Cambridge O Level

Cambridge Assessment International Education
Cambridge Ordinary Level

ADDITIONAL MATHEMATICS **4037/12**

Paper 1 **October/November 2017**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

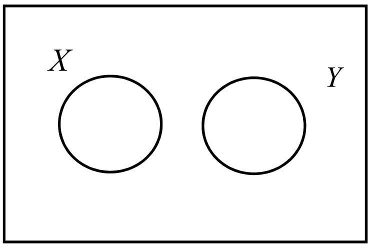
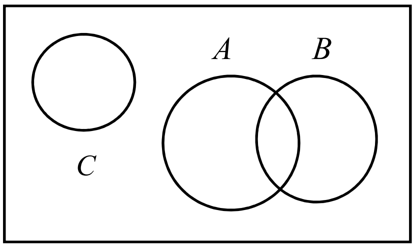
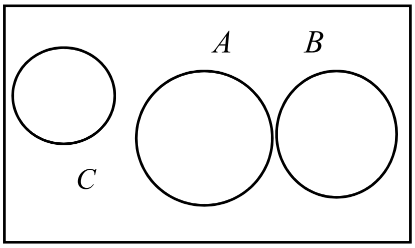
A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

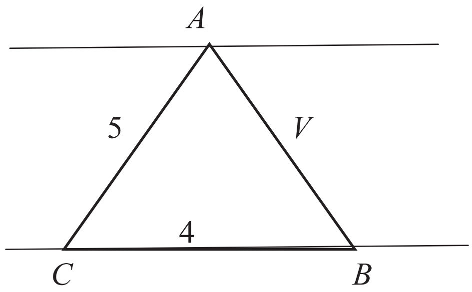
Abbreviations

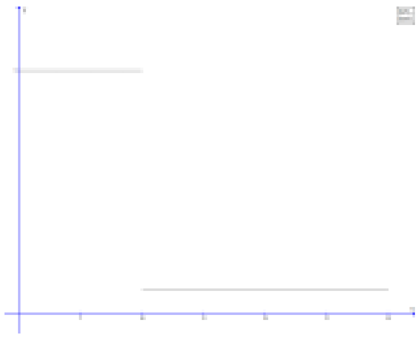
awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1(i)		1	
1(ii)	Either  Or 	2	**B1** for C with no intersection with either A or B (allow if C is not represented by a circle) **B1** for all correct, C must be represented by a circle
2	$a = 4$	**B1**	
	$b = 6$	**B1**	
	$c = -2$	**M1, A1**	**M1** for use of $\left(\frac{\pi}{12}, 2\right)$ to obtain c, using *their* values of a and of b
3(i)	$32 - 20x^2 + 5x^4$	**B3**	**B1** for each correct term
3(ii)	$\left(32 - 20x^2 + 5x^4\right)\left(\frac{1}{x^2} \ldots\ldots\ldots\ldots + \frac{9}{x^4}\right)$	**B1**	$\frac{1}{x^2}$ and $\frac{9}{x^4}$
	Independent of x: $-20 + 45$	**M1**	attempt to deal with 2 terms independent of x, must be looking at terms in x^2 and $\frac{1}{x^2}$ and terms in x^4 and $\frac{1}{x^4}$
	$= 25$	**A1**	**FT** *their* answers from (i) (*their* -20×1) + (*their* 5×9)

Question	Answer	Marks	Guidance
4	correct differentiation of $\ln(3x^2+2)$	B1	
	attempt to differentiate a quotient or a product	M1	
	$\frac{dy}{dx} = \frac{(x^2+1)\left(\frac{6x}{3x^2+2}\right) - 2x\ln(3x^2+2)}{(x^2+1)^2}$	A1	all other terms correct.
	When $x = 2$, $\frac{dy}{dx} = \frac{5\left(\frac{12}{14}\right) - 4\ln 14}{25}$	M1	**M1dep** for substitution and attempt to simplify
	$= \frac{6}{35} - \frac{4}{25}\ln 14$	A2	**A1** for each correct term, must be in simplest form
5(i)	**Either** Gradient $= -0.2$	B1	
	$\lg y = -0.2x + c$	B1	$\lg y = mx + c$ soi
	correct attempt to find c	M1	must have previous B1
	$\lg y = 0.42 - 0.2x$ or $\lg y = \frac{21}{50} - \frac{x}{5}$	A1	line in either form, allow equivalent fractions
	Or $0.3 = 0.6m + c$	B1	
	$0.2 = 1.1m + c$	B1	
	attempt to solve for both m and c	M1	must have at least one of the previous B marks
	Leading to $\lg y = 0.42 - 0.2x$ or $\lg y = \frac{21}{50} - \frac{x}{5}$	A1	line in either form, allow equivalent fractions

