UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS **4037/01**

Paper 1

October/November 2005

2 hours

Additional Materials: Answer Booklet/Paper
Graph paper
Mathematical tables

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Booklet/Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

UNIVERSITY *of* CAMBRIDGE
International Examinations

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$
$$\sec^2 A = 1 + \tan^2 A.$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$
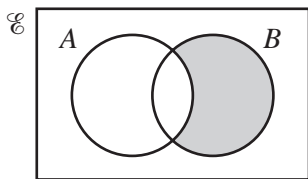
Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$
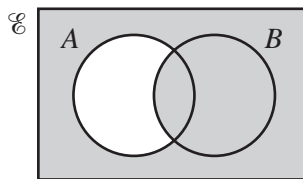
$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2}bc \sin A.$$

1   Find the set of values of x for which $(x - 6)^2 > x$.   [3]
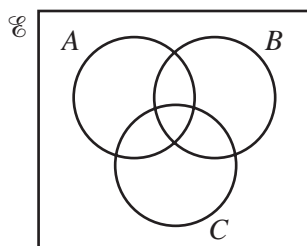
2   **(a)   (i)**

(ii)

For each of the Venn diagrams above, express the shaded region in set notation.   [2]

(b)

   (i)   Copy the Venn diagram above and shade the region that represents $A \cap B \cap C'$.   [1]

   (ii)   Copy the Venn diagram above and shade the region that represents $A' \cap (B \cup C)$.   [1]

3   Find the values of the constant c for which the line $2y = x + c$ is a tangent to the curve $y = 2x + \dfrac{6}{x}$.   [4]

4   A cuboid has a square base of side $(2 - \sqrt{3})$ m   and a volume of $(2\sqrt{3} - 3)$ m^3. Find the height of the cuboid in the form $(a + b\sqrt{3})$ m,   where a and b are integers.   [4]

5   The diagram, which is not drawn to scale, shows a horizontal rectangular surface. One corner of the surface is taken as the origin O and **i** and **j** are unit vectors along the edges of the surface.

A fly, F, starts at the point with position vector $(\mathbf{i} + 12\mathbf{j})$ cm and crawls across the surface with a velocity of $(3\mathbf{i} + 2\mathbf{j})$ cm s^{-1}. At the instant that the fly starts crawling, a spider, S, at the point with position vector $(85\mathbf{i} + 5\mathbf{j})$ cm, sets off across the surface with a velocity of $(-5\mathbf{i} + k\mathbf{j})$ cm s^{-1}, where k is a constant. Given that the spider catches the fly, calculate the value of k.   [6]

6   A particle starts from rest at a fixed point O and moves in a straight line towards a point A. The velocity, v ms^{-1}, of the particle, t seconds after leaving O, is given by $v = 6 - 6e^{-3t}$. Given that the particle reaches A when $t = \ln 2$, find

    (i)   the acceleration of the particle at A,   [3]

    (ii)   the distance OA.   [4]

7   **(a)**   Solve $\log_7(17y + 15) = 2 + \log_7(2y - 3)$.   [4]

    (b)   Evaluate $\log_p 8 \times \log_{16} p$.   [3]

8   A curve has the equation $y = (x + 2)\sqrt{x - 1}$.

    (i)   Show that $\dfrac{dy}{dx} = \dfrac{kx}{\sqrt{x - 1}}$, where k is a constant, and state the value of k.   [4]

