CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Ordinary Level

MARK SCHEME for the October/November 2015 series

4037 ADDITIONAL MATHEMATICS

4037/23 Paper 2, maximum raw mark 80

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Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied
www	without wrong working

1	$y = x^3 + 3x^2 - 5x - 7$ $\frac{dy}{dx} = 3x^2 + 6x - 5$ $x = 2 \rightarrow \frac{dy}{dx} = 19$ $y = 3$ eqn of tangent: $\frac{y-3}{x-2} = 19 \quad \rightarrow \quad (y = 19x - 35)$	M1 A1 A1FT B1 A1FT	Differentiate on *their* $\frac{dy}{dx}$
2	$2x + k + 2 = 2x^2 + (k+2)x + 8$ $2x^2 + kx + 6 - k \quad (= 0)$ $b^2 - 4ac = k^2 - 4 \times 2(6-k)$ $k^2 + 8k - 48 \quad (> 0)$ $(k+12)(k-4) \quad (> 0)$ $k < -12$ or $k > 4$	M1 A1 M1 DM1 A1 A1	eliminate y or x correct quadratic use discriminant attempt to solve 3 term quadratic $k = -12$ and $k = 4$
3 (a)	$\frac{dy}{dx} = \frac{(2-x^2)3x^2 - x^3(-2x)}{(2-x^2)^2} = \left(\frac{6x^2 - x^4}{(2-x^2)^2}\right)$	M1 A2,1,0	For quotient rule (or product rule on correct y)
(b)	$\frac{dy}{dx} = x \times \frac{1}{2}(4x+6)^{-0.5} \times 4 + (4x+6)^{0.5}$ $= \frac{6(x+1)}{(4x+6)^{0.5}} \quad \rightarrow \quad k = 6$	M1 A1 A1	product rule
4	$x(4 - \sqrt{3}) = 13$ $x = \frac{13(4+\sqrt{3})}{(4-\sqrt{3})(4+\sqrt{3})}$ $= 4 + \sqrt{3}$ $y = 1 - 2\sqrt{3}$	M1 A1 M1 A1 A1	eliminate y or x simplified rationalisation

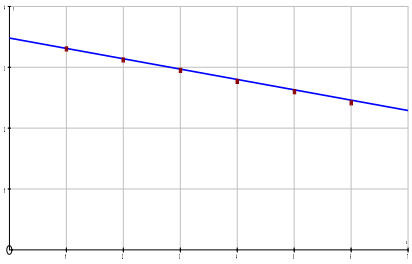
5	$(x-3)(x-3)(x-1)=0$ $x^3-7x^2+15x-9=0$ $a=-7$ $b=15$ $c=-9$	M1 A1 A1 A1	 **AG** for c
6	$\log_x 2=\frac{\log_2 2}{\log_2 x}$ $2\log_2 x=\log_2 x^2$ $3=\log_2 8$ $8x^2-29x+15\ (=0)$ $\rightarrow(8x-5)(x-3)\ (=0)$ $x=\frac{5}{8}$ or $x=3$	B1 B1 B1 M1 A1	 obtain quadratic and attempt to solve
7 **(i)**	$a=-\frac{20}{(t+2)^3}$ $t=3 \rightarrow a=-0.16$ m/s^2	M1 A1 A1FT	$k(t+2)^{-3}$ oe $k=-20$
(ii)	$\frac{10}{(t+2)^2}$ is never zero.	B1	
(iii)	$s=-\frac{10}{t+2}+5$	M1 A1 A1	integrate $\frac{k}{t+2}$ $k=-10$ +5
(iv)	$s=\left[-\frac{10}{t+2}\right]_3^8=-1+2$ $=1$	M1 A1	insert limits and subtract

