CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Ordinary Level

MARK SCHEME for the October/November 2015 series

4037 ADDITIONAL MATHEMATICS

4037/13 Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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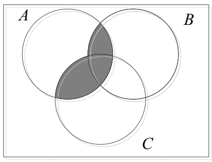
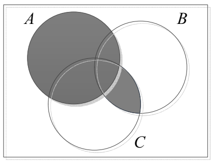
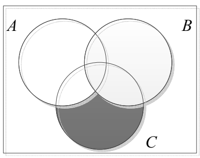
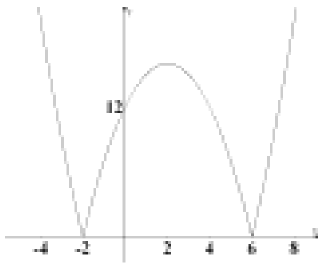
CAMBRIDGE
International Examinations

Abbreviations

Awrt	answers which round to
Cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied
www	without wrong working

1 (i)		B1	
(ii)		B1	
(iii)		B1	
2	$\cos\left(3x - \frac{\pi}{4}\right) = (\pm)\frac{1}{\sqrt{2}}$ oe	M1	division by 2 and square root
	$3x - \frac{\pi}{4} = -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$		
	$x = \left(-\frac{\pi}{4} + \frac{\pi}{4}\right) \div 3, \left(\frac{\pi}{4} + \frac{\pi}{4}\right) \div 3, \left(\frac{3\pi}{4} + \frac{\pi}{4}\right) \div 3$ oe	DM1	correct order of operations in order to obtain a solution
	$x = 0$ and $\frac{\pi}{6}$ (or 0 and 0.524) $x = \frac{\pi}{3}$ (or 1.05)	A2/1/0	A2 for 3 solutions and no extras in the range A1 for 2 solutions A0 for one solution or no solutions

3	(a)	$\begin{pmatrix} 12 & 16 & 4 \\ 30 & 32 & 10 \end{pmatrix}$	B2,1,0	B2 for 6 elements correct, B1for 5 elements correct
	(b)	$\begin{pmatrix} 28 & -24 \\ -8 & 76 \end{pmatrix} = m\begin{pmatrix} 4 & 6 \\ 2 & -8 \end{pmatrix} + n\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	B2,1,0	B2 for 4 correct elements in $\mathbf{X}^2$ B1 for 3 correct elements in $\mathbf{X}^2$
		$-24 = 6m$ or $-8 = 2m$ giving $m = -4$	B1	For $m = -4$ using correct **I**
		$28 = 4m + n$ or $76 = -8m + n$ $n = 44$	M1 A1	complete method to obtain n
	(c)	$a^2 - 6 = 0$ so $a = \pm\sqrt{6}$	B2,1,0	B2 for $a = \pm\sqrt{6}$ or $a = \pm 2.45$, with no incorrect statements seen or B1 for $a = \pm\sqrt{6}$ or $a = \pm 2.45$ seen or B1 for $a = \sqrt{6}$ and no incorrect working
4	(i)	$\frac{1}{2}\left(4\sqrt{3}+1\right) \times BC = \frac{47}{2}$	B1	correct use of the area
		$BC = \frac{47}{\left(4\sqrt{3}+1\right)} \times \frac{\left(4\sqrt{3}-1\right)}{\left(4\sqrt{3}-1\right)}$	M1	correct rationalisation
		$BC = 4\sqrt{3} - 1$	A1	Dependent on all method being seen
		Alternative method		
		$\frac{1}{2}\left(4\sqrt{3}+1\right) \times BC = \frac{47}{2}$ $\left(4\sqrt{3}+1\right)\left(a\sqrt{3}+b\right) = 47$	B1	
		Leading to $12a + b = 47$ and $a + 4b = 0$ Solution of simultaneous equations	M1	
		$BC = 4\sqrt{3 - 1}$	A1	Dependent on all method seen including solution of simultaneous equations
	(ii)	$\left(4\sqrt{3}+1\right)^2 + \left(4\sqrt{3}-1\right)^2$		
		$= \left(48 + 8\sqrt{3} + 1\right) + \left(48 - 8\sqrt{3} + 1\right)$	B1FT	6 correct FT terms seen
		$AC^2 = 98$ $AC = 7\sqrt{2}$ **or** $p = 7$	B1cao	98 and $7\sqrt{2}$ or 98 and $p = 7$

