



# Cambridge O Level

CANDIDATE  
NAME

CENTRE  
NUMBER

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CANDIDATE  
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**ADDITIONAL MATHEMATICS**

**4037/21**

Paper 2

**May/June 2020**

**2 hours**

You must answer on the question paper.

No additional materials are needed.

## INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

## INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

**Mathematical Formulae****1. ALGEBRA***Quadratic Equation*

For the equation  $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial Theorem*

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where  $n$  is a positive integer and  $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

*Arithmetic series*      $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

*Geometric series*      $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

**2. TRIGONOMETRY***Identities*

$$\begin{aligned} \sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A \end{aligned}$$

*Formulae for  $\triangle ABC$* 

$$\begin{aligned} \frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A \end{aligned}$$

- 1 Variables  $x$  and  $y$  are such that, when  $\sqrt[4]{y}$  is plotted against  $\frac{1}{x}$ , a straight line graph passing through the points  $(0.5, 9)$  and  $(3, 34)$  is obtained. Find  $y$  as a function of  $x$ . [4]

- 2 (a) Write  $9x^2 - 12x + 5$  in the form  $p(x - q)^2 + r$ , where  $p$ ,  $q$  and  $r$  are constants. [3]

- (b) Hence write down the coordinates of the minimum point of the curve  $y = 9x^2 - 12x + 5$ . [1]

**3 DO NOT USE A CALCULATOR IN THIS QUESTION.**

$$p(x) = 15x^3 + 22x^2 - 15x + 2$$

(a) Find the remainder when  $p(x)$  is divided by  $x + 1$ . [2]

(b) (i) Show that  $x + 2$  is a factor of  $p(x)$ . [1]

(ii) Write  $p(x)$  as a product of linear factors. [3]

- 4 (a) In an examination, candidates must select 2 questions from the 5 questions in section A and select 4 questions from the 8 questions in section B. Find the number of ways in which this can be done. [2]

- (b) The digits of the number 6378129 are to be arranged so that the resulting 7-digit number is even. Find the number of ways in which this can be done. [2]

- 5 The vectors  $\mathbf{a}$  and  $\mathbf{b}$  are such that  $\mathbf{a} = \alpha\mathbf{i} + \mathbf{j}$  and  $\mathbf{b} = 12\mathbf{i} + \beta\mathbf{j}$ .

- (a) Find the value of each of the constants  $\alpha$  and  $\beta$  such that  $4\mathbf{a} - \mathbf{b} = (\alpha + 3)\mathbf{i} - 2\mathbf{j}$ . [3]

- (b) Hence find the unit vector in the direction of  $\mathbf{b} - 4\mathbf{a}$ . [2]

- 6 Find the values of  $k$  for which the line  $y = kx - 7$  and the curve  $y = 3x^2 + 8x + 5$  do not intersect. [6]

- 7 (a) Solve the simultaneous equations

$$\begin{aligned}10^{x+2y} &= 5, \\10^{3x+4y} &= 50,\end{aligned}$$

giving  $x$  and  $y$  in exact simplified form.

[4]

- (b) Solve  $2x^{\frac{2}{3}} - x^{\frac{1}{3}} - 10 = 0$ .

[3]

8 (a) Expand  $(2-x)^5$ , simplifying each coefficient.

[3]

(b) Hence solve  $\frac{e^{(2-x)^5} \times e^{80x}}{e^{10x^4+32}} = e^{-x^5}$ .

