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Cambridge Ordinary Level

ADDITIONAL MATHEMATICS **4037/12**

Paper 1 **October/November 2016**

MARK SCHEME

Maximum Mark: 80

Published

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Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied
www	without wrong working

Question	Answer	Marks	Part Marks
1 (a) (i)	10	B1	
(ii)	22	B1	
(iii)	4	B1	
(b) (i)	$Q \subset R$	B1	
(ii)	$P \cap Q = \varnothing$, or $\{\ \}$	B1	
2	$a = 1,\ \ b = -3,\ \ c = -1$	B3	**B1** for each
3	$3y^2 + 5y - 2 = 0$	B1, B1	**B1** for $5y$ or $5\log_3 x$, **B1** for -2
	$y = \frac{1}{3},\ \ y = -2$	M1	for correct attempt at the solution of *their* quadratic equation
	$x = 3^{\frac{1}{3}},\ \ x = 3^{-2}$	M1	for dealing with one base 3 logarithm correctly
	$x = 1.44,\ \ x = \frac{1}{9}$	A1, A1	**A1** for each
4 (i)	$32x^{10} - \frac{80}{3}x^7 + \frac{80}{9}x^4$	B3	**B1** for each term, powers of x must be simplified
(ii)	Coefficients needed:		
	$\left(3 \times \textit{their} - \frac{80}{3}\right) + (1 \times \textit{their}\ 32)$	M1	for dealing with 2 terms
	$= -48$	A1	Allow **A1** for $-48x^7$

Question	Answer	Marks	Part Marks
5 (i)	$\frac{dy}{dx} = \frac{3}{2(3x+2)}$	B1	for correct derivative of log function
	When $x = -\frac{1}{3}$, $y = 0$, $\frac{dy}{dx} = \frac{3}{2}$	B1	for $y = 0$
	Equation of normal: $y = -\frac{2}{3}\left(x + \frac{1}{3}\right)$	M1 A1	**M1** for attempt at a gradient of a perpendicular from differentiation and the equation of the normal
(ii)	$Q\left(0, -\frac{2}{9}\right)$ or $(0, 0.22)$ or better	B1 ft	Follow through on *their c* from part (i)
	$R\left(0, \frac{1}{2}\ln 2\right)$ or $(0, 0.35)$ or better	B1	
	Area of PQR $= \frac{1}{2}\left(\frac{1}{2}\ln 2 + \frac{2}{9}\right) \times \frac{1}{3}$ $= 0.0948$	B1	Allow 0.095
6 (a)	**YX**, **XZ**	B2	**B2** for both with no extras **B1** for 1 correct with or without extras **B1** for both correct with extras **B0** for anything else
(b) (i)	$\frac{1}{18}\begin{pmatrix} 7 & 1 \\ -4 & 2 \end{pmatrix}$	B1, B1	**B1** for $\frac{1}{18}$, **B1** for $\begin{pmatrix} 7 & 1 \\ -4 & 2 \end{pmatrix}$
(ii)	$\mathbf{C} = \mathbf{A}^{-1}\mathbf{B}$ $= \frac{1}{18}\begin{pmatrix} 7 & 1 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} -4 & 2 \\ 10 & 4 \end{pmatrix}$	M1	for pre-multiplication
	$= \begin{pmatrix} -1 & 1 \\ 2 & 0 \end{pmatrix}$	A1, A1	**A1** for any correct pair of elements, but must be from correct matrices

Question	Answer	Marks	Part Marks
7 (i)	$(0, \sqrt{3})$ or $(0, 1.73)$ or better	B1	
(ii)	$\left(\frac{\pi}{6}, 2\right)$ or $(0.524, 2)$ or better	B1, B1	B1 for each
(iii)	$\cos\left(x - \frac{\pi}{6}\right) = 0$	M1	for correct attempt to solve trigonometric equation
	$x = \frac{2\pi}{3}$ oe or 2.09 or better	A1	
(iv)	$2\sin\left(x - \frac{\pi}{6}\right)$ (+c)	B1	
(v)	Area $= \left[2\sin\left(x - \frac{\pi}{6}\right)\right]_0^{\frac{2\pi}{3}}$ $= 2 + 1$ $= 3$	M1 A1	for correct use of **their** limits, in radians, into $k\sin\left(x - \frac{\pi}{6}\right)$.
8 (i)	$47 - 24 = 12\theta$		
	$\theta = \frac{23}{12}$, so $\theta = 1.917$ or better	M1	for complete correct method to get $\theta =$
	$\theta = 1.92$ to 2dp	A1	must have evidence of working to more than 2 dp, allow if 1.916 seen (truncated)
(ii)	$\sin\frac{\theta}{2} = \frac{CD/2}{12}$	M1	for a complete method, may use cosine rule to get CD
	CD = awrt 19.6 or 19.7	A1	
(iii)	Area of sector = awrt 138	B1	for sector area, allow unsimplified
	Area of triangle AOB = awrt 67 or 68	M1	for a correct attempt at area
	Area of segment = awrt 70 or 71	M1	for segment area (*their* sector area – *their* triangle area)
	$AD \times AB$ + segment area = 425	M1	for complete method to find AD
	leading to AD = awrt 18.1 or 18.0	A1	Allow **A1** for 18
	Alternative method:		
	Area of sector = awrt 138	B1	for sector area
	Difference in length between BC (or AD) and OM where M is the midpoint of CD = 6.88, allow awrt 6.9	M1	for attempt to find difference between parallel sides
	Remaining area consists of two trapezia each of width 9.85 and each of area 143.4 $\frac{1}{2}(2BC - 6.88) \times 9.85 = 143.4$ oe	M1	for area of one trapezium $\frac{1}{2}(2BC - \textit{their}\ 6.88) \times \textit{their}\ 9.85$ oe
	leading to AD = awrt 18.1 or 18.0	M1 A1	for attempt to find either BC or AD