    (ii)   Hence evaluate $\displaystyle\int_2^5 \dfrac{x}{\sqrt{x - 1}}\, dx$.   [4]
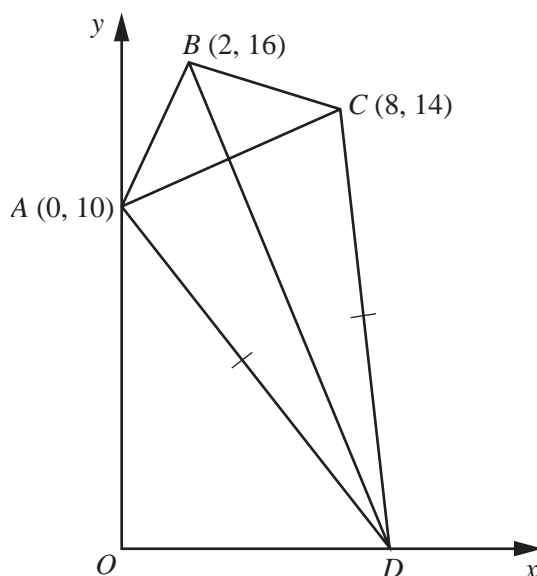
9   **(a)**   Find all the angles between $0°$ and $360°$ which satisfy the equation

$$3\cos x = 8\tan x.$$ [5]

    (b)   Given that $4 \leqslant y \leqslant 6$, find the value of y for which

$$2\cos\left(\frac{2y}{3}\right) + \sqrt{3} = 0.$$ [3]

10 **Solutions to this question by accurate drawing will not be accepted.**

The diagram, which is not drawn to scale, shows a quadrilateral *ABCD* in which *A* is (0, 10), *B* is (2, 16) and *C* is (8, 14).

(i) Show that triangle *ABC* is isosceles. [2]

The point *D* lies on the *x*-axis and is such that *AD* = *CD*. Find

(ii) the coordinates of *D*, [4]

(iii) the ratio of the area of triangle *ABC* to the area of triangle *ACD*. [3]

11 A function f is defined by $f : x \mapsto |2x - 3| - 4$,   for $-2 \leqslant x \leqslant 3$.

(i) Sketch the graph of $y = f(x)$. [2]

(ii) State the range of f. [2]

(iii) Solve the equation $f(x) = -2$. [3]

A function g is defined by $g: x \mapsto |2x - 3| - 4$,   for $-2 \leqslant x \leqslant k$.

(iv) State the largest value of *k* for which g has an inverse. [1]

(v) Given that g has an inverse, express g in the form $g: x \mapsto ax + b$,   where *a* and *b* are constants. [2]

12 Answer only **one** of the following two alternatives.

EITHER

Variables x and y are related by the equation $yx^n = a$, where a and n are constants. The table below shows measured values of x and y.

x	1.5	2	2.5	3	3.5
y	7.3	3.5	2.0	1.3	0.9

(i) On graph paper plot $\lg y$ against $\lg x$, using a scale of $2\,$cm to represent 0.1 on the $\lg x$ axis and $1\,$cm to represent 0.1 on the $\lg y$ axis. Draw a straight line graph to represent the equation $yx^n = a$. [3]

(ii) Use your graph to estimate the value of a and of n. [4]

(iii) On the same diagram, draw the line representing the equation $y = x^2$ and hence find the value of x for which $x^{n+2} = a$. [3]

OR
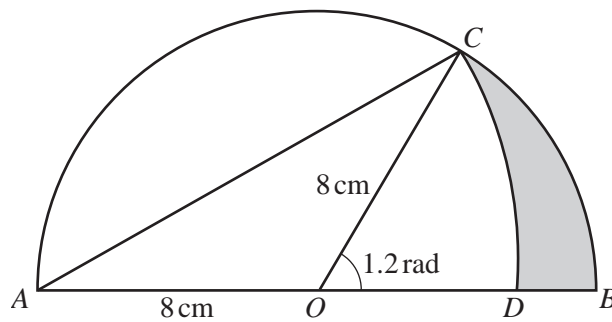
The diagram shows a semicircle, centre O, of radius $8\,$cm. The radius OC makes an angle of 1.2 radians with the radius OB. The arc CD of a circle has centre A and the point D lies on OB. Find the area of

(i) sector COB, [2]

(ii) sector CAD, [5]

(iii) the shaded region. [3]

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