CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the October/November 2012 series

4037 ADDITIONAL MATHEMATICS

4037/13 Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2012 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

The following abbreviations may be used in a mark scheme or used on the scripts:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

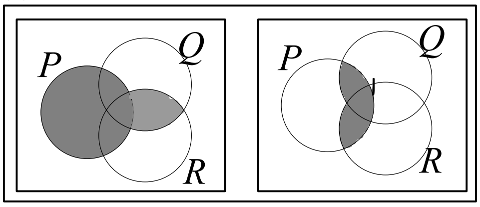
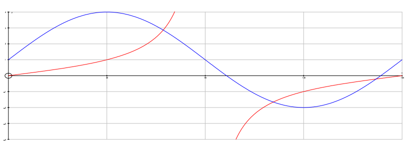
MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

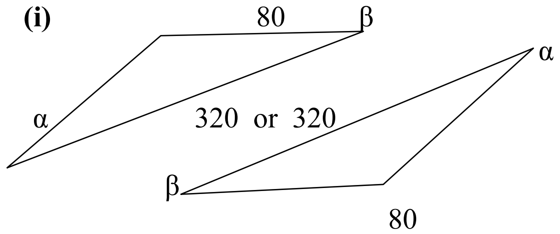
OW –1,2 This is deducted from A or B marks when essential working is omitted.

PA –1 This is deducted from A or B marks in the case of premature approximation.

S –1 Occasionally used for persistent slackness – usually discussed at a meeting.

EX –1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

1 **(a)** 	B1 B1 [2]	
(b) **(i)** $F \subset B$, $B \supset F$, $F \subseteq B$ and $B \supseteq F$, $F \cap B = F$ or $F \cup B = B$	B1 [1]	
(ii) $S \cap F = \varnothing$, $S \cap F = \{\}$ or $n(S \cap F) = 0$	B1 [1]	
2 **(i)** 3 or $\frac{3}{1}$	B1 [1]	
(ii) $\frac{dy}{dx} = \frac{3\sin t}{4\cos^2 t}$ $\left(= \frac{3\sin t}{3}\right)$	M1	M1 correct substitution in $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ o.e.
$= \frac{3\sin\frac{\pi}{6}}{3}$	DM1	
$= 0.5$	A1 [3]	DM1 for use of their '3' and substitution of $\frac{\pi}{6}$.
3 **(i)** ${}^{15}C_7 = 6435$	B1 [1]	
(ii) ${}^{6}C_2 \times {}^{9}C_5 = 1890$	M1,A1 [2]	M1 for a correct method
(iii) No women: ${}^{9}C_7 = 36$ 6435 – 36 = 6399	B1 M1 A1 [3]	B1 for ${}^{9}C_7 = 36$ M1 for a complete, correct method
4 **(i)** 	B1 B1, B1 [3]	B1 for $y = \tan x$ $y = 1 + 3\sin 2x$ B1 for shape of curve B1 for a 'curve' starting at 1 and finishing at 1 and going between 4 and –2.
(ii) $\left(\frac{\pi}{4}, 4\right)$ and $\left(\frac{3\pi}{4}, -2\right)$	B1, B1 [2]	B1 for each or B1 for both x coordinates correct
(iii) 3	B1ft [1]	Ft from their **(i)** or correct

5 (i)		B1	B1 for correct triangle Could be implied by subsequent working.
	$\frac{320}{\sin 120°} = \frac{80}{\sin \alpha}$	M1	M1 for complete method (sine rule and/or cosine rule) to find α or β
	α = 12.5° (or β = 47.5°)	A1	A1 for α (or β)
	Bearing = 042.5° or 043°	A1 [4]	A1 for bearing
(ii)	$\frac{v_r}{\sin 47.5°} = \frac{320}{\sin 120°}$, $v_r = 272.4$	M1	M1 for use of complete method (sine rule and/or cosine rule) to find v_r or x
	or $\frac{x}{\sin 120°} = \frac{450}{\sin 47.5°}$	A1	For either $v = 272$ or $x = 529$
	Time = $\frac{450}{272.4}$ or $\frac{528.6}{320}$	DM1	DM1 for $\frac{450}{\text{their velocity}}$ or their $\frac{x}{320}$
	= 1.65	A1 [4]	
6	$(p+x)^6 = p^6 + 6p^5x + 15p^4x^2 + 20p^3x^3 \ldots$		
(i)	$15p^4 = \frac{3}{2} \times 20p^3$,	B1, B1 M1	B1 for $15p^4$, B1 for $20p^3$ M1 for correct attempt to equate
	$p = 2$	A1 [4]	
(ii)	need $p^6(1) + 6p^5(-2) + 15p^4(1)$	B1	B1 for both $p^6, 6p^5$ (allow in **(i)**)
	= – 80	M1 A1 [3]	M1 for attempt using 3 terms for $\left(1 - \frac{1}{x}\right)^2$ and identifying and adding at least two terms independent of x