Question	Answer	Marks	Part Marks
9 (i)	$\text{p}\left(\frac{3}{2}\right)$: $\frac{27a}{8}-\left(4\times\frac{9}{4}\right)+\frac{3b}{2}+18\ \ (=0)$	M1	for attempt at $\text{p}\left(\frac{3}{2}\right)$
	$\text{p}'\left(\frac{3}{2}\right)=\left(3a\times\frac{9}{4}\right)-\left(8\times\frac{3}{2}\right)+b\ \ (=0)$	M1	for differentiation and attempt at $\text{p}'\left(\frac{3}{2}\right)$
	leading to $9a+4b+24=0$ oe		
	and $27a+4b-48=0$ oe	M1	for solution of simultaneous equations, to get either a or b
	leading to $a=4,\ b=-15$	A1	for both
(ii)	$(x+2)(2x-3)^2$ oe	M1, A1	**M1** for attempt at long division or factorisation
(iii)	$(x+2)(2x-3)^2=x+2$		
	$x+2=0,\ x=-2$	B1	Must be using $(x+2)$ correctly using part (ii) to get $x=-2$
	$(2x-3)^2=1$	M1	for solution of the quadratic equation
	leading to $x=1,\ x=2$	A1	
10 (a) (i)	$20U+\frac{1}{2}\left(U+\frac{U}{2}\right)10=165$	M1	for realising that area under the graph is needed and attempt to find an area
		DM1	for equating their area to 165 and attempt to solve
	leading to $U=6$	A1	
(ii)	Gradient of line: -0.3	M1, A1	**M1** for use of the gradient, must be negative
(b) (i)	27	B1	
(ii)	$t^2=8\ln 4$	M1	for a correct attempt to solve $\text{e}^{\frac{t^2}{8}}=4$
	$t=3.33$ or better	A1	
(iii)	acceleration $=3\frac{2t}{8}\text{e}^{\frac{t^2}{8}}\left(\text{e}^{\frac{t^2}{8}}-4\right)^2$	M1, A1	**M1** for a correct attempt to differentiate using the chain rule
	When $t=1,\ a=6.98$	M1, A1	**M1**for use of $t=1$ in their acceleration

Question	Answer	Marks	Part Marks
11 (i)	$\ln y = \ln A + x \ln b$	B1	may be implied, if equation not seen specifically, by correct values for A and b
	Gradient: $\ln b = -\dfrac{0.12}{8}$, $= -0.015$	M1	for use of gradient to obtain ln b
	$b = 0.985$	A1	Allow **A1** for $e^{-0.015}$
	Intercept: $\ln A = 0.26$	DM1	for use of one of the given points correctly
	$A = 1.30$	A1	Allow **A1** for $e^{0.26}$ or 1.3
	Alternative 1		
	$\ln y = \ln A + x \ln b$	B1	
	$0.2 = 4 \ln b + \ln A$	M1	for one correct equation
	$0.08 = 12 \ln b + \ln A$	DM1	for attempt to obtain either ln$A$ or ln$b$ from simultaneous equations
	$A = 1.30$ and $b = 0.985$	A1, A1	Allow **A1** for $b = e^{-0.015}$ and $a = e^{0.26}$ or 1.3
	Alternative 2		
	$1.22 = Ab^4$	B1	
	$1.08 = Ab^{12}$	B1	
		M1	for correct attempt to obtain b or A, must already have **B2**
	$A = 1.30$ and $b = 0.985$	A1, A1	Allow **A1** for $b = e^{-0.015}$ and $a = e^{0.26}$ or 1.3
(ii)	When $x = 6$, $\ln y = 0.17$	M1	for $\ln y = their \ln A + 6\ their \ln b$ or $y = their\ A \times (their\ b)^6$
	$y = 1.19$	A1	allow awrt 1.18 to 1.20
(iii)	When $y = 1.1$, $\ln y = 0.095$	M1	for $\ln 1.1 = their \ln A + x\ their \ln b$ or $1.1 = their A \times (their b)^x$
	$x = 11$	A1	allow 10.5 to 11.5