

CANDIDATE
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CENTRE
NUMBER

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CANDIDATE
NUMBER

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ADDITIONAL MATHEMATICS

4037/21

Paper 2

May/June 2019

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 80.

This document consists of **15** printed pages and **1** blank page.

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

- 1 Find the values of x for which $x(6x + 7) \geq 20$.

[2]

- 2 Two variables x and y are such that $y = \frac{\ln x}{x^3}$ for $x > 0$.

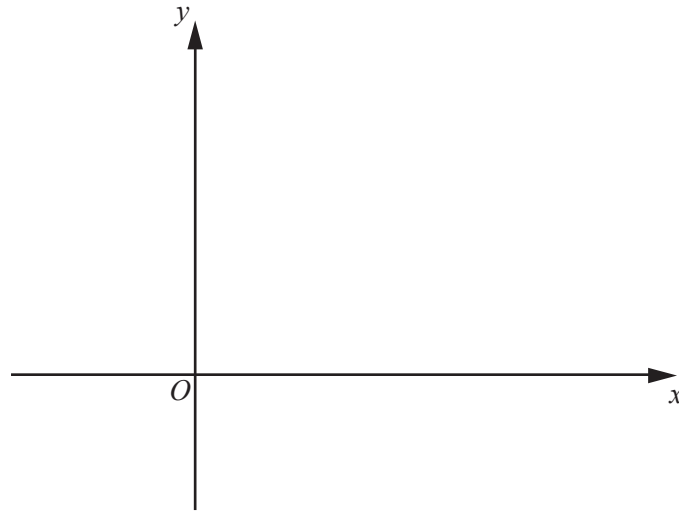
(i) Show that $\frac{dy}{dx} = \frac{1 - 3 \ln x}{x^4}$.

[3]

- (ii) Hence find the approximate change in y as x increases from e to $e + h$, where h is small.

[2]

- 3 (i) Sketch the graph of $y = |5x - 3|$ on the axes below, showing the coordinates of the points where the graph meets the coordinate axes.



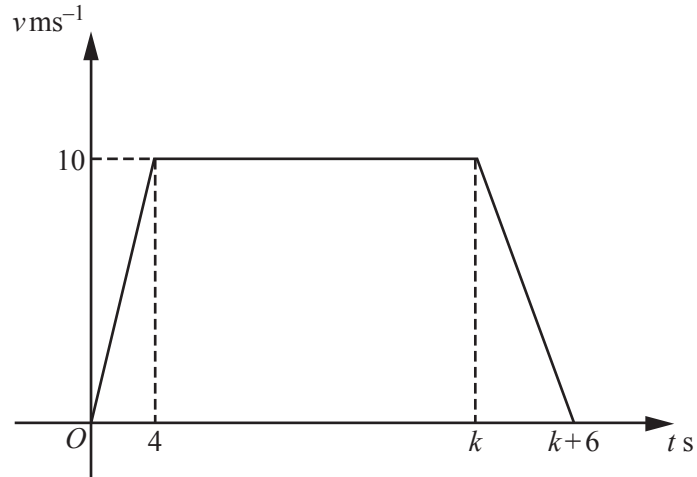
[3]

- (ii) Solve the equation $|5x - 3| = 2 - x$.

[3]

- 4 Without using a calculator, express $\frac{(\sqrt{5} - 3)^2}{\sqrt{5} + 1}$ in the form $p\sqrt{5} + q$, where p and q are integers. [4]

5



The velocity-time graph represents the motion of a particle travelling in a straight line.

- (i) Find the acceleration during the last 6 seconds of the motion. [1]
- (ii) The particle travels with constant velocity for 23 seconds. Find the value of k . [1]
- (iii) Using your answer to **part (ii)**, find the total distance travelled by the particle. [3]

6 (a) $\mathbf{A} = \begin{pmatrix} x+3 & -x \\ 2x & x-3 \end{pmatrix}$

Given that \mathbf{A} does not have an inverse, find the exact values of x .

[3]

(b) $\mathbf{B} = \begin{pmatrix} 0 & 3 \\ -4 & 1 \\ 5 & 2 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 0 & 1 & 2 \\ 3 & -4 & 5 \end{pmatrix}$

(i) Write down the order of matrix \mathbf{B} .

[1]