Question	Answer	Marks	Guidance
5(ii)	**Either** $y = 10^{(0.42-0.2x)}$	M1	dealing with the index, using their answer to **(i)**
	$y = 10^{0.42}\left(10^{-0.2x}\right)$ $y = 2.63\left(10^{-0.2x}\right)$	A2	**A1** for each
	Or $y = A\left(10^{bx}\right)$ leads to $\lg y = \lg A + bx$ Compare this form with their equation from (i)	M1	comparing their answer to **(i)** with $\lg y = \lg A + bx$ may be implied by one correct term from correct work
	$\lg A = 0.42$ so $A = 2.63$	A1	
	$b = -0.2$	A1	A1 for each
6(i)	$y \in \mathbb{R}$ oe	B1	Must have correct notation i.e. no use of x
6(ii)	$y > 3$ oe	B1	Must have correct notation i.e. no use of x
6(iii)	$f^{-1}(x) = e^x$ or $g(4) = 35$	B1	First B1 may be implied by correct answer or by use of 35
	$f^{-1}g(4) = e^{35}$	B1	
6(iv)	$\frac{y-3}{2} = x^2$ or $\frac{x-3}{2} = y^2$	M1	valid attempt to obtain the inverse
	$g^{-1}(x) = \sqrt{\frac{x-3}{2}}$	A1	correct form, must be $g^{-1}(x) =$ or $y =$
	Domain $x > 3$	B1	Must have correct notation
7(i)	$p\left(\frac{1}{2}\right)$: $\frac{a}{8} + 2 + \frac{b}{2} + 5 = 0$	M1	substitution of $x = \frac{1}{2}$ and equating to zero (allow unsimplified)
	$p(-2)$: $-8a + 32 - 2b + 5 = -25$	M1	substitution of $x = -2$ and equating to –25 (allow unsimplified)
	leading to $a + 4b + 56 = 0$ $4a + b - 31 = 0$ oe	M1	**M1dep** for solution of simultaneous equations to obtain a and b
	$a = 12,\ b = -17$	A2	**A1** for each

Question	Answer	Marks	Guidance
7(ii)	$12x^3 + 8x^2 - 17x = 0$ $x = 0$	**B1**	for $x = 0$
	$x = -\frac{1}{3} \pm \frac{\sqrt{55}}{6}$ oe	**B1**	
8			
8(i)	$\angle ABC = 67.4°$	**B1**	
	$\frac{4}{\sin BAC} = \frac{5}{\sin 67.4°}$	**M1**	attempt at the sine rule, using 4 and 5 (or e.g. use of cosine rule followed by sine rule on triangle shown)
	$\angle BAC = 47.6°$	**A1**	may be implied by later work
	Angle required = $180° - 47.6° - 67.4° = 65°$	**A1**	Answer Given
8(ii)	$V^2 = 5^2 + 4^2 - (2 \times 5 \times 4 \times \cos 65°)$	**M1**	attempt at the cosine rule or sine rule to obtain V – allow if seen in (i)
	$V = 4.91$ or $\frac{4}{\sin BAC} = \frac{V}{\sin 65}$	**A1**	
	Distance to travel: $\frac{120}{\sin 67.4°}$	**M1**	distance to travel – allow if seen in **(i)**
	130 or $\sqrt{120^2 + 50^2}$	**A1**	
	Time taken: $\frac{130}{4.91}$	**M1**	**M1dep** for correct method to find the time, must have both of the previous M marks
	26.5	**A1**	

Question	Answer	Marks	Guidance
	Alternative method $AC = \frac{120}{\cos 25}$ oe	M1	correct attempt at AC
	$= 132.4$	A1	Allow 132
	Speed for this distance = 5	M1A1	**M1dep A1** for speed, it must be 5 exactly for A1, must have first M mark
	Time taken $= \frac{132.4}{5}$	M1	**M1dep** for a correct method to find the time, must have both of the previous M marks
	= 26.5	A1	
9(a)		B3	**B1** for line joining $(0,5)$ and $(10,5)$ **B1** for a line joining $(10,0.5)$ and $(30,0.5)$ **B1** all correct with **no solid** line joining $(10,5)$ to $(10,0.5)$
9(b)(i)	3	B1	
9(b)(ii)	$\frac{dv}{dt} = -15e^{-5t} + \frac{3}{2}$	M1	attempt to differentiate, must be in the form $ae^{-5t} + b$
	When $\frac{dv}{dt} = 0$, $e^{-5t} = 0.1$	M1	**M1dep** for equating to zero and attempt to solve, must be of the form $ae^{-5t} = b$, $b > 0$ to obtain an equation in the form $-5t = k$ where k is a logarithm or < 0
	$t = 0.461$	A1	

