Cambridge O Level

Cambridge Assessment International Education
Cambridge Ordinary Level

ADDITIONAL MATHEMATICS **4037/13**

Paper 1 **October/November 2017**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1	Using $\tan^2\theta+1=\sec^2\theta$ to obtain $y=2(\tan^2\theta+1)$ or $(x+5)^2=\sec^2\theta-1$ $(x+5)^2+1=\frac{y}{2}$	**M1**	use of correct identity
	$y=2((x+5)^2+1)$ oe	**A1**	
2	$\frac{dy}{dx}=10e^{5x}+3$ an attempt at integration in form $ae^{5x}+bx$	**M1**	
	$y=\frac{10}{5}e^{5x}+3x\ \ (+c)$	**A1**	condone omission of c
	attempt to find c using $x=0, y=9$	**M1**	**M1dep**
	$y=2e^{5x}+3x+7$	**A1**	
3	$9<4k(k-4)$ $4k^2-16k-9$	**M1**	use of the discriminant with correct values
	$(2k-9)(2k+1)$	**M1**	**M1dep** for solution of *their* quadratic to obtain critical values
	Critical values $\frac{9}{2},\ \ -\frac{1}{2}$	**A1**	
	$k<-\frac{1}{2},\ \ k>\frac{9}{2}$	**A1**	
4	$a=3$	**B1**	
	$b=8$	**B1**	
	$\frac{5}{2}=3\cos\left(8\times\frac{\pi}{12}\right)+c$	**M1**	substitution of $x=\frac{\pi}{12}$ and $y=\frac{5}{2}$ to find c
	$c=4$	**A1**	
5(i)	$\frac{5}{14}(7x-10)^{\frac{2}{5}}$	**B2**	**B1** for $k(7x-10)^{\frac{2}{5}}$

Question	Answer	Marks	Guidance
5(ii)	$\frac{5}{14}\left[(7x-10)^{\frac{2}{5}}\right]_6^a = \frac{25}{14}$ $\frac{5}{14}(7a-10)^{\frac{2}{5}} - \frac{5}{14}(7\times 6-10)^{\frac{2}{5}} = \frac{25}{14}$ $(7a-10)^{\frac{2}{5}} - 4 = 5$	M1	correct application of limits for $k(7x-10)^{\frac{2}{5}}$
	$a = \frac{9^{\frac{5}{2}}+10}{7}$	M1	**M1dep** for evaluation of $(7\times 6-10)^{\frac{2}{5}}$ and correct order of operations to find *a*, including dealing with power.
	$a = \frac{253}{7}$ or $36\frac{1}{7}$	A1	
6(i)	Gradient $= \frac{2.4-0.9}{0.2-0.8}$ $(=-2.5)$	B1	
	$\ln y = -\frac{5}{2}x^2 + c$	M1	straight line form and correct substitutions to find *c*
	$\ln y = -\frac{5}{2}x^2 + 2.9$ oe	A1	
	<u>Alternative method</u> $2.4 = p(0.2)+q$ $0.9 = p(0.8)+q$	B1	
	Correct method of solution to find *p* and *q* from two correct equations	M1	**M1dep**
	$\ln y = -\frac{5}{2}x^2 + 2.9$	A1	
6(ii)	$y = e^{\left(-\frac{5}{2}x^2+2.9\right)}$	M1	dealing with ln
	$y = e^{-\frac{5}{2}x^2} \times e^{2.9}$	M1	**M1dep** for dealing with the index
	$y = 18.2z^{-\frac{5}{2}}$	A1	

