



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS

4037/23

Paper 2

October/November 2010

2 hours

Additional Materials: Answer Booklet/Paper
 Graph paper (1 sheet)



READ THESE INSTRUCTIONS FIRST

- If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
- Write your Centre number, candidate number and name on all the work you hand in.
- Write in dark blue or black pen.
- You may use a soft pencil for any diagrams or graphs.
- Do not use staples, paper clips, highlighters, glue or correction fluid.

- Answer **all** the questions.
- Write your answers on the separate Answer Booklet/Paper provided.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
- The use of an electronic calculator is expected, where appropriate.
- You are reminded of the need for clear presentation in your answers.

- At the end of the examination, fasten all your work securely together.
- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 80.

This document consists of **7** printed pages and **1** blank page.



Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} .$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

3

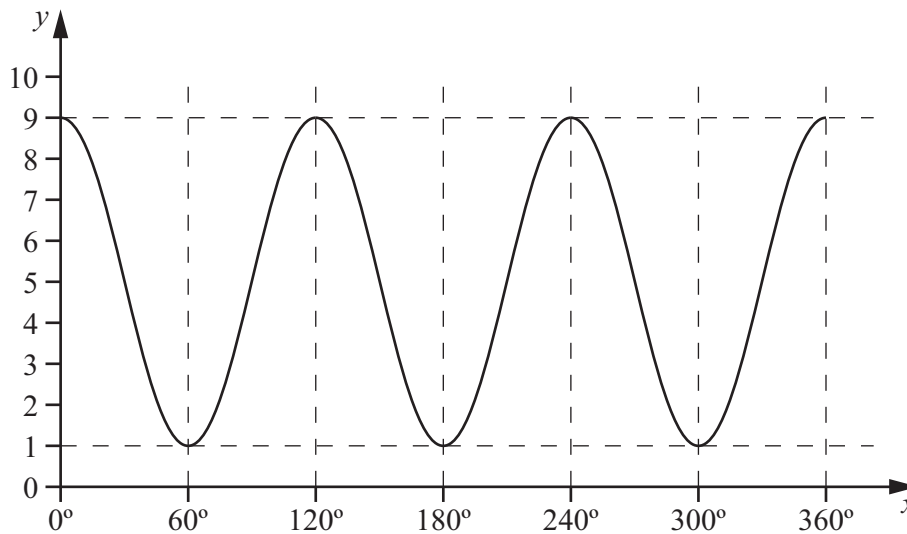
1 The two variables x and y are such that $y = \frac{10}{(x+4)^3}$.

(i) Find an expression for $\frac{dy}{dx}$. [2]

(ii) Hence find the approximate change in y as x increases from 6 to $6+p$, where p is small. [2]

2 Find the equation of the curve which passes through the point (4, 22) and for which $\frac{dy}{dx} = 3x(x-2)$. [4]

3 (a)



The diagram shows the curve $y = A \cos Bx + C$ for $0^\circ \leq x \leq 360^\circ$. Find the value of

(i) A , (ii) B , (iii) C . [3]

(b) Given that $f(x) = 6 \sin 2x + 7$, state

(i) the period of f , [1]

(ii) the amplitude of f . [1]

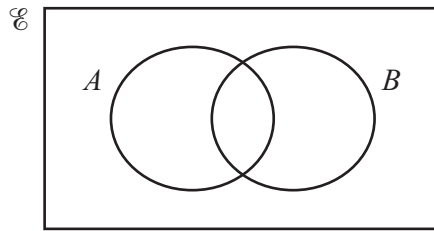
- 4 (i) Find, in ascending powers of x , the first 4 terms of the expansion of $(1 + x)^6$. [2]
- (ii) Hence find the coefficient of p^3 in the expansion of $(1 + p - p^2)^6$. [3]

5 (a) Given that $\mathbf{A} = \begin{pmatrix} 2 & -4 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 3 & -1 \\ 0 & 5 \\ -2 & 7 \end{pmatrix}$, find the matrix product \mathbf{AB} . [2]

(b) Given that $\mathbf{C} = \begin{pmatrix} 3 & 5 \\ -2 & -4 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} 6 & -4 \\ 2 & 8 \end{pmatrix}$, find

- (i) the inverse matrix \mathbf{C}^{-1} , [2]
- (ii) the matrix \mathbf{X} such that $\mathbf{CX} = \mathbf{D}$. [2]

- 6 (a)



Copy the diagram above and shade the region which represents the set $A' \cup B$. [1]

- (b) The sets P , Q and R are such that

$$P \cap Q = \emptyset \text{ and } P \cup Q \subset R.$$

Draw a Venn diagram showing the sets P , Q and R . [2]

- (c) In a group of 50 students F denotes the set of students who speak French and S denotes the set of students who speak Spanish. It is given that $n(F) = 24$, $n(S) = 18$, $n(F \cap S) = x$ and $n(F' \cap S') = 3x$. Write down an equation in x and hence find the number of students in the group who speak neither French nor Spanish. [3]

- 7 The line $y = 2x - 6$ meets the curve $4x^2 + 2xy - y^2 = 124$ at the points A and B . Find the length of the line AB . [7]

8 (i) Show that $(5 + 3\sqrt{2})^2 = 43 + 30\sqrt{2}$.