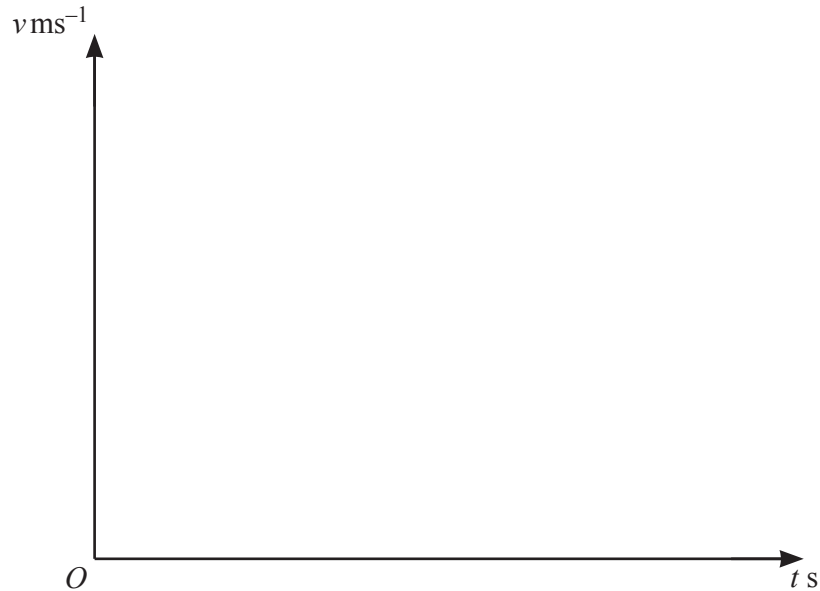
[4]



- 9 A particle travels in a straight line. As it passes through a fixed point  $O$ , the particle is travelling at velocity of  $3 \text{ ms}^{-1}$ . The particle continues at this velocity for 60 seconds then decelerates at a constant rate for 15 seconds to a velocity of  $1.6 \text{ ms}^{-1}$ . The particle then decelerates again at a constant rate for 5 seconds to reach point  $A$ , where it stops.

(a) Sketch the velocity-time graph for this journey on the axes below.

[3]

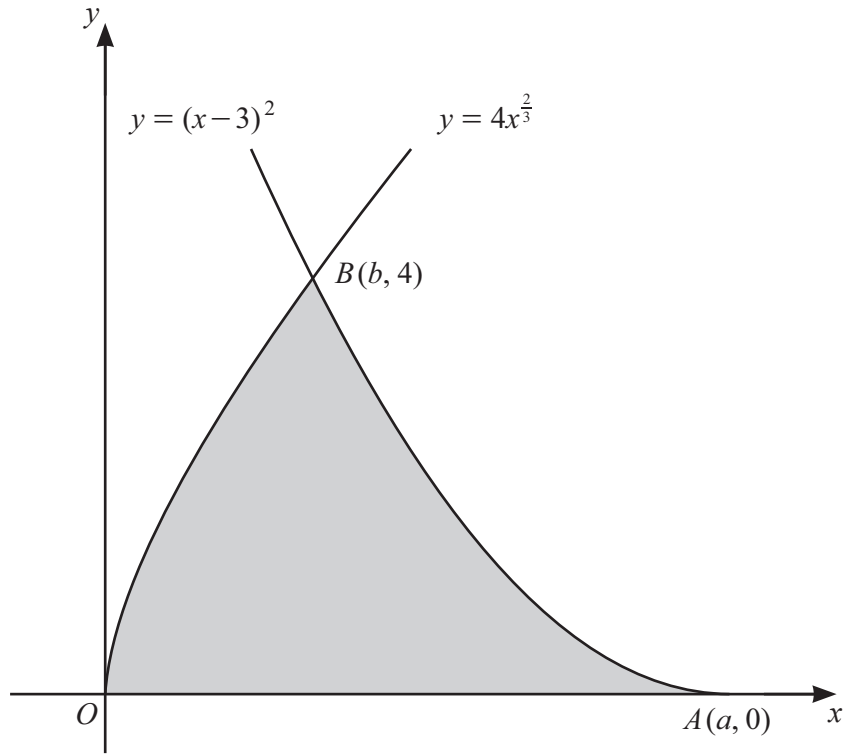


(b) Find the distance between  $O$  and  $A$ .

