

MARK SCHEME for the October/November 2013 series

4037 ADDITIONAL MATHEMATICS

4037/13

Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

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Mark Scheme Notes

Marks are of the following three types:

- M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B Accuracy mark for a correct result or statement independent of method marks.

When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

The symbol \surd implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

Note: B2 or A2 means that the candidate can earn 2 or 0.
B2, 1, 0 means that the candidate can earn anything from 0 to 2.

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<p>1 (i) ${}^6C_2 (2^4) (px)^2$ or $\binom{6}{2} 2^4 (px)^2$</p> $240p^2 = 60$ $p = \frac{1}{2}$ <p>(ii) coefficients of the terms needed</p> $(-1) {}^6C_1 (2)^5 p + (3 \times 60)$ $= 84$	<p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p> <p>M1</p> <p>B1</p> <p>A1</p> <p>[3]</p>	<p>Seen or implied, unsimplified</p> <p>M1 for their coefficient of $x^2 = 60$ and attempt to solve</p> <p>M1 for realising that 2 terms are involved</p> <p>B1 for $(-1) {}^6C_1 (2)^5 p$ or $-192p$, using their p.</p>
<p>2 $\lg \frac{y^2}{5y+60} = \lg 10$</p> <p>Or $\lg y^2 = \lg 10 (5y + 60)$</p> $y^2 - 50y - 600 = 0$ <p>leading to $y = -10, 60$</p> <p>y must be positive so $y = 60$</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>DM1</p> <p>A1</p> <p>[5]</p>	<p>B1 for $2 \lg y = \lg y^2$</p> <p>B1 for $1 = \lg 10$ or equivalent, allow when seen</p> <p>M1 for use of $\log A - \log B = \log A/B$ or $\log A + \log B = \log AB$</p> <p>DM1 for forming a 3 term quadratic equation and an attempt to solve</p> <p>A1 for $y = 60$ only</p>