5	When $x = \frac{\pi}{4}$, $y = 2$	B1	$y = 2$
	$\frac{dy}{dx} = 5\sec^2 x$	B1	$5\sec^2 x$
	When $x = \frac{\pi}{4}$, $\frac{dy}{dx} = 10$	B1	10 from differentiation
	Equation of normal $y - 2 = -\frac{1}{10}\left(x - \frac{\pi}{4}\right)$	M1	$y - their2 = -\frac{1}{their10}\left(x - \frac{\pi}{4}\right)$
	$10y + x - 20 - \frac{\pi}{4} = 0$ or $10y + x - 20.8 = 0$ oe	A1	allow unsimplified
6 (i)		B1 B1 B1	shape intercepts on x-axis intercept on y-axis for a curve with a maximum and two arms
(ii)	$(2, 16)$	M1 A1	(2, ±16) seen or $(2, k)$ where $k > 0$ (2, 16) or $x = 2$ **and** $y = 16$ only
(iii)	$k = 0$	B1	
	$k > 16$	B1	

7	$\frac{dy}{dx} = 2\sin 3x \quad (+c)$	B1	$2\sin 3x$
	$4\sqrt{3} = 2\frac{\sqrt{3}}{2} + c$	M1	finding constant using $\frac{dy}{dx} = k\sin 3x + c$ making use of $\frac{dy}{dx} = 4\sqrt{3}$ and $x = \frac{\pi}{9}$
	$\frac{dy}{dx} = 2\sin 3x + 3\sqrt{3}$	A1	Allow with $c = 5.20$ or $\sqrt{27}$
	$y = -\frac{2}{3}\cos 3x + 3\sqrt{3}x \quad (+d)$	B1FT	FT integration of *their* $k\sin 3x$
	$-\frac{1}{3} = -\frac{2}{3}\cos\frac{\pi}{3} + 3\sqrt{3}\left(\frac{\pi}{9}\right) + d$	M1	finding constant d for $k\cos 3x + cx + d$
	$y = -\frac{2}{3}\cos 3x + 3\sqrt{3}x - \frac{\sqrt{3}}{3}\pi$	A1	Allow $y = -0.667\cos 3x + 5.20x - 0.577\pi$ or better
8 **(a)**	$(2 + kx)^8 = 256 + 1024kx + 1792k^2x^2 + 1792k^3x^3$		
	$k = \frac{1}{4}$	B1	
	$p = 112$	B1FT	FT 1792 multiplied by *their* k^2
	$q = 28$	B1FT	FT 1792 multiplied by *their* k^3
(b)	${}^9C_3 x^6\left(-\frac{2}{x^2}\right)^3$	M1	correct term seen
	$84x^6\left(-\frac{8}{x^6}\right)$ leading to	DM1	Term selected and 2^3 and ${}^9C_3$ correctly evaluated
	-672	A1	

9 (a) (i)	Number of arrangements with Maths books as one item = 4! or 4 × 3!	M1	$4!(\times 2)$ or $4\times 3!(\times 2)$ oe
	or Maths books can be arranged 2! ways and History 3! ways = 2! × 3!		$2!\times 3!(\times 4)$ or $2\times 3!(\times 4)$ oe
	2 × 4! or 2 × 4 × 3! or 4 × 2 × 3! = 48	A1	A1 for 48
(ii)	$5! - 48$ or $6\times 2\times 3!$	M1	5! – *their* answer to (i) or for $6\times 2\times 3$
	72	A1	
(b) (i)	3003	B1	
(ii)	$3003 - 6 - 135$	M1 B1	*their* answer to (i) $- 6 - {}^6C_4 \times 9$ 135 subtracted
	2862	A1	
	or 2M 3W = 720 3M 2W = 1260 4M 1W = 756 5M = 126 2862	M1 B1 A1	complete correct method using 4 cases, may be implied by working. Must have at least one correct any 3 correct