Question	Answer	Marks	Guidance
7(i)	$64-48x^2+15x^4$	**B3**	**B1** for each correct term in final line of response
7(ii)	$\left(64-48x^2+15x^4\right)\left(\frac{1}{x^2}+2+x^2\right)$	**B1**	**B1**for $\frac{1}{x^2}+2+x^2$ oe
	at least two correctly obtained products leading to terms in x^2	**M1**	
	Term in x^2 : $64+15-96$	**A1**	**FT** for correct evaluation of *their* 64 + (2 × *their*–48) + *their* 15
	$=-17$	**A1**	
8(i)	attempt to differentiate a product	**M1**	
	$\frac{dy}{dx}=\left((x-4)\times\frac{5}{3}\times3(3x-1)^{\frac{2}{3}}\right)+(3x-1)^{\frac{5}{3}}$	**A2**	**A1** for $(+)\left((x-4)\times\frac{5}{3}\times3(3x-1)^{\frac{2}{3}}\right)$ **A1** for $(+)(3x-1)^{\frac{5}{3}}$
	$=(3x-1)^{\frac{2}{3}}\left((5x-20)+(3x-1)\right)$	**M1**	use of $(3x-1)^{\frac{5}{3}}=(3x-1)^{\frac{2}{3}}(3x-1)$
	$=(3x-1)^{\frac{2}{3}}(8x-21)$	**A1**	
8(ii)	When $x=3$, $\frac{dy}{dx}=8^{\frac{2}{3}}\times3$	**M1**	$(3\times3-1)^{\frac{2}{3}}\times k$ or $(9-1)^{\frac{2}{3}}\times k$ or $4\times k$ (where k is any number)
	$\partial y=8^{\frac{2}{3}}\times3\times h$	**M1**	**M1dep** for *their* $\left((9-1)^{\frac{2}{3}}\times k\right)\times h$
	$\partial y=12h$	**A1**	
9(a)(i)	720	**B1**	
9(a)(ii)	240	**B1**	
9(a)(iii)	$k\times4!\ \times2$ or $240-k\times4!\ \times2$ or correct equivalents with no extra terms added or subtracted	**B1**	
	$4\times4!\ \times p$ or correct equivalents with no extra terms added or subtracted	**B1**	
	192	**B1**	

Question	Answer	Marks	Guidance
9(b)(i)	6435	**B1**	
9(b)(ii)	With twins: ${}^{13}C_6$ or 1716 Without twins: ${}^{13}C_8$ or 1287	**B2**	**B1** for ${}^{13}C_6$ or 1716 or ${}^{13}C_8$ or 1287 **B1** for (${}^{13}C_6$ and ${}^{13}C_8$) or (1716 and 1287) with no multiples and no extra terms
	Total: 1716 + 1287 = 3003	**B1**	3003 from a correct method
10(a)	matrix multiplication, must have at least 2 correct elements	**M1**	
	$\mathbf{AB} = \begin{pmatrix} 13 & 8 \\ 2a-5b & 3a+4b \end{pmatrix}$	**A1**	
	$2a-5b=18$ $3a+4b=4$	**M1**	formation and solution of simultaneous equations
	leading to $a=4,\ b=-2$	**A1**	
	<u>Alternate scheme</u> $\mathbf{AB} = \begin{pmatrix} 13 & 8 \\ 18 & 4 \end{pmatrix}$ $\mathbf{ABB}^{-1} = \begin{pmatrix} 13 & 8 \\ 18 & 4 \end{pmatrix} \mathbf{B}^{-1}$	**M1**	Correct plan
	Correct inverse	**B1**	
	$\mathbf{A} = \begin{pmatrix} 4 & -1 \\ a & b \end{pmatrix} = \frac{1}{23}\begin{pmatrix} 13 & 8 \\ 18 & 4 \end{pmatrix}\begin{pmatrix} 4 & -3 \\ 5 & 2 \end{pmatrix}$	**M1**	Correct order and method of multiplication with at least two correct elements
	leading to $a=4,\ b=-2$	**A1**	
10(b)(i)	$-\frac{1}{17}\begin{pmatrix} 1 & 5 \\ 4 & 3 \end{pmatrix}$ oe	**B2**	**B1** for $-\frac{1}{17}$ **B1** for $\begin{pmatrix} 1 & 5 \\ 4 & 3 \end{pmatrix}$
10(b)(ii)	$\mathbf{Z} = -\frac{1}{17}\begin{pmatrix} 1 & 5 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} -1 & 2 \\ 4 & 0 \end{pmatrix}$	**M1**	pre-multiplication with two elements correct
	$= -\frac{1}{17}\begin{pmatrix} 19 & 2 \\ 8 & 8 \end{pmatrix}$ oe	**A2**	**A1** for four correct of $-\frac{1}{17}$, 19, 2, 8, 8

Question	Answer	Marks	Guidance
11(i)	1.48	**B1**	
11(ii)	$\frac{1}{2}\times 10^2 \times \theta = 21.8$	**M1**	correct use of sector area
	$\theta = 0.436$	**A1**	
11(iii)	$\angle BOC = \frac{2\pi - 1.48 - 0.436}{2} \quad (= 2.18(4))$	**B1**	2.18(4) or unsimplified
	$BC = 20\sin\left(\frac{1}{2}\angle BOC\right)$ or $BC = \frac{10\times \sin BOC}{\sin\left(\frac{\pi - BOC}{2}\right)}$ or $BC = \sqrt{(200 - 200\cos BOC}$ $BC = 17.7(5)$	**M2**	**M1** for a complete correct method to find *BC* using *their* angle *BOC* **M1** for a correct plan using 14.8, *their BC* and 10 × *their* answer to (ii)
	Perimeter = $14.8 + (2\times 17.7(5)) + 4.36$ $= 54.7$ or 54.6	**A1**	awrt 54.7 or awrt 54.6