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<p>3 $\tan^2 \theta - \sin^2 \theta = \frac{\sin^2 \theta}{\cos^2 \theta} - \sin^2 \theta$</p> $= \frac{\sin^2 \theta - \sin^2 \theta \cos^2 \theta}{\cos^2 \theta}$ $= \frac{\sin^2 \theta (1 - \cos^2 \theta)}{\cos^2 \theta}$ $= \frac{\sin^4 \theta}{\cos^2 \theta}$ $= \sin^4 \theta \sec^2 \theta$ <p>Alt solution 1</p> <p>Using $\tan^2 \theta = \sin^2 \theta \sec^2 \theta$</p> $\begin{aligned} \text{LHS} &= \sin^2 \theta \sec^2 \theta - \sin^2 \theta \\ &= \sin^2 \theta (\sec^2 \theta - 1) \\ &= \sin^2 \theta \tan^2 \theta \\ &= \sin^4 \theta \sec^2 \theta \end{aligned}$ <p>Alt solution 2</p> $\begin{aligned} \text{RHS} &= \sin^4 \theta \sec^2 \theta \\ &= \frac{\sin^2 \theta \sin^2 \theta}{\cos^2 \theta} \\ &= \frac{\sin^2 \theta (1 - \cos^2 \theta)}{\cos^2 \theta} \\ &= \frac{\sin^2 \theta - \sin^2 \theta \cos^2 \theta}{\cos^2 \theta} \\ &= \frac{\sin^2 \theta}{\cos^2 \theta} - \frac{\sin^2 \theta \cos^2 \theta}{\cos^2 \theta} \\ &= \tan^2 \theta - \sin^2 \theta \end{aligned}$	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[4]</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p>	<p>Marks are awarded only if they can lead to a complete proof for the methods other than those shown below</p> <p>M1 for dealing with tan and a fraction</p> <p>M1 for factorising</p> <p>M1 for use of identity $\cos^2 \theta + \sin^2 \theta = 1$</p> <p>A1 for all correct</p> <p>M1 use of $\tan^2 x = \sin^2 x \sec^2 x$</p> <p>M1 for factorising</p> <p>M1 for use of identity</p> <p>A1 for all correct</p> <p>M1 for splitting $\sin^4 \theta$ and use of identity</p> <p>M1 for multiplication</p> <p>M1 for writing as two terms and cancelling</p> <p>A1 for all correct</p>
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<p>4 (i) $\frac{dy}{dx} = \frac{(x+3)^2 2e^{2x} - e^{2x} 2(x+3)}{(x+3)^4}$</p> $= \frac{2e^{2x}(x+2)}{(x+3)^3}, A = 2$ <p>Alt solution</p> $\frac{dy}{dx} = e^{2x} (-2(x+3)^{-3}) + 2e^{2x}(x+3)^{-2}$ $= \frac{2e^{2x}(x+2)}{(x+3)^3}, A = 2$ <p>(ii) $x = -2, y = e^{-4}$</p>	<p>M1 A2, 1, 0 A1 [4]</p> <p>M1 A2,1,0 A1</p> <p>B1, B1 [2]</p>	<p>M1 for attempt at quotient rule -1 for each error Must be convinced of correct simplification e.g. sight of $(x+3-1)$ or $(x+2)(x+3)$</p> <p>M1 for attempt at product rule -1 for each error Must be convinced of correct simplification e.g. sight of $(x+3-1)$ or $(x+2)(x+3)$</p> <p>Accept $1/e^4$</p>
<p>5 (i) $f^2(x) = f(2x^3)$</p> $= 2(2x^3)^3 \text{ or } 2\left(2\left(\frac{1}{2}\right)^3\right)^3$ $= 2^{-5}$ <p>Alt method</p> $f\left(\frac{1}{2}\right) = \frac{1}{4} \quad f\left(\frac{1}{4}\right) = 2^{-5}$ <p>(ii) $f'(x) = g'(x)$ $6x^2 = 4 - 10x$</p> <p>Leading to $(3x - 1)(x + 2) = 0$</p> $x = \frac{1}{3}, -2$	<p>M1 A1 [2]</p> <p>M1 A1</p> <p>B1 B1</p> <p>M1 A1 [4]</p>	<p>M1 for $= 2(2x^3)^3$ or $2\left(2\left(\frac{1}{2}\right)^3\right)^3$</p> <p>For 2^{-5} only</p> <p>M1 for f of their $f\left(\frac{1}{2}\right)$ For 2^{-5} only</p> <p>B1 for $6x^2$ B1 for $4 - 10x$</p> <p>M1 for solution of quadratic equation obtained from differentiation of both A1 for both</p>

