CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Ordinary Level

MARK SCHEME for the October/November 2015 series

4037 ADDITIONAL MATHEMATICS

4037/12 Paper 1, maximum raw mark 80

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Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied
www	without wrong working

1	$kx^2 + (2k-8)x + k = 0$	M1	for attempt to obtain a 3 term quadratic in the form $ax^2 + bx + c = 0$,where b contains a term in k and a constant
	$b^2 - 4ac > 0$ so $(2k-8)^2 - 4k^2 (>0)$	DM1	for use of $b^2 - 4ac$
	$4k^2 - 32k + 64 - 4k^2 (>0)$	DM1	for attempt to simplify and solve for k
	leading to $k < 2$ only	A1	A1 must have correct sign
2	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = -5x (+c)$	M1	for attempt to integrate, do not penalise omission of arbitrary constant.
	When $x = -1$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 2$ leading to		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -5x - 3$	A1	Must have $\frac{\mathrm{d}y}{\mathrm{d}x} = \ldots$
	$y = -\frac{5x^2}{2} - 3x + d$	DM1	for attempt to integrate *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$, but penalise omission of arbitrary constant.
	When $x = -1, y = 3$ leading to		
	$y = \frac{5}{2} - \frac{5x^2}{2} - 3x$	A1	
	Alternative scheme:		
	$y = ax^2 + bx + c$ so $\frac{\mathrm{d}y}{\mathrm{d}x} = 2ax + b$	M1	for use of $y = ax^2 + bx + c$, differentiation and use of conditions to give an equation in a and b
	When $x = -1$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 2$		
	so $-2a + b = 2$	A1	for a correct equation
	$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 2a$	DM1	for a second differentiation to obtain a
	so $a = -\frac{5}{2}$, $b = -3$, $c = \frac{5}{2}$	A1	for a, b and c all correct

3	$\sqrt{(\sec^2\theta - 1)} + \sqrt{(\operatorname{cosec}^2\theta - 1)} = \sec\theta\operatorname{cosec}\theta$		
	LHS $= \tan\theta + \cot\theta$	B1	may be implied by the next line
	$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$	B1	for dealing with $\tan\theta$ and $\cot\theta$ in terms of $\sin\theta$ and $\cos\theta$
	$= \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$	M1	for attempt to obtain as a single fraction
	$= \dfrac{1}{\sin\theta\cos\theta}$	M1	for the use of $\sin^2\theta + \cos^2\theta = 1$ in correct context
	$= \sec\theta\operatorname{cosec}\theta$	A1	Must be convinced as AG
	Alternate scheme:		
	LHS $= \tan\theta + \cot\theta$		
	$= \tan\theta + \dfrac{1}{\tan\theta}$	B1	may be implied by subsequent work
	$= \dfrac{\tan^2\theta + 1}{\tan\theta}$	M1	for attempt to obtain as a single fraction
	$= \dfrac{\sec^2\theta}{\tan\theta}$	B1	for use of the correct identity
	$= \dfrac{\sec\theta}{\tan\theta} \times \sec\theta$	M1	for 'splitting' $\sec^2\theta$
	$= \operatorname{cosec}\theta\sec\theta$	A1	Must be convinced as AG
4 (a) (i)	28	B1	
(ii)	20160	B1	
(iii)	$6 \times (5 \times 4 \times 3)$ oe to give 360 $6 \times (5 \times 4 \times 3) \times 2$	B1	for realising that the music books can be arranged amongst themselves and consideration of the other 5 books
	$= 720$	B1	for the realisation that the above arrangement can be either side of the clock.
(b)	Either ${}^{10}C_6 - {}^{7}C_6 = 210 - 7$	B1, B1	B1 for ${}^{10}C_6$, B1 for ${}^{7}C_6$
	$= 203$	B1	
	Or 1W 5M = 63 2W 4M = 105	B1	for 1 case correct, must be considering more than 1 different case, allow C notation
	3W 3M = 35	B1	for the other 2 cases, allow C notation
	Total = 203	B1	for final result

