**Cambridge International Examinations**
Cambridge Ordinary Level

**ADDITIONAL MATHEMATICS** **4037/13**

Paper 1 **October/November 2016**

MARK SCHEME

Maximum Mark: 80

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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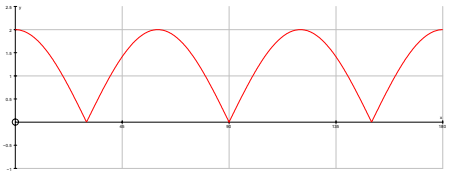
**Abbreviations**

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied
www without wrong working

| Question | Answer | Marks | Part Marks |
|---|---|---|---|
| 1 |  | B1 | for symmetrical shape as in the diagram with curved maxima of equal height and cusps on the $x$-axis |
| | | B1 | for a complete 'curve' with all low points on the $x$-axis and all high points on $y = 2$ |
| | | B1 | for a complete 'curve' meeting the $x$-axis at $x = 30^o$, $90^o$, $150^o$ only. |
| 2 | $= \dfrac{4m^2 - 9}{2m + 3}$ | M1 | for multiplying each term by $\sqrt{m}$, using a common denominator of $\sqrt{m}$ or for multiplying numerator and denominator by $2\sqrt{m} - \dfrac{3}{\sqrt{m}}$ |
| | $= \dfrac{(2m-3)(2m+3)}{2m+3}$ | A1 | for a correct expression that will cancel $\dfrac{(2m-3)(2m+3)}{2m+3}$, $\dfrac{(4m^2-9)(2m-3)}{(4m^2-9)}$ $\dfrac{(2m-3)(2m+3)(2m-3)}{(2m+3)(2m-3)}$, or equivalents |
| | $= 2m - 3$ | A1 | for $2m - 3$ or $A = 2,\ B = -3$ |
| | **Alternative Method** | | |
| | $(4m\sqrt{m} - \dfrac{9}{\sqrt{m}})$ | M1 | for correct expansion |
| | $= (2\sqrt{m} + \dfrac{3}{\sqrt{m}})(Am + B)$ | | |
| | Comparing coefficients<br>$2A = 4,\ 3A + 2B = 0,\ 3B = -9$ | A1<br>A1 | for correct comparisons to obtain $A$ and $B$<br>for $2m - 3$ or $A = 2,\ B = -3$ |

| Question | Answer | Marks | Part Marks |
|---|---|---|---|
| 3 (i) | $3x^2 - 2xp + (p+3) = 0$<br>$(-2p)^2 - 4\times 3\times (p+3) \geqslant 0$ oe | M1 | for obtaining a 3-term quadratic in the form $ax^2 + bx + c(=0)$ |
| | | DM1 | for correct substitution of *their* $a$, $b$ and $c$ into ' $b^2 - 4ac$ 'and use of discriminant. |
| | $p^2 \geqslant 3(p+3)$ or $4p^2 - 12p - 36 \geqslant 0$<br>$p^2 - 3p - 9 \geqslant 0$ | A1 | for full correct working, $\geqslant$ the only sign used, $\geqslant$ used before division by 4 and $\geqslant$ used in answer line and penultimate line. |
| (ii) | Correct method of solution<br>$p^2 - 3p - 9 = 0$ leading to critical values | M1 | for correct substitution in the quadratic formula or for correct attempt to complete the square. (allow 1 sign error in either method) |
| | $p = \frac{3 \pm 3\sqrt{5}}{2}$ | A1 | for both correct critical values |
| | $p \leqslant \frac{3-3\sqrt{5}}{2}$, $p \geqslant \frac{3+3\sqrt{5}}{2}$ | A1 | for correct range |
| 4 (i) | $64 - 48x + 15x^2$ | B3 | for each correct term |
| (ii) | $(4\times '64') + (2\times '-48') + (3\times '15')$ | M1 | for correctly obtaining three products using *their* coefficients in (i) |
| | | A1 | for two correct out of three products (unsimplified) cao |
| | = 205 cao | A1 | for 205 selected as final answer |
| 5 (i) | $\log_9 xy = \log_9 x + \log_9 y$ | M1 | for use of log $AB$ = log$A$ + log$B$ |
| | $= \frac{\log_3 x}{\log_3 9} + \frac{\log_3 y}{\log_3 9}$ | M1 | for correct method for change of base. Division by $\log_3 9$ should be seen and not implied. |
| | $= \frac{\log_3 x}{2} + \frac{\log_3 y}{2} \quad = \frac{5}{2}$ | | |
| | $\log_3 x + \log_3 y = 5$ | A1 | for dealing with 2 correctly and 'finishing off' |
| | **Alternative method** | | |
| | $\log_9 xy = \frac{5}{2}$ | M1 | for obtaining $xy$ as a power of 3 |
| | $xy = 9^{\frac{5}{2}} = 3^5$<br>$\log_3 xy = 5$<br>$\log_3 x + \log_3 y = 5$ | M1<br>A1 | for correct use of $\log_3$<br>for using law for logs and arriving at correct answer |