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<p>6 Area under the curve:</p> $\int_0^{\sqrt{2}} 4 - x^2 \, dx = \left[4x - \frac{x^3}{3} \right]_0^{\sqrt{2}}$ $= \left(4\sqrt{2} - \frac{2\sqrt{2}}{3} \right) - (0)$ $= \frac{10\sqrt{2}}{3}$ <p>Area of trapezium =</p> $\frac{1}{2}(4 + 2)(\sqrt{2}) = 3\sqrt{2}$ <p>Shaded area = $\frac{10\sqrt{2}}{3} - 3\sqrt{2}$</p> <p>Shaded area = $\frac{\sqrt{2}}{3}$</p> <p>Or: Equation of chord:</p> $y = 4 - \sqrt{2}x$ <p>Shaded area = $\int_0^{\sqrt{2}} 4 - x^2 - 4 + \sqrt{2}x \, dx$</p> $\left[\frac{\sqrt{2}}{2}x^2 - \frac{x^3}{3} \right]_0^{\sqrt{2}} = \frac{\sqrt{2}}{3}$	<p>M1 A1</p> <p>DM1</p> <p>B1</p> <p>M1</p> <p>A1 [6]</p> <p>B1</p> <p>M1 M1</p> <p>√A1 DM1 A1 [6]</p>	<p>M1 for attempt to integrate</p> <p>DM1 for application of limits</p> <p>B1 for area of trapezium, allow unsimplified</p> <p>M1 for subtraction of the two areas</p> <p>Must be in this form</p> <p>B1 for the equation of the chord unsimplified</p> <p>M1 for subtraction M1 for attempt to integrate</p> <p>√A1 for $\left[-m \frac{x^2}{2} - \frac{x^3}{3} \right]$ or equivalent, where m is the gradient of their chord DM1 for application of limits</p>
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<p>7 (i) $2t^2 - 2(t^2 - t + 1)$</p> <p>Leading to, $t = \frac{3}{2}$</p> <p>(ii) $\mathbf{A} = \begin{pmatrix} 6 & 2 \\ 7 & 3 \end{pmatrix}, \mathbf{A}^{-1} = \frac{1}{4} \begin{pmatrix} 3 & -2 \\ -7 & 6 \end{pmatrix}$</p> $\begin{pmatrix} 6 & 2 \\ 7 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 11 \end{pmatrix}$ $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 3 & -2 \\ -7 & 6 \end{pmatrix} \begin{pmatrix} 10 \\ 11 \end{pmatrix}$ $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \text{ leading to } x = 2, y = -1$	<p>B1</p> <p>M1 A1 [3]</p> <p>B1, B1</p> <p>B1</p> <p>M1</p> <p>A1 [5]</p>	<p>Correct determinant seen unsimplified</p> <p>M1 for simplification and solution A1 for solution of $\det \mathbf{A} = 1$ only, not $1/\det \mathbf{A} = 1$</p> <p>B1 for $\frac{1}{4}$, B1 for matrix</p> <p>B1 for dealing correctly with the factor of 2</p> <p>M1 for pre-multiplying their $\begin{pmatrix} 10 \\ 11 \end{pmatrix}$ by their \mathbf{A}^{-1} to obtain a column matrix</p> <p>Allow $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ for A1</p>
<p>8 (i) $\frac{1}{2}(4^2)\sin \theta = 7.5$</p> <p>$\sin \theta = \frac{15}{16}, \theta = 1.215 \dots$</p> <p>(ii) $\sin \frac{\theta}{2} = \frac{1}{2} \frac{CD}{4}, (CD = 4.567)$</p> <p>Arc length = $6(1.215)$</p> <p>Perimeter = $2 + 2 + 6(1.215) + \text{their } CD$</p> <p>= awrt 15.9</p> <p>(iii) Area = $\frac{1}{2} 6^2 (1.215) - 7.5$</p> <p>= 14.4 (awrt)</p>	<p>M1</p> <p>A1 [2]</p> <p>M1</p> <p>B1</p> <p>M1</p> <p>A1 [4]</p> <p>B1 M1</p> <p>A1 [3]</p>	<p>M1 for attempt to find the area of the triangle and equate to 7.5</p> <p>A1 for solution to obtain the given answer Solution must include 1.2153... or 1.2154</p> <p>M1 for attempt to find CD</p> <p>B1 for arc length</p> <p>M1 for sum of 4 appropriate lengths</p> <p>B1 for sector area M1 for subtraction of the 2 areas</p>