Question	Answer	Marks	Guidance
9(b)(iii)	**Either**		
	attempt to integrate, must be in the form $ce^{-5t}+dt^2$	M1	
	$s=-\frac{3}{5}e^{-5t}+\frac{3}{4}t^2 \quad (+c)$	A1	
	When $t=0,\ s=0$ so $c=\frac{3}{5}$	M1	**M1dep** for attempt to find c and substitute $t=0.5$
	$s=0.738$	A1	
	Or		
	attempt to integrate, must be in the form $ce^{-5t}+dt^2$	M1	
	$\left[-\frac{3}{5}e^{-5t}+\frac{3}{4}t^2\right]_0^{0.5}$	A1	
	correct use of limits	M1	**M1dep**
	leading to $s=0.738$	A1	
10(i)	$5\angle BAC=6.2$, $\angle BAC=1.24$	B1	
10(ii)	$\sin 0.62=\frac{BD}{5}$, $BD=2.905$, 2.91	B1	valid method to find BD
	Arc BFC: $\pi\times BD \quad (=9.13)$	M1	attempt to find arc length BFC, using *their* BD
	Perimeter: $9.13+6.2=15.3$	A1	
10(iii)	Area: $\left(\frac{1}{2}\times\pi\times2.91^2\right)-\left(\left(\frac{1}{2}\times5^2\times1.24\right)-\left(\frac{1}{2}\times5^2\times\sin1.24\right)\right)$	B3	**B1** for area of semi circle (= 13.3) **B1** for area of sector (= 15.5) **B1** for area of triangle (= 11.8)
	$9.58\leqslant$ Area $\leqslant 9.62$	B1	final answer

Question	Answer	Marks	Guidance
11(a)	$\tan(\phi + 35^\circ) = \frac{2}{5}$	M1	dealing correctly with cot and an attempt at solution of $\tan(\phi + 35) = c$, order must be correct, to obtain a value for $\phi + 35$
	$\phi + 35^\circ = 21.8^\circ,\ 201.8^\circ,\ 381.8^\circ$	M1	**M1dep** for an attempt at a second solution in the range, (180° + *their* first solution in the range oe)
	$\phi = 166.8^\circ,\ 346.8^\circ$	A2	**A1** for each
11(b)(i)	**Either** $\dfrac{\frac{1}{\cos\theta}}{\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}}$	M1	expressing all terms in terms of $\sin\theta$ and $\cos\theta$ where necessary
	$= \frac{1}{\cos\theta}\left(\frac{\sin\theta\cos\theta}{\cos^2\theta + \sin^2\theta}\right)$	M1	dealing with the fractions correctly to get $\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$ in denominator or as in left hand column
	$= \frac{\sin\theta}{(1)}$	A1	use of identity, together with a complete and correct solution, withhold A1 for incorrect use of brackets
	Or $\dfrac{\sec\theta}{\frac{1}{\tan\theta} + \tan\theta}$ $= \dfrac{\sec\theta}{\frac{1 + \tan^2\theta}{\tan\theta}}$	M1	dealing with fractions in the denominator correctly to get $\frac{1 + \tan^2\theta}{\tan\theta}$ in the denominator, allow $\tan\theta$ taken to the numerator
	$= \frac{\sec\theta\tan\theta}{\sec^2\theta}$	M1	use of the identity to get $\sec^2\theta$
	$= \frac{\tan\theta}{\sec\theta} = \frac{\sin\theta}{\cos\theta} \times \cos\theta = \sin\theta$	A1	expressing all terms in terms of $\sin\theta$ and $\cos\theta$ and simplification to the given answer, withhold A1 for incorrect use of brackets

Question	Answer	Marks	Guidance
11(b)(ii)	$\sin 3\theta = -\frac{\sqrt{3}}{2}$	**M1**	correct attempt to solve for θ, order must be correct, may be implied by one correct solution
	$3\theta = -\frac{2\pi}{3},\ -\frac{\pi}{3},\ \frac{4\pi}{3}$ $\theta = -\frac{2\pi}{9},\ -\frac{\pi}{9},\ \frac{4\pi}{9}$	**A3**	**A1** for each