10	(i)	$10^2 = 6^2 + 6^2 - 2\times 6\times 6\times \cos ABC$ or $\sin\left(\frac{ABC}{2}\right) = \frac{5}{6}$ or $ABC = \pi - \sin^{-1}\frac{10\sqrt{11}}{36}$	M1	correct cosine rule statement or correct statement for $\sin\frac{ABC}{2}$ or equating areas oe
		$ABC = 1.9702$	A1	1.9702 or better
	(ii)	$XY = 2$	B1	for XY (may be implied by later work, allow on diagram)
		Arc length $6\left(\frac{\pi - 1.970}{2}\right)$ oe	B1	correct arc length (unsimplified)
		Perimeter = $2 + 2\left(6\left(\frac{\pi - 1.970}{2}\right)\right)$	M1	*their* 2 + 2 × 6 × *their* angle C
		$= 9.03$	A1	
	(iii)	$\left(\frac{1}{2}\times 6^2\left(\frac{\pi - 1.970}{2}\right) - \frac{1}{2}\times 5\times\sqrt{11}\right)\times 2$	M1 M1	sector area using *their* C area of $\Delta\, ABM$ where M is the midpoint of AC, or (Δs ABY and BXY) or ΔABC
		= 4.50 or 4.51 or better	A1	Answers to 3sf or better

11	$x^2 - 2x - 3 = 0$ or $y^2 - 6y + 5 = 0$	M1	substitution and simplification to obtain a three term quadratic equation in one variable
	leading to (3, 5) and (–1, 1)	A1,A1	A1 for each 'pair' from a correct quadratic equation, correctly obtained.
	Midpoint (1, 3)	B1cao	midpoint
	(Gradient – 1) Perpendicular bisector $y = 4 - x$	M1	perpendicular bisector, must be using *their* perpendicular gradient and *their* midpoint
	Meets the curve again if $x^2 + 10x - 15 = 0$ or $y^2 - 18y + 41 = 0$	M1	substitution and simplification to obtain a three term quadratic equation in one variable.
	leading to $x = -5 \pm 2\sqrt{10}$, $y = 9 \mp 2\sqrt{10}$	A1,A1	A1 for each 'pair'
	$CD^2 = \left(4\sqrt{10}\right)^2 + \left(4\sqrt{10}\right)^2$	M1	Pythagoras using *their* coordinates from solution of second quadratic. $(x_1 - x_2)^2 + (y_1 - y_2)^2$ must be seen if not using correct coordinates.
	$CD = 8\sqrt{5}$	A1	A1 for $8\sqrt{5}$ from $\sqrt{320}$ and all correct so far.

12 **(a)**	$2^{2x-1} \times 2^{2(x+y)} = 2^7$ and $\dfrac{3^{2(2y-x)}}{3^{3(y-4)}} = 1$	M1	expressing 4^{x+y}, 128 as powers of 2 and 9^{2y-x}, 27^{y-4} as powers of 3
	$2x - 1 + 2(x + y) = 7$ oe	A1	Correct equation from correct working
	$2(2y - x) = 3(y - 4)$ oe	A1	Correct equation from correct working
	leading to $x = 4$, $y = -4$	A1	for both
	<u>Example of Alternative method</u>		
	Method mark as above	M1	As before
	$2x - 1 + 2(x + y) = 7$	A1	One of the correct equations in x and y
	leading to $y = \dfrac{(8-4x)}{2}$		
	Correctly substituted in $\dfrac{3^{2(2y-x)}}{3^{3(y-4)}} = 1$		
	Leading to $2\left(\dfrac{2(8-4x)}{2} - x\right) = 3\left(\dfrac{(8-4x)}{2} - 4\right)$	A1	Correct, unsimplified, equation in x or y only
	Leading to $x = 4$ and $y = -4$	A1	Both answers
(b)	$(2(5^z) - 1)(5^z + 1) = 0$	M1	solution of quadratic
	leading to $2.5^z = 1$ $\quad (5^z = -1)$	A1	correct solution
	$5^z = 0.5$	DM1	correct attempt to solve $2.5^z = k$, where k is positive
	$z = \dfrac{\log 0.5}{\log 5}$ or $z = -0.431$ or better	A1	must have one solution only