8	**(i)**	$\sec^2 x + \text{cosec}^2 x = \dfrac{1}{\cos^2 x} + \dfrac{1}{\sin^2 x}$	B1	
		$= \dfrac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}$	B1	add fractions
		$= \dfrac{1}{\sin^2 x \cos^2 x}$	B1	use of $\sin^2 x + \cos^2 x = 1$
		$= \sec^2 x \text{cosec}^2 x$	B1	fully correct solution
	(ii)	$\dfrac{1}{\cos^2 x \sin^2 x} = 4\dfrac{\sin^2 x}{\cos^2 x}$	M1	
		$\rightarrow \quad 4\sin^2 x = 1$	A1	correct simplified equation
		$\sin x = \pm\dfrac{1}{\sqrt{2}}$		
		$x = 135°, 225°$	A1, A1	
9	**(i)**	$f(x) = 3x^2 + 12x + 2 = 3(x+2)^2 - 10$		
		$a = 3$	B1	
		$b = 2$	B1	
		$c = -10$	B1	
	(ii)	minimum $f(x) = -10$	B1FT	
		at $x = -2$	B1FT	
	(iii)	$f\left(\dfrac{1}{y}\right) = 0 \quad \rightarrow \quad \left(\dfrac{1}{y}\right) = (\pm)\sqrt{\dfrac{10}{3}} - 2$	M1	obtain explicit expression for $\dfrac{1}{y}$ or y
		$y = -5.74, \quad -0.26$	A1, A1	

10	**(i)**	$\frac{d}{dx}(e^{2-x^2}) = -2xe^{2-x^2}$	B1	$k = -2$
	(ii)	$-\frac{3e^{2-x^2}}{2} + c$	M1 A1FT	De^{2-x^2} $D = \frac{-3}{2}$ or $\frac{3}{k}$
	(iii)	$\left[-\frac{3e^{2-x^2}}{2}\right]_1^{\sqrt{2}} = -\frac{3}{2} + \frac{3}{2}e$ 2.58	M1 A1	insert limits on *their* (ii) and subtract
	(iv)	$y = 3xe^{2-x^2}$	M1 A1	product rule
		$\frac{dy}{dx} = 3x(-2xe^{2-x^2}) + 3e^{2-x^2}$ $\frac{dy}{dx} = 0 \rightarrow x = \pm\frac{1}{\sqrt{2}} = \pm 0.707$	A1	both x or a pair
		$y = \pm\frac{3}{\sqrt{2}}e^{1.5} = \pm 9.51$	A1	both y
11	**(i)**	$\log N = \log A - t\log b$	B1	
	(ii)			

t	1	2	3	4	5	6
$\log N$	3.30	3.11	2.95	2.77	2.60	2.41
$\ln N$	7.60	7.17	6.79	6.38	5.98	5.56

M1 find logs of N



M1 plot $\log N$ or $\ln N$ against t or $-t$

A1 straight line passing through five points

	(iii)	gradient $= -\log b = \frac{2.415 - 3.3}{5} \rightarrow b = 1.5$ intercept $= \log A = 3.47 \rightarrow A = 2950$	DM1 DM1 A1	set gradient $= -\log b$ and solve set intercept $= \log A$ and solve both values correct
	(iv)	$t = 10 \rightarrow N = \frac{2950}{1.5^{10}} = 51$	B1	
	(v)	$N = 10 \rightarrow 1.5^t = 295 \rightarrow t = \frac{\log 295}{\log 1.5}$ $= 14$ years	M1 A1	substitute $N = 10$, *their* A, b into given or transformed equation

12	$v_p = \begin{pmatrix} 250\cos 20^\circ \\ 250\sin 20^\circ \end{pmatrix}$, $v_r = \begin{pmatrix} V\cos 30^\circ \\ V\sin 30^\circ \end{pmatrix}$, $v_w = \begin{pmatrix} 0 \\ w \end{pmatrix}$	B1	
	$v_r = v_p + v_w$ $\begin{pmatrix} V\cos 30^\circ \\ V\sin 30^\circ \end{pmatrix} = \begin{pmatrix} 250\cos 20^\circ \\ 250\sin 20^\circ \end{pmatrix} + \begin{pmatrix} 0 \\ w \end{pmatrix}$		
	$V = \dfrac{250\cos 20^\circ}{\cos 30^\circ}$ $= 271\text{km/hr}$	M1 A1	equate x components and solve
	$w = V\sin 30^\circ - 250\sin 20^\circ$ $= 50.1\text{km/hr}$	M1 A1	equate y components and solve
	OR triangle with sides 250 V w opposite angles 60° 110° 10°	B1	
	sine rule: $\dfrac{w}{\sin 10^\circ} = \dfrac{250}{\sin 60^\circ}$ $w = 50.1\text{km/hr}$	M1 A1	apply to correct triangle and solve
	$\dfrac{V}{\sin 110^\circ} = \dfrac{250}{\sin 60^\circ}$ $V = 271\text{km/hr}$	M1 A1	apply to correct triangle and solve