| Question | Answer | Marks | Part Marks |
|---|---|---|---|
| (ii) | $\log_3 x(5-\log_3 x)=-6$ | | |
| | $-(\log_3 x)^2+5\log_3 x=-6$ | M1 | for substitution, correct expansion of brackets and manipulation to get a 3 term quadratic |
| | $(\log_3 x)^2-5\log_3 x-6=0$ | A1 | for a correct quadratic equation in the form $ax^2+bx+c=0$ |
| | leading to $\log_3 x=6,\quad \log_3 x=-1$ | A1 | for both solutions |
| | | DM1 | for method of solution of $\log_3 x=k$ or $\log_3 y=k$ |
| | $x=729,\quad x=\frac{1}{3}$ | | |
| | $y=\frac{1}{3},\quad y=729$ | A1 | for all $x$ and $y$ correct |
| 6 (i) | $\frac{6x}{3x^2-11}$ | M1<br>A1 | **M1** for $\frac{mx}{3x^2-11}$ |
| (ii) | $p=\frac{1}{6}$ | B1 | **FT** for $p=\frac{1}{m}$ |
| (iii) | $\frac{1}{6}\ln(3a^2-11)-\frac{1}{6}\ln 1=\ln 2$ | M1 | for correct use of limits in $p\ln(3x^2-11)$ May be implied by following equation |
| | $\ln(3a^2-11)=\ln 2^6$ | DM1 | for dealing with logs correctly |
| | $3a^2-11=64$ | DM1 | for solution of $3a^2-11=k$ |
| | $a=5$ only | A1 | for 5 obtained from an exact method |

| Question | Answer | Marks | Part Marks |
|---|---|---|---|
| 7 (i) | $\ln y = \ln A + \frac{b}{x}$ | B1 | for equation, may be implied, must be using ln unless recovered |
| | Gradient: $b = -0.8$ | B1 | for $b = -0.8$ oe |
| | Intercept or use of equation:<br>$\ln A = 4.7$ | B1 | for ln A = 4.7 oe, allow 4.65 to 4.75 |
| | $A = 110$ | B1 | for A = 110, allow 105 to 116<br>Allow $A$ in terms of e |
| | **Alternative Method**<br>$3.5 = \ln A + 1.5b$ and<br>$1.5 = \ln A + 4b$ | B1 | for one equation |
| | leading to $b = -0.8$ | B1 | for $b = -0.8$ |
| | $\ln A = 4.7$ | B1 | for $\ln A = 4.7$ |
| | and $A = 110$ | B1 | for $A = 110$ or $e^{4.7}$ |
| | **Alternative Method**<br>$e^{1.5} = Ae^{4b}$ | B1 | for $e^{1.5} = Ae^{4b}$ or $4.48 = Ae^{4b}$ |
| | $e^{3.5} = Ae^{1.5b}$ | B1 | for $e^{3.5} = Ae^{1.5b}$ or $33.1 = Ae^{1.5b}$ |
| | leading to $b = -0.8$ | B1 | for $b = -0.8$ |
| | and $A = 110$ | B1 | for $A = 110$ or $e^{4.7}$ |
| (ii) | When $x = 0.32, \frac{1}{x} = 3.125, \ln y = 2.2$ | M1 | for a complete method to obtain $y$, using either the graph, using *their* values in the equation for ln$y$ or using *their* values in the equation for $y$. |
| | $y = 9$ ( allow 8.5 to 9.5) or $e^{2.2}$ | A1 | |
| (iii) | When $y = 20$, $\ln y = 3$ , $\frac{1}{x} = 2.125$ | M1 | for a complete method to obtain $x$, using either the graph, using *their* values in the equation for ln$y$ or using *their* values in the equation for $y$. |
| | so $x = 0.47$ ( allow 0.45 to 0.49) | A1 | |