7 **(i)** $\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{(t^2+1)-t(2t)}{(t^2+1)^2}$	M1	M1 for attempt to differentiate a quotient or product
	A1	A1 all correct, allow unsimplified
When $\frac{\mathrm{d}x}{\mathrm{d}t}=0,\ t=1$ so $x=\frac{1}{2}$	DM1	DM1 for equating to zero and attempt to solve to find t.
	A1 [4]	A1 for $x=\frac{1}{2}$
(ii) $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=\frac{(t^2+1)^2(-2t)-(1-t^2)4t(t^2+1)}{(t^2+1)^4}$	M1	M1 for attempt to differentiate a quotient or product to find acceleration
	A1	A1 correct unsimplified
When t = 1, acceleration = – 0.5	A1 [3]	
8 **(i)** f(2) = 24 + 20 + 2p + 8 = 0	M1	M1 for use of 2 and equating to zero, or use of comparing coefficients or algebraic long division
$p = -26$	A1	
$a=3,\ b=11,\ c=-4$	B3 [5]	B1 for each of a, b and c
(ii) $(x-2)(3x-1)(x+4)$	M1 A1 [2]	M1 for attempt to obtain 3 factors
9 **(i)** $AD^2=20^2+10^2-2(20)(10)\cos\frac{5\pi}{6}$	M1	M1 finding AD using cosine rule including square root.
	B1	B1 for either arc length
Perimeter = $\frac{10\pi}{6}+\frac{20\pi}{6}+2(29.1)$	DM1	DM1 for correct plan before evaluation using correct arc lengths and AD
= 73.9	A1 [4]	Awrt 73.9
(ii) Area = $\frac{1}{2}10^2\left(\frac{\pi}{6}\right)+\frac{1}{2}20^2\left(\frac{\pi}{6}\right)+2\left(\frac{1}{2}(10)(20)\sin\frac{5\pi}{6}\right)$	M1	M1 for area of triangle using the sine rule, or complete correct method
	B1	B1 for ½ $10^2(\pi/6)$ or ½ $20^2(\pi/6)$
	DM1	DM1 for correct plan before evaluation using correct sector and triangle areas.
= 231	A1 [4]	Awrt 231

10 (i) $(\sec^2 x - 1) - 2\sec x + 1 = 0$ $\sec x(\sec x - 2) = 0$ $\cos x = 0.5,\ x = 60°, 300°$ Alt scheme: $\frac{\sin^2 x}{\cos^2 x} - \frac{2}{\cos x} + 1 = 0$ $\sin^2 x - 2\cos x + \cos^2 x = 0,$ $\cos x = 0.5, x = 60°, 300°$	M1 M1 A1, A1 [4]	M1 for use of correct identity M1 for solution of quadratic in sec or cos A1 for one correct solution M1 for dealing with tan and sec correctly and for use of correct identity M1 for solution to obtain cos x
(ii) $\tan^2 3y = \frac{1}{5},\ \tan 3y = (\pm)\frac{1}{\sqrt{5}}$ (or $\sin 3y = (\pm)\frac{1}{\sqrt{6}},\ \cos 3y = (\pm)\frac{\sqrt{5}}{\sqrt{6}}$) $3y = 0.42,\ 2.72$, etc. $y = 0.140,\ 0.907,\ 1.19,\ 1.95$	M1 M1 A1, A1 [4]	M1 for correctly obtaining in terms of 1 trig ratio and square rooting M1 for dealing with '3' correctly A1 for first A1 for others
(iii) $\sin\left(z + \frac{\pi}{4}\right) = \frac{2}{5}$ $z + \frac{\pi}{4} = 0.4115,\ \ 2.730,\ \ 6.695$ $z = 1.94,\ \ 5.91$	M1 DM1 A1,A1 [4]	M1 for dealing with '2' and cosec correctly DM1 for dealing with $\frac{\pi}{4}$ correctly
11 EITHER **(i)** $\frac{dy}{dx} = 5e^x - 3e^{-x}$ When $x = \ln\frac{3}{5},\ \frac{dy}{dx} = -2$ When $x = \ln\frac{3}{5},\ y = 8$ Tangent: $y - 8 = -2\left(x - \ln\frac{3}{5}\right)$ When $y = 0,\ x = 4 + \ln\frac{3}{5}$ (3.49)	B1 B1 B1 M1 A1 [5]	B1 For correct derivative B1 for grad = –2 from correct working B1 for $y = 8$ Equation of a tangent using their gradient and their 8
(ii) $\int_0^a 5e^x + 3e^{-x}\,dx = 12$ $\left[5e^x - 3e^{-x}\right]_o^a = 12$ $5e^a - 3e^{-a} - 2 = 12$ $5e^{2a} - 14e^a - 3 = 0$	B1 M1 A1 [3]	B1 for correct integration M1 for correct use of limits Answer given so need to see some manipulation
(iii) $(5e^a + 1)(e^a - 3) = 0$ $a = \ln 3$, 1.1 or 1.10	M1 M1 A1 [3]	M1 for recognising and dealing with quadratic M1 for correct method of solution to obtain a

11 OR **(i)** $\frac{dy}{dx}=\frac{(1+e^{2x})6e^{2x}-3e^{2x}(2e^{2x})}{(1+e^{2x})^2}$	M1 A2,1,0	M1 for attempt to differentiate a quotient or product –1 each error
$\frac{6e^{2x}}{(1+e^{2x})^2}$ $\therefore A=6$	A1 [4]	For 6 obtained from correct working.
(ii) When $x=0,\ y=\frac{3}{2}$	B1	B1 for $y=\frac{3}{2}$
$\frac{dy}{dx}=\frac{3}{2}$	B1ft	B1 for grad = $\frac{A}{4}$
$\therefore\ y-\frac{3}{2}=\frac{3}{2}x$	B1ft [3]	Ft their y_0 and $\frac{A}{4}$
(iii) $\int\frac{e^{2x}}{(1+e^{2x})^2}\,dx=\frac{1}{2}\left(\frac{e^{2x}}{(1+e^{2x})}\right)\ (+c)$	M1 A1ft	M1 for attempt at 'reverse differentiation' Ft on their A, i.e. $\frac{3}{A}$ for a correct statement
$\frac{1}{2}\left[\frac{e^{2x}}{(1+e^{2x})}\right]_0^{\ln 3}=\frac{1}{2}\left(\frac{9}{10}-\frac{1}{2}\right)$	M1	M1 for correct use of limits
$=0.2$	A1ft [4]	Ft $\frac{A}{30}$