Cambridge Assessment International Education

# Cambridge O Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

*2340714436*

**ADDITIONAL MATHEMATICS** **4037/24**

Paper 2 **May/June 2021**

**2 hours**

You must answer on the question paper.

No additional materials are needed.

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

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## *Mathematical Formulae*

### 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial Theorem*

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

*Arithmetic series* $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

*Geometric series* $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

### 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

*Formulae for $\Delta ABC$*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

**1** Find the exact solution of the equation $\dfrac{p^{\frac{3}{2}}+p^{\frac{1}{2}}}{p^{-\frac{1}{2}}}=4$. [3]

**2** Find $\displaystyle\int\left(\frac{1}{2x-3}+\sqrt{x}\right)\mathrm{d}x$. [3]

**3** Variables $x$ and $y$ are such that when $\lg y$ is plotted against $\lg x$ a straight line passing through the points $(-1, -4)$ and $(2, 11)$ is obtained. Show that $y = ax^n$, where $a$ and $n$ are integers. [6]

**4** The normal to the curve $y = x^5 - 2x^3 + x^2 + 3$ at the point on the curve where $x = -1$, cuts the $x$-axis at the point $P$. Find the equation of the normal and the coordinates of $P$. [7]

**5** Solve the simultaneous equations $3y = x - 20$ and $x^2 + y^2 - 2x + 6y = 0$. [4]

**6** The variables $x$ and $y$ are such that $y = \sqrt[3]{x^3 - 91}$.

**(a)** Find an expression for $\dfrac{dy}{dx}$. [2]

**(b)** Hence, find the approximate change in $y$ as $x$ increases from 6 to $6 + h$, where $h$ is small. [2]

**7** **(a)** Write the expression $4x^2 - 4x + 7$ in the form $p(x+q)^2 + r$, where $p$, $q$ and $r$ are constants. [3]

**(b)** Hence find the greatest value of $\frac{1}{4x^2 - 4x + 7}$ and state the value of $x$ at which this occurs. [2]

**8** **(a)** **(i)** Show that $\dfrac{\cos^2 2x}{1+\sin 2x} = 1-\sin 2x$. [2]

**(ii)** Hence solve $\dfrac{3\cos^2 2x}{1+\sin 2x} = 1$ for $0° \leqslant x \leqslant 90°$. [4]

**(b)** Solve $\cot\left(y-\dfrac{\pi}{2}\right) = \sqrt{3}$ for $0 \leqslant y \leqslant \pi$ radians. [3]

**9** A function f is defined, for all real values of $x$, by $\mathrm{f}(x) = 3 + \mathrm{e}^{5x}$.

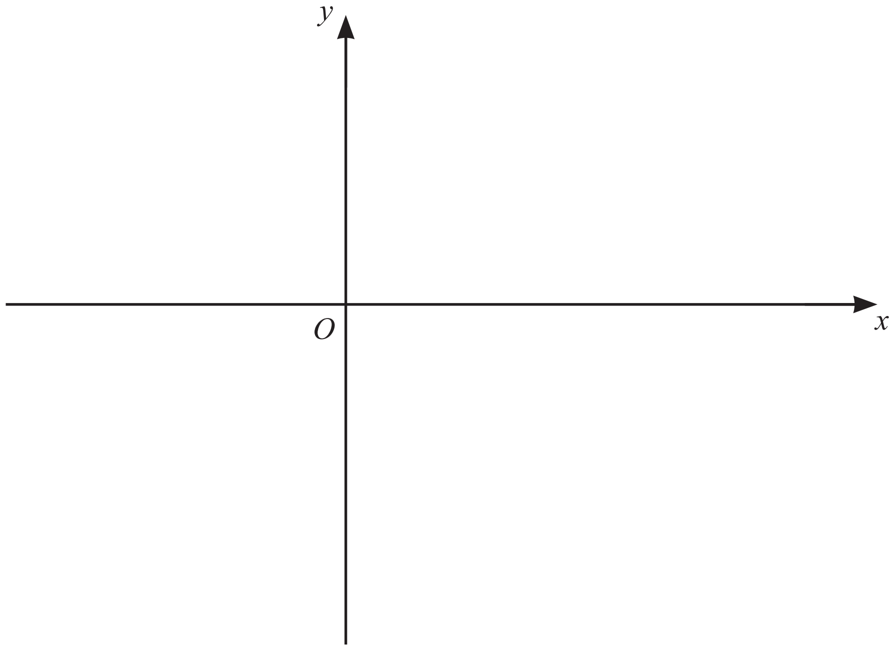
**(a)** Find the range of f. [1]