| Question | Answer | Marks | Part Marks |
|---|---|---|---|
| 8 (a) (i) | $\dfrac{\text{cosec}\,\theta}{\text{cosec}\,\theta-\sin\theta}=\dfrac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta}-\sin\theta}$ | M1 | for using $\text{cosec}\,\theta=\dfrac{1}{\sin\theta}$ and either attempt to multiply top and bottom by sin $\theta$ or an attempt to combine terms in denominator. |
| | $=\dfrac{1}{1-\sin^2\theta}$ or $=\dfrac{\frac{1}{\sin\theta}}{\frac{(1-\sin^2\theta)}{\sin\theta}}$ | DM1 | for correct use of $1-\sin^2\theta=\cos^2\theta$ |
| | $=\dfrac{1}{\cos^2\theta}$<br>$=\sec^2\theta$ | A1 | for completing the proof |
| | **Alternative Method using cosec** | | |
| | $\dfrac{\text{cosec}\,\theta}{\text{cosec}\,\theta-\sin\theta}=\dfrac{\text{cosec}\,\theta}{\text{cosec}\,\theta-\frac{1}{\text{cosec}\,\theta}}$ | | |
| | $=\dfrac{\text{cosec}^2\theta}{\text{cosec}^2\theta-1}$ | M1 | for using $\sin\theta=\dfrac{1}{\text{cosec}\,\theta}$ and an attempt to combine terms in denominator. |
| | $=\dfrac{1+\cot^2\theta}{\cot^2\theta}$ | DM1 | for use of $1+\cot^2\theta=\text{cosec}^2\theta$ |
| | $=\tan^2\theta+1=\sec^2\theta$ | A1 | for completing the proof |
| (ii) | $\cos^2\theta=\dfrac{1}{4},\ \cos\theta=\pm\dfrac{1}{2}$<br>or $\tan^2\theta=3,\ \tan\theta=\pm\sqrt{3}$<br>or $\sin^2\theta=\dfrac{3}{4},\ \sin\theta=\pm\dfrac{\sqrt{3}}{2}$ | M1 | for using (i) to obtain a value for $\cos^2\theta$, $\tan^2\theta$ or $\sin^2\theta$ and then taking the square root. |
| | $\theta=60^\circ, 120^\circ, 240^\circ, 300^\circ$ | A1<br>A1 | for two correct values<br>for two further correct values and no extras in range. |
| (b) | $\tan\left(x+\dfrac{\pi}{4}\right)=\dfrac{1}{\sqrt{3}}$ | M1 | for correct order of operations, can be implied by $x=-\dfrac{\pi}{12}$ |
| | $x=\dfrac{\pi}{6}-\dfrac{\pi}{4},\ \dfrac{7\pi}{6}-\dfrac{\pi}{4},\ \dfrac{13\pi}{6}-\dfrac{\pi}{4}$<br>$x=\left(-\dfrac{\pi}{12}\right),\ \dfrac{11\pi}{12},\ \dfrac{23\pi}{12}$ | A1,A1 | **A1** for $x=\dfrac{11\pi}{12}$<br>**A1** for $x=\dfrac{23\pi}{12}$<br>If there are extra solutions in range in addition to the two correct ones then **A1A0** |