5 **(i)**	$\frac{dy}{dx} = (x-3)\frac{4x}{2x^2+1} + \ln(2x^2+1)$ when $x = 2$, $\frac{dy}{dx} = -\frac{8}{9} + \ln 9$ oe or 1.31 or better	B1 M1 A1 A1	for correct differentiation of ln function for attempt to differentiate a product for correct product, terms must be bracketed where appropriate for correct final answer
(ii)	$\partial y \approx$ (answer to **(i)**) × 0.03 $= 0.0393$, allow awrt 0.039	M1 A1FT	for attempt to use small changes follow through on *their* numerical answer to **(i)** allow to 2 sf or better
6 **(i)**	$A \cap B = \{3\}$	B1	
(ii)	$A \cup C = \{1, 3, 5, 6, 7, 9, 11, 12\}$	B1	
(iii)	$A' \cap C = \{1, 5, 7, 11\}$	B1	
(iv)	$(D \cup B)' = \{1, 9\}$	B1	
(v)	Any set containing up to 5 positive even numbers $\leqslant 12$	B1	
7 **(i)**	Gradient $= \frac{0.2}{0.8} = 0.25$ $b = 0.25$ Either $6 = 0.25(2.2) + c$ Or $5.8 = 0.25(1.4) + c$ leading to $A = 233$ or $e^{5.45}$ **Alternative schemes**: Either $6 = b(2.2) + c$, $5.8 = b(1.4) + c$ Or $e^6 = A(e^{2.2})^b$, $e^{5.8} = A(e^{1.4})^b$ Leading to $A = 233$ or $e^{5.45}$ and $b = 0.25$	M1 A1 M1 A1 M1 DM1 A1, A1	for attempt to find the gradient for a correct substitution of values from either point and attempt to obtain c or solution by simultaneous equations dealing with $c = \ln A$ for 2 simultaneous equations as shown for attempt to solve to get at least one solution for one unknown A1 for each
(ii)	Either $y = 233 \times 5^{0.25}$ Or $\ln y = 0.25 \ln 5 + \ln 233$ leading to $y = 348$	M1 A1	for correct use of either equation in attempt to obtain y using *their* value of A and of b found in **(i)**

8	$\frac{dy}{dx} = \frac{2(x^2+5)^{\frac{1}{2}} - \frac{1}{2}(2x)(x^2+5)^{-\frac{1}{2}}(2x-1)}{x^2+5}$ or $\frac{dy}{dx} = 2(x^2+5)^{-\frac{1}{2}} - \frac{1}{2}(2x)(x^2+5)^{-\frac{3}{2}}(2x-1)$	B1 M1 A1	for $\frac{1}{2}(2x)(x^2+5)^{-\frac{1}{2}}$ for a quotient or $-\frac{1}{2}(2x)(x^2+5)^{-\frac{3}{2}}$ for a product allow if either seen in separate working for attempt to differentiate a quotient or a **correct** product for all correct, allow unsimplified
	When $x=2, y=1$ and $\frac{dy}{dx}=\frac{4}{9}$ (allow 0.444 or 0.44)	B1, B1	B1 for each
	Equation of tangent: $y-1=\frac{4}{9}(x-2)$ $(9y=4x+1)$	M1 A1	for attempt at straight line, must be tangent using *their* gradient and y allow unsimplified.
9 (i)	$\frac{2}{3}(4+x)^{\frac{3}{2}}\ (+c)$	B1,B1	B1 for $k(4+x)^{\frac{3}{2}}$ only, B1 for $\frac{2}{3}(4+x)^{\frac{3}{2}}$ only Condone omission of c
(ii)	Area of trapezium $=\left(\frac{1}{2}\times5\times5\right)$ $=12.5$	M1 A1	for attempt to find the area of the trapezium
	Area $=\left[\frac{2}{3}(4+x)^{\frac{3}{2}}\right]_0^5 - \left(\frac{1}{2}\times5\times5\right)$ $=\left(\frac{2}{3}\times27\right)-\frac{16}{3}-\frac{25}{2}$ $=\frac{1}{6}$ or awrt 0.17	M1 A1 A1	for correct use of limits using $k(4+x)^{\frac{3}{2}}$ only (must be using 5 and 0) for $18-\frac{16}{3}$ or equivalent
	Alternative scheme: Equation of AB $y=\frac{1}{5}x+2$	M1	for a correct attempt to find the equation of AB
	Area $=\int_0^5 \sqrt{4+x} - \left(\frac{1}{5}x+2\right)dx$ $=\left[\frac{2}{3}(4+x)^{\frac{3}{2}} - \frac{x^2}{10} - 2x\right]_0^5$ $=\left(\frac{2}{3}\times27\right)-\frac{16}{3}-\frac{25}{2}$ $=\frac{1}{6}$ or awrt 0.17	M1 A1 A1 A1	for correct use of limits using $k(4+x)^{\frac{3}{2}}$ only (must be using 5 and 0) for $18-\frac{16}{3}$ or equivalent for 12.5 or equivalent

