i> 5 into <i>their</i> $\left[ \frac{x^3}{3} - \frac{4x^2}{2} + 10x \right]$	<b>M1</b>	dep on at least <b>M1</b> being earned; Condone + c as long as <i>their</i> c is <b>not</b> numerical
	$\frac{1}{2} (10 + 15) \times 5 \text{ oe}$ or $\int_0^5 (x + 10) dx = \left[ \frac{x^2}{2} + 10x \right]_0^5$ $= \frac{(5)^2}{2} + 10(5) \text{ oe}$	<b>B2</b>	or <b>M1</b> for $\frac{1}{2}(\text{their } 10 + \text{their } 15) \times \text{their } 5 \text{ oe}$ or <b>B1</b> for $\int (x + 10) dx = \frac{x^2}{2} + 10x$
	<i>their</i> $\left( \frac{125}{2} - \frac{125}{3} \right)$	<b>M1</b>	
$\frac{125}{6}$ or $20\frac{5}{6}$	<b>A1</b>	<b>Answer only scores 0/8.</b>	

Question	Answer	Marks	Partial Marks (Subtracting and using integration once)
11	<b>Method 2</b>		
	$[x_B = x_C = ]5$ soi	<b>B1</b>	
	$\int (-x^2 + 5x) dx$	<b>B1</b>	Condone omission of dx
	$\left[ -\frac{x^3}{3} + \frac{5x^2}{2} \right]$ oe or $\left[ \frac{x^3}{3} - \frac{5x^2}{2} \right]$ oe	<b>M3</b>	or <b>M2</b> for $\int (px^2 + qx) dx = \frac{px^3}{3} + \frac{qx^2}{2}$ oe either with $p = \pm 1$ or $q = \pm 5$ or <b>M1</b> for $\int (px^2 + qx) dx = \frac{px^3}{3} + \frac{qx^2}{2}$ with non-zero constants $p$ and $q$ , with $p \neq \pm 1$ and $q \neq \pm 5$
	Correct or correct <b>FT</b> substitution of limits 0 and <i>their</i> 5 into <i>their</i> $\left[ -\frac{x^3}{3} + \frac{5x^2}{2} \right]$	<b>M2</b>	Condone omission of lower limit
$\frac{125}{6}$ or $20\frac{5}{6}$	<b>A1</b>	<b>Answer only scores 0/8.</b>	