**(b)** Find an expression for $\mathrm{f}^{-1}(x)$ and state its domain. [3]

**(c)** Solve $\mathrm{f}^{-1}(x) = 0$. [2]

**(d)** Sketch the graph of $y = \mathrm{f}(x)$. Hence, on the same axes, sketch the graph of $y = \mathrm{f}^{-1}(x)$. Give the coordinates of any points where the graphs cross the axes. [4]



**10 (a)** A particle $P$ travels in a straight line so that, $t$ seconds after passing through a fixed point $O$, its displacement, $s$ metres from $O$, is given by

$$s = \frac{31}{3} - \frac{e^t}{3} - 10e^{-t}.$$

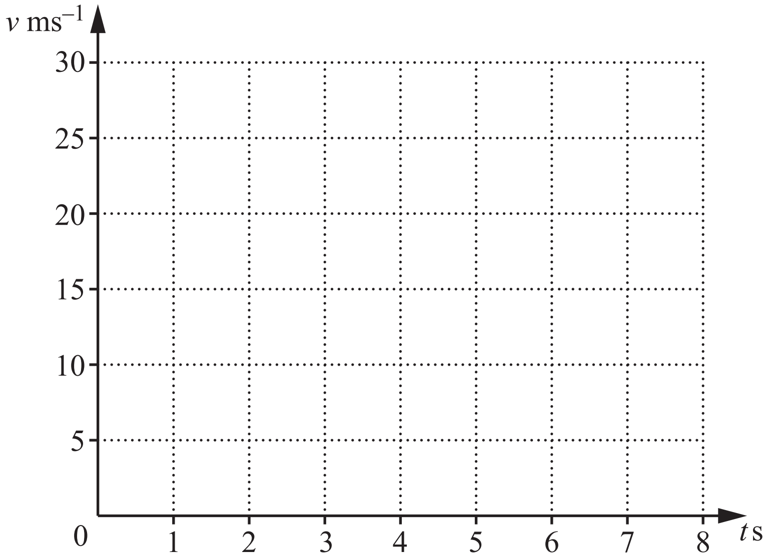
**(i)** Find the value of $t$ when $P$ is at instantaneous rest, giving your answer correct to 2 significant figures. [4]

**(ii)** Find the distance travelled in the first two seconds. [3]

**(b)** A particle $Q$ travels in a straight line so that $t$ seconds after leaving a fixed point $O$, its velocity, $v\ \text{ms}^{-1}$, is given by

$$v = 2t \qquad \text{for } \ 0 \leqslant t \leqslant 5\ ,$$
$$v = t^2 - 8t + 25 \quad \text{for } \ t > 5\ .$$

**(i)** On the axes below, sketch the velocity-time graph for the first 8 seconds of the motion of particle $Q$. [2]




**(ii)** Showing all your working, find the distance travelled by $Q$ in the first 8 seconds of its motion. [5]

**11** $OAB$ is a triangle. The position vectors of points $A$ and $B$ relative to the origin $O$ are $\mathbf{a}$ and $\mathbf{b}$ respectively.

The side $AB$ is extended to point $C$ such that $AB = \frac{1}{4}AC$.

**(a)** Show that $\overrightarrow{OC} = 4\mathbf{b} - 3\mathbf{a}$. [2]

**(b)** The point $D$ lies on $OA$ such that $OD : DA$ is $3 : 2$. The line $CD$ meets $OB$ at the point $E$. Find the position vector of the point $E$. [5]

**Question 12 is printed on the next page.**

**12** **(a)** The first term of an arithmetic progression is –5 and the fifth term is 7. Find the sum of the first 40 terms of this progression. [4]

**(b)** A geometric progression has third term of 8 and sixth term of 0.064. Find the sum to infinity of this progression. [4]