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<p>9 (a) (i) $6(1 - \cos^2 x) = 5 + \cos x$ $6 \cos^2 x + \cos x - 1 = 0$ $(3 \cos x - 1)(2 \cos x + 1) = 0$</p> <p>$x = 70.5^\circ \quad x = 120^\circ$</p> <p>(ii) $\cos x = \sin y$</p> <p>$\sin y = \frac{1}{3}$ only so $y = 19.5^\circ, 160.5^\circ$</p> <p>(b) $\cot z(4 \cot z - 3) = 0$</p> <p>$\cot z = 0, \quad z = \frac{\pi}{2}$</p> <p>$\cot z = \frac{3}{4}, \quad \tan z = \frac{4}{3}$ so $z = 0.927$</p>	<p>M1 M1</p> <p>A1, A1 [4]</p> <p>DM1</p> <p>$\sqrt{A1}, \sqrt{A1}$ [3]</p> <p>M1</p> <p>B1</p> <p>M1 A1 [4]</p>	<p>M1 for use of $\sin^2 x = (1 - \cos^2 x)$ correctly M1 for solution of a 3 term quadratic in \cos and attempt at solution of a trig equation</p> <p>A1 for each correct solution</p> <p>DM1 for relating $\cos x$ and $\sin y$ or other correct method of solution</p> <p>M1 for attempt to use a factor</p> <p>B1 for $\frac{\pi}{2}$ (1.57)</p> <p>M1 dealing with \cot and attempt at solution</p>										
<p>10 (i) $\lg s$</p> <p>(ii)</p> <table border="1" data-bbox="247 1220 686 1299"> <tr> <td>lgs</td> <td>0.3</td> <td>0.6</td> <td>0.78</td> <td>0.9</td> </tr> <tr> <td>lgt</td> <td>1.4</td> <td>0.8</td> <td>0.44</td> <td>0.19</td> </tr> </table> <p>(iii) <u>No marks in this part unless lgt v lgs graph is used</u> Gradient : $n = -2$ (allow $-2.1 \rightarrow -1.9$)</p> <p>Intercept : $\log k$, or other method $k = 100$ (allow $90 \rightarrow 120$)</p> <p>Alt method Using simultaneous equations, points used must lie on the plotted line.</p> <p>(iv) When $t = 4$, $\lg t = 0.6$ so $\lg s = 0.69$ $s = 4.9$ (allow $4.8 \rightarrow 5.2$)</p>	lgs	0.3	0.6	0.78	0.9	lgt	1.4	0.8	0.44	0.19	<p>B1 [1]</p> <p>M1 DM1 A1 [3]</p> <p>M1A1</p> <p>M1, A1 [4]</p> <p>M2 A1, A1</p> <p>M1 A1 [2]</p>	<p>Allow in table or on graph if no contradiction</p> <p><u>No marks for graph unless lgt against lgs (or lnt against lns)</u></p> <p>M1 for 3 or more points correct DM1 for a line through 3 or 4 correct points A1 all points correct with a straight line extending at least from first point to last point</p> <p>M1 calculates gradient A1 for $n = -2$</p> <p>M1 for use of intercept and dealing with logarithm correctly (can use another point)</p> <p>Must attempt to solve 2 valid equations. $k = 100$ and $n = -2$</p> <p>M1 for valid method using either the correct graph or using $\lg t = n \lg s + \lg k$ or $t = ks^n$ using their n and their k</p>
lgs	0.3	0.6	0.78	0.9								
lgt	1.4	0.8	0.44	0.19								

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<p>11 (i) $\left[e^{2x} + \frac{5}{4}e^{-2x} \right]_0^k$</p> $\left(e^{2k} + \frac{5}{4}e^{-2k} \right) - \left(1 + \frac{5}{4} \right) = 3$ $e^{2k} + \frac{5}{4}e^{-2k} - \frac{12}{4} = 0$ $4e^{4k} - 12e^{2k} + 5 = 0$	<p>B1, B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[5]</p>	<p>B1 for each term integrated correctly, allow unsimplified</p> <p>M1 for application of limits to an integral of the form $Ae^{2x} \pm Be^{-2x}$</p> <p>M1 for equating to $\frac{3}{4}$ and attempt to rearrange to obtain a 3 term equation. Must be using an integral of the form $Ae^{2x} \pm Be^{-2x}$</p> <p>Answer given, so must be convinced</p>
<p>(ii) $4y^2 - 12y + 5 = 0$</p> <p>leading to $e^{2k} = \frac{5}{2}$, $e^{2k} = \frac{1}{2}$</p> <p>$k = 0.458, -0.347$</p>	<p>M1</p> <p>M1</p> <p>A1, A1</p> <p>[4]</p>	<p>M1 for solution of quadratic equation</p> <p>M1 for solving equations involving exponentials</p> <p>A1 for each</p>