[3]

(c) Find the deceleration in the last 5 seconds.

[1]



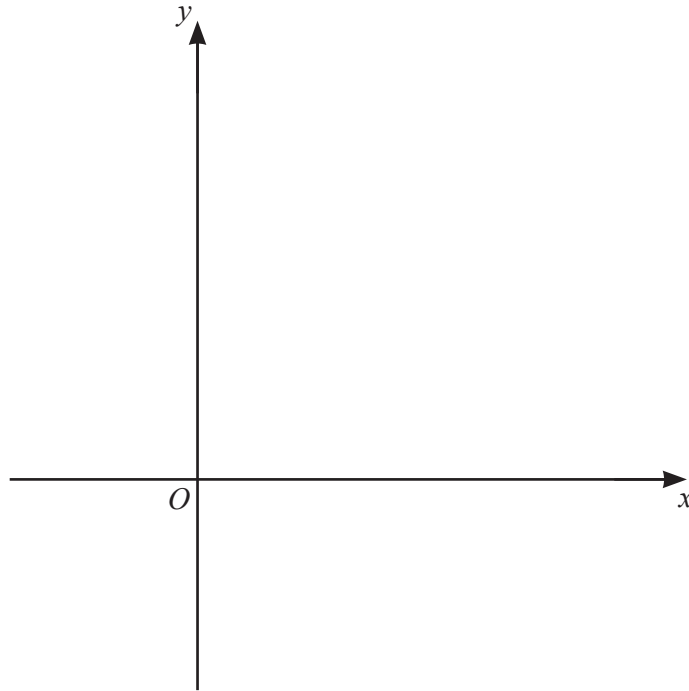
The diagram shows part of the graphs of  $y = 4x^{\frac{2}{3}}$  and  $y = (x-3)^2$ . The graph of  $y = (x-3)^2$  meets the  $x$ -axis at the point  $A(a, 0)$  and the two graphs intersect at the point  $B(b, 4)$ .

- (a) Find the value of  $a$  and of  $b$ . [2]

(b) Find the area of the shaded region.

11 The function  $f$  is defined by  $f(x) = \ln(2x + 1)$  for  $x \geq 0$ .

(a) Sketch the graph of  $y = f(x)$  and hence sketch the graph of  $y = f^{-1}(x)$  on the axes below. [3]



The function  $g$  is defined by  $g(x) = (x - 4)^2 + 1$  for  $x \leq 4$ .

(b) (i) Find an expression for  $g^{-1}(x)$  and state its domain and range. [4]

(ii) Find and simplify an expression for  $fg(x)$ .

[2]

(iii) Explain why the function  $gf$  does not exist.

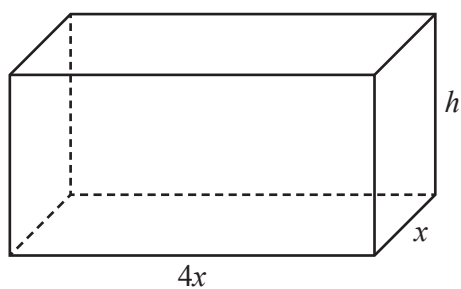
[1]

- 12 (a) Find the  $x$ -coordinates of the stationary points of the curve  $y = e^{3x}(2x+3)^6$ .

[6]

- (b) A curve has equation  $y = f(x)$  and has exactly two stationary points. Given that  $f''(x) = 4x - 7$ ,  $f'(0.5) = 0$  and  $f'(3) = 0$ , use the second derivative test to determine the nature of each of the stationary points of this curve. [2]

(c) In this question all lengths are in centimetres.



The diagram shows a solid cuboid with height  $h$  and a rectangular base measuring  $4x$  by  $x$ . The volume of the cuboid is  $40 \text{ cm}^3$ . Given that  $x$  and  $h$  can vary and that the surface area of the cuboid has a minimum value, find this value. [5]

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