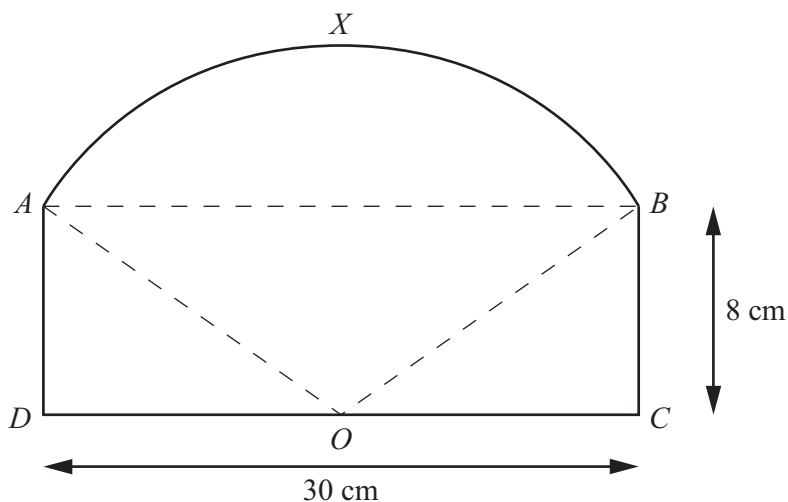
Hence find, **without using a calculator**, the positive square root of

(ii) $86 + 60\sqrt{2}$, giving your answer in the form $a + b\sqrt{2}$, where a and b are integers, [2]

(iii) $43 - 30\sqrt{2}$, giving your answer in the form $c + d\sqrt{2}$, where c and d are integers, [1]

(iv) $\frac{1}{43 + 30\sqrt{2}}$, giving your answer in the form $\frac{f + g\sqrt{2}}{h}$, where f , g and h are integers. [3]

9



The diagram shows a rectangle $ABCD$ and an arc AXB of a circle with centre at O , the mid-point of DC . The lengths of DC and BC are 30 cm and 8 cm respectively. Find

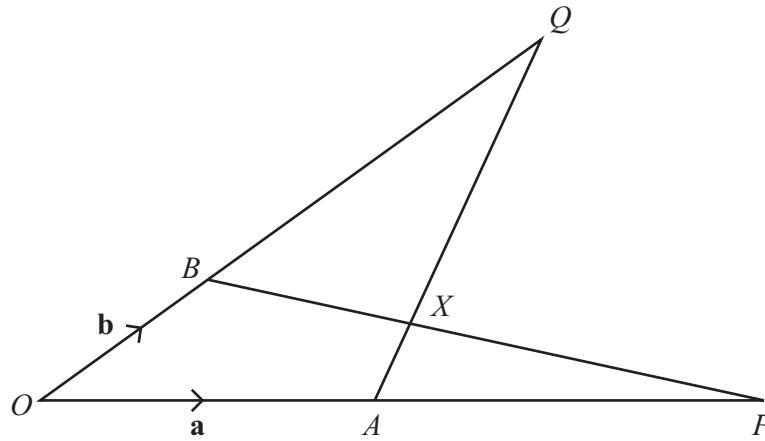
(i) the length of OA , [2]

(ii) the angle AOB , in radians, [2]

(iii) the perimeter of figure $ADOCBXA$, [2]

(iv) the area of figure $ADOCBXA$. [2]

10 The equation of a curve is $y = x^2e^x$. The tangent to the curve at the point $P(1, e)$ meets the y -axis at the point A . The normal to the curve at P meets the x -axis at the point B . Find the area of the triangle OAB , where O is the origin. [9]



In the diagram $\vec{OA} = \mathbf{a}$, $\vec{OB} = \mathbf{b}$, $\vec{OP} = 2\mathbf{a}$ and $\vec{OQ} = 3\mathbf{b}$.

- (i) Given that $\vec{AX} = \mu \vec{AQ}$, express \vec{OX} in terms of μ , \mathbf{a} and \mathbf{b} . [3]
- (ii) Given that $\vec{BX} = \lambda \vec{BP}$, express \vec{OX} in terms of λ , \mathbf{a} and \mathbf{b} . [3]
- (iii) Hence find the value of μ and of λ . [3]

12 Answer only **one** of the following two alternatives.

EITHER

The table shows values of the variables v and p which are related by the equation $p = \frac{a}{v^2} + \frac{b}{v}$, where a and b are constants.

v	2	4	6	8
p	6.22	2.84	1.83	1.35

(i) Using graph paper, plot $v^2 p$ on the y -axis against v on the x -axis and draw a straight line graph. [2]

(ii) Use your graph to estimate the value of a and of b . [4]

In another method of finding a and b from a straight line graph, $\frac{1}{v}$ is plotted along the x -axis. In this case, and without drawing a second graph,

(iii) state the variable that should be plotted on the y -axis, [2]

(iv) explain how the values of a and b could be obtained. [2]

OR

The table shows experimental values of two variables r and t .

t	2	8	24	54
r	22	134	560	1608

(i) Using the y -axis for $\ln r$ and the x -axis for $\ln t$, plot $\ln r$ against $\ln t$ to obtain a straight line graph. [2]

(ii) Find the gradient and the intercept on the y -axis of this graph and express r in terms of t . [6]

Another method of finding the relationship between r and t from a straight line graph is to plot $\lg r$ on the y -axis and $\lg t$ on the x -axis. Without drawing this second graph, find the value of the gradient and of the intercept on the y -axis for this graph. [2]

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