Question	Answer	Marks	Guidance
11(iv)	Area = $\left(\frac{1}{2}\times 10^2\times 1.48\right)+21.8+2\left(\frac{1}{2}\times 10^2\sin 2.18(4)\right)$	**B2**	**B1** for $\left(\frac{1}{2}\times 10^2\times 1.48\right)+21.8$ **B1** for $2\left(\frac{1}{2}\times 10^2\sin 2.18(4)\right)$
	= 178	**B1**	awrt 178 from correct working
	<u>Alternative method 1</u> Segment area $=\frac{1}{2}\left(10^2\left(2.18-\sin 2.18\right)\right)$	**B1**	**B1** for $2\times\frac{1}{2}\left(10^2\left(2.18(4)-\sin 2.18(4)\right)\right)$
	Area required = $100\pi-2\times\frac{1}{2}\left(10^2\left(2.18(4)-\sin 2.18(4)\right)\right)$	**B1**	
	= 178	**B1**	awrt 178 from correct working
	<u>Alternative method 2</u> Area of trapezium = $\frac{1}{2}\left((13.5+4.33)(17.1)\right)$	**B1**	correct area of trapezium *ABCD* (allow unsimplified)
	Area of segments = $\frac{1}{2}\left(10^2\left(1.48-\sin 1.48\right)\right)$ + $\frac{1}{2}\left(10^2\left(0.436-\sin 0.436\right)\right)$	**B1**	correct area of both segments (allow unsimplified)
	= 178	**B1**	awrt 178 from correct working

Question	Answer	Marks	Guidance
12(i)	$2x^2+5x-12=0$ or $y^2+3y-28=0$	M1	attempt to get in terms of one variable
	$(2x-3)(x+4)=0$ or $(y+7)(y-4)=0$	M1	**M1dep** for solution of a three term quadratic
	leading to $x=-4,\ y=-7$ and $x=\frac{3}{2},\ y=4$	A2	**A1** for each 'pair'
	Midpoint $M\left(\frac{\frac{3}{2}-4}{2},\frac{4+(-7)}{2}\right)\left(=\left(-\frac{5}{4},-\frac{3}{2}\right)\right)$	A1	correctly obtained midpoint
	Gradient of $PQ=2$	B1	may be implied
	Perp gradient $=-\frac{1}{2}$	M1	$\frac{-1}{\textit{their}\text{ gradient of }PQ}$
	Perp bisector: $y+\frac{3}{2}=-\frac{1}{2}\left(x+\frac{5}{4}\right)$	M1	**M1dep** for equation of perp bisector using *their* perp gradient and *their* midpoint. (unsimplified)
	$y=-\frac{1}{2}(-10)-\frac{17}{8}=\frac{23}{8}$ or $\frac{23}{8}=-\frac{1}{2}x-\frac{17}{8}\to x=-10$	A1	all correct so far and for verification using a correct equation

Question	Answer	Marks	Guidance
12(ii)	Area $= \frac{1}{2} \times \left(\frac{17}{8} + 1\right) \times \frac{5}{4}$	M1	finding R, S and RS
	correct method for finding area	M1	**M1dep**
	$= \frac{125}{64}$ or 1.95 or $1\frac{61}{64}$	A1	
	<u>Alternative method 1</u> Area $= \frac{1}{2} \times \frac{\sqrt{125}}{4} \times \frac{\sqrt{125}}{8}$	M1	finding R, S, RM and MS
	correct method for finding area	M1	**M1dep**
	$= \frac{125}{64}$ or 1.95 or $1\frac{61}{64}$	A1	
	<u>Alternative method 2</u> Area $= \frac{1}{2}\begin{vmatrix} 0 & 0 & \frac{-5}{4} & 0 \\ 1 & \frac{-17}{8} & \frac{-3}{2} & 1 \end{vmatrix}$	M1	finding R and S to obtain *their* $\frac{1}{2}\begin{vmatrix} 0 & 0 & \frac{-5}{4} & 0 \\ 1 & \frac{-17}{8} & \frac{-3}{2} & 1 \end{vmatrix}$
	$= \frac{1}{2}\left\|-\frac{5}{4} - \frac{85}{32}\right\|$ oe	M1	**M1dep** for correct method of evaluation
	$= \frac{125}{64}$ or 1.95 or $1\frac{61}{64}$	A1	