| Question | Answer | Marks | Part Marks |
|---|---|---|---|
| 9 (a) (i) | ${}^{18}C_5 = 8568$ mmm | B1 | |
| (ii) | **Either** | | |
| | ${}^{10}C_4 \times {}^{8}C_1 = 1680$ | B1 | for a correct plan |
| | ${}^{10}C_3 \times {}^{8}C_2 = 3360$ | B2,1,0 | **B2** 4 correct numbers with no extras |
| | ${}^{10}C_2 \times {}^{8}C_3 = 2520$ | | **B1** 3 correct numbers (out of 3 or 4) |
| | ${}^{10}C_1 \times {}^{8}C_4 = 700$ | | |
| | Total = 8260 | B1 | for correct total |
| | **Or** | | |
| | *their* ${}^{18}C_5 - \left({}^{10}C_5 + {}^{8}C_5\right)$ | B1 | for correct plan |
| | $8568 - (252 + 56)$ | B1 | for 252 subtracted |
| | | B1 | for 56 subtracted |
| | Total =8260 | B1 | for correct total |
| (b) (i) | ${}^{10}P_6 = 151200$ | B1 | |
| (ii) | $4 \times {}^{8}P_4 \times 3$ | M1 | for correct unsimplified |
| | = 20160 | A1 | for correct numerical answer |
| (iii) | Answer to (i) - ${}^{7}P_6$ | M1 | for correct plan |
| | | A1 | for correct unsimplified |
| | =146160 | A1 | for correct numerical answer |
| | Alternative: | | |
| | 1 symbol: 45360 | B2,1,0 | **B2** for all 3 correct |
| | 2 symbols: 75600 | | **B1** for 2 correct (out of 2 or 3) |
| | 3 symbols: 25200 | | |
| | Total: 146160 | B1 | for correct sum |

| Question | Answer | Marks | Part Marks |
|---|---|---|---|
| 10 (i) | $f(x) = 3x^2 - 4e^{2x}$ $(+c)$ | M1 | for one correct term |
| | | A1 | for one correct term $3x^2$ or $-4e^{2x}$ |
| | | A1 | for a second correct term with no extras |
| | passing through $(0,-3)$ | DM1 | for correct method to find $c$. |
| | $-3 = 3\times 0 - 4e^0 + c$ | | |
| | $f(x) = 3x^2 - 4e^{2x} + 1$ | A1 | for correct equation |
| (ii) | $f'(0) = -8$ | B1 | for $m = \frac{1}{8}$ |
| | Normal: $y + 3 = \frac{1}{8}x$ | M1 | for equation of normal using $m = \frac{1}{8}$ |
| | $8y + 24 = x$ | DM1 | for solving normal equation simultaneously with $y = 2 - 3x$ to get a value of $x$ |
| | $y = 2 - 3x$ | | |
| | leads to $x = \frac{8}{5}$ oe | A1 | for $x = \frac{8}{5}$, 1.6 oe |
| | Area $= = \frac{1}{2}\times 3\times \frac{8}{5} = 2.4$ oe | B1 | **FT** for a numerical answer equal to $\left\lvert \frac{1}{2}\times 3\times \text{their } x \right\rvert$ |
| 11 (i) | $a = 8t - 8$ | B1 | for $8t - 8$ |
| | When $t = 3$ , $a = 16$ | B1 | for 16 |
| (ii) | 0.5, 1.5 | B1,B1 | **B1** for each |
| (iii) | $s = \frac{4}{3}t^3 - 4t^2 + 3t$ | M1 | for at least two terms correct |
| | | A1 | all correct |
| | when $t = \frac{1}{2}$, $s = \frac{2}{3}$ | DM1 | for calculating displacement when either $t = \frac{1}{2}$ or $t = \frac{3}{2}$ |
| | when $t = \frac{3}{2}$, $s = 0$ | DM1 | for calculating displacement at $t = \frac{1}{2}$ and doubling. |
| | total distance travelled $= \frac{4}{3}$ | A1 | for $\frac{4}{3}$ oe allow 1.33 |
| | **Alternative method** | M1A1 | As before |
| | | DM1 | **DM1** for calculating displacement when $t = 0.5$ or for calculating distance travelled between $t = 0.5$ and $t = 1.5$ |
| | | DM1 | **DM1** for doubling distance travelled between $t = 0.5$ and $t = 1.5$ or for adding that distance to displacement at $t = 0.5$ |
| | | A1 | **A1** for $\frac{4}{3}$ oe allow 1.33 |