

Cambridge Assessment International Education Cambridge Ordinary Level

	CANDIDATE NAME			
	CENTRE NUMBER		CANDIDATE NUMBER	
× л л л л л л л о о о		MATHEMATICS		4037/13
	Paper 1		Oc	tober/November 2019
0				2 hours
ν 	Candidates ans	swer on the Question Paper.		
	No Additional M	laterials are required.		

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READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs. Do not use staples, paper clips, glue or correction fluid. DO NOT WRITE IN ANY BARCODES.

Answer all the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question. The use of an electronic calculator is expected, where appropriate. You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together. The number of marks is given in brackets [] at the end of each question or part question. The total number of marks for this paper is 80.

This document consists of **15** printed pages and **1** blank page.



Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \; .$

Binomial Theorem

where *n*

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\csc^2 A = 1 + \cot^2 A$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

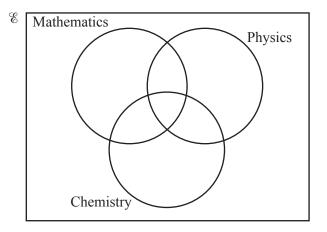
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WWW. MYMEHSCIOUU.com 1 In a group of 145 students, the numbers studying mathematics, physics and chemistry are given below. A students study at least one of the three subjects.

x students study all 3 subjects

24 students study both mathematics and chemistry

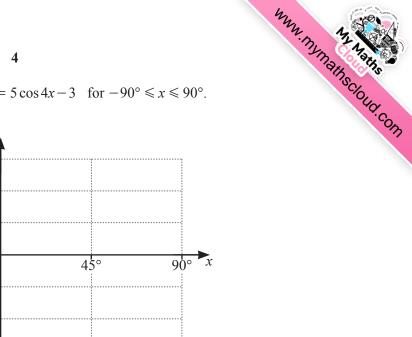
- 23 students study both physics and chemistry
- 28 students study both mathematics and physics
- 50 students study chemistry
- 75 students study physics
- 80 students study mathematics
- (i) Using the Venn diagram, find the value of x.



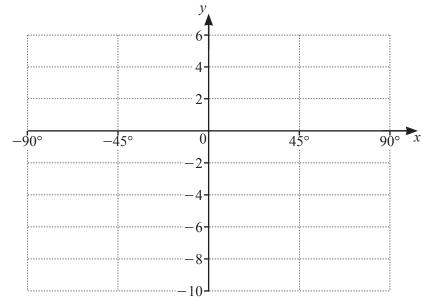
(ii) Find the number of students who study mathematics only.

3

[4]



2 (i) On the axes below, sketch the graph of $y = 5\cos 4x - 3$ for $-90^\circ \le x \le 90^\circ$.



(ii) Write down the amplitude of *y*.

(iii) Write down the period of y.

[1]

[1]



3 (i) Differentiate $y = (3x^2 - 1)^{-\frac{1}{3}}$ with respect to x.

(ii) Find the approximate change in y as x increases from $\sqrt{3}$ to $\sqrt{3} + p$, where p is small. [1]

5

(iii) Find the equation of the normal to the curve $y = (3x^2 - 1)^{-\frac{1}{3}}$ at the point where $x = \sqrt{3}$. [3]



4 It is given that
$$\mathbf{A} = \begin{pmatrix} 5 & 2 \\ 4 & -1 \end{pmatrix}$$
.
(i) Find \mathbf{A}^{-1} .

(ii) Hence find, in radians, the acute angles x and y such that

6

$$5 \tan x + 2 \tan y = 12,$$

 $4 \tan x - \tan y = 7.$ [5]



5 (i) Differentiate $(x^2+3)\ln(x^2+3)$ with respect to x.

7

(ii) Hence find $\int x \ln(x^2 + 3) dx$.

[2]

x	1	1.5	2	2.5	3
У	6	14.3	48	228	1536

The table shows values of the variables x and y such that $y = Ab^{x^2}$, where A and b are constants.

(i) Draw a straight line graph to show that $y = Ab^{x^2}$.

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(ii) Use your graph to find the value of A and of b.

(iii) Estimate the value of x when y = 100.

[2]

- www.mymathscloud.com 7 (a) A 5-digit code is to be chosen from the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9. Each digit may be use once only in any 5-digit code. Find the number of different 5-digit codes that may be chosen if
 - (i) there are no restrictions,
 - (ii) the code is divisible by 5,

(iii) the code is even and greater than 70000.

(b) A team of 6 people is to be chosen from 8 men and 6 women. Find the number of different teams that may be chosen if

(i) there are no restrictions,	[1]
--------------------------------	-----

- (ii) there are no women in the team, [1]
- (iii) there are a husband and wife who must not be separated. [3]

[1]

[3]

[1]



[2]

[1]

- 8 (a) Given that $\log_a x = p$ and $\log_a y = q$, find, in terms of p and q,
 - (i) $\log_a axy^2$,

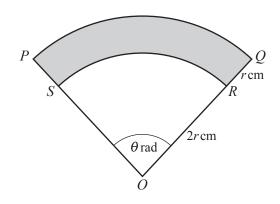
(ii)
$$\log_a\left(\frac{x^3}{ay}\right)$$
,

(iii) $\log_x a + \log_y a$.

(b) Using the substitution $m = 3^x$, or otherwise, solve $3^x - 3^{1+2x} + 4 = 0$. [3]

12

9



The diagram shows a sector *OPQ* of the circle centre *O*, radius 3r cm. The points *S* and *R* lie on *OP* and *OQ* respectively such that *ORS* is a sector of the circle centre *O*, radius 2r cm. The angle $POQ = \theta$ radians. The perimeter of the shaded region *PQRS* is 100 cm.

(i) Find θ in terms of r.

[2]

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(ii) Hence show that the area, $A \text{ cm}^2$, of the shaded region *PQRS* is given by $A = 50r - r^2$. [2]



(iv) Given that A is increasing at the rate of $3 \text{ cm}^2 \text{s}^{-1}$ when r = 10, find the corresponding rate of change of r. [3]

13

(v) Find the corresponding rate of change of θ when r = 10.

[3]

14

[2]

The velocity-time graph for a particle P is shown by the two straight lines in the diagram.

(i) Find the deceleration of *P* for $5 \le t \le 10$.

 $v\,{\rm ms}^{-1}$

20

0

 -20^{-10}

(ii) Write down the value of t when the speed of P is zero. [1]

(iii) Find the distance *P* has travelled for $0 \le t \le 10$. [2]

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10 (a)

- www.mymathscloud.com (b) A particle Q has a displacement of x m from a fixed point O, ts after leaving O. The velocity, $v \text{ ms}^{-1}$, o Q at time ts is given by $v = 6e^{2t} + 1$.
 - (i) Find an expression for x in terms of t.

[3]

[3]

(ii) Find the value of t when the acceleration of Q is 24 ms^{-2} .



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