10 (i)	All sides are equal to the radii of the circles which are also equal	B1	for a convincing argument
(ii)	Angle $CBE = \frac{2\pi}{3}$	B1	must be in terms of π, allow 0.667π, or better
(iii)	$DE = 10\sqrt{3}$	M1 A1	for correct attempt to find *DE* using *their* angle *CBE* for correct *DE*, allow 17.3 or better
	Arc $CE = 10 \times \frac{2\pi}{3}$	M1	for attempt to find arc length with *their* angle *CBE* (20.94)
	Perimeter $= 20 + 10\sqrt{3} + \frac{20\pi}{3}$	M1	for 10 + 10 + *DE* + an arc length
	$= 58.3$ or 58.2	A1	allow unsimplified
(iv)	Area of sector: $\frac{1}{2} \times 10^2 \times \frac{2\pi}{3} = \frac{100\pi}{3}$	M1	for sector area using *their* angle *CBE* allow unsimplified, may be implied
	Area of triangle: $\frac{1}{2} \times 10^2 \times \sin\frac{2\pi}{3} = 25\sqrt{3}$	M1	for triangle area using *their* angle *DBE* which must be the same as *their* angle *CBE*, allow unsimplified, may be implied
	Area $= \frac{100\pi}{3} + 25\sqrt{3}$ or awrt 148	A1	allow in either form

11 (a) (i)	$(x+3)^2-5$	B1, B1	B1 for 3, B1 for -5
(ii)	$y \geqslant 4$ or $\mathrm{f} \geqslant 4$	B1	Correct notation or statement must be used
(iii)	$y=\sqrt{x+5}-3$	M1 A1	for a correct attempt to find the inverse function must be in the correct form and positive root only
	Domain $x \geqslant 4$	B1FT	Follow through on *their* answer to **(ii),** must be using x
(b)	$\mathrm{h}^2\mathrm{g}(x)=\mathrm{h}^2(\mathrm{e}^x)$ $=\mathrm{h}(5\mathrm{e}^x+2)$ $=25\mathrm{e}^x+12$ $25\mathrm{e}^x+12=37,$ leading to $x=0$	M1 M1 DM1 A1	for correct order for dealing with h^2 for solution of equation (dependent on both previous M marks)
	Alternative scheme 1: $\mathrm{hg}(x)=\mathrm{h}^{-1}(37)$ $\mathrm{h}^{-1}(37)=7$ $5\mathrm{e}^x+2=7,$ leading to $x=0$	 M1 M1 DM1 A1	 for correct order for dealing with $\mathrm{h}^{-1}(37)$ for solution of equation (dependent on both previous M marks)
	Alternative scheme 2: $\mathrm{g}(x)=\mathrm{h}^{-2}(37)$ $\mathrm{h}^{-2}(37)=1$ $\mathrm{e}^x=1,$ leading to $x=0$	 M1 M1 DM1 A1	 for correct order for dealing with $\mathrm{h}^{-2}(37)$ for solution of equation (dependent on both previous M marks)

12	$x^2 + 6x - 16 = 0$ or $y^2 + 10y - 75 = 0$ leading to $(x+8)(x-2) = 0$ or $(y-5)(y+15) = 0$	M1 DM1	for attempt to obtain a 3 term quadratic in terms of one variable only for attempt to solve quadratic equation
	so $x = 2, y = 5$ and $x = -8, y = -15$	A1, A1	A1 for each 'pair' of values.
	Midpoint $(-3, -5)$	B1	
	Gradient $= 2$, so perpendicular gradient $= -\frac{1}{2}$ Perpendicular bisector: $y + 5 = -\frac{1}{2}(x+3)$ $(2y + x + 13 = 0)$	M1	for attempt at straight line equation, must be using midpoint and perpendicular gradient
	Point C $(-13, 0)$	M1	for use of $y = 0$ in *their* line equation (but not $2x - y + 1 = 0$)
	Area $= \frac{1}{2}\begin{vmatrix} -13 & 2 & -8 & -13 \\ 0 & 5 & -15 & 0 \end{vmatrix}$	M1	for correct attempt to find area, may be using *their* values for A, B and C (C must lie on the x-axis)
	$= 125$	A1	
	Alternative method for area: $CM^2 = 125$, $AB^2 = 500$ Area $= \frac{1}{2} \times \sqrt{125} \times \sqrt{500}$	M1	for correct attempt to find area may be using *their* values for A, B and C
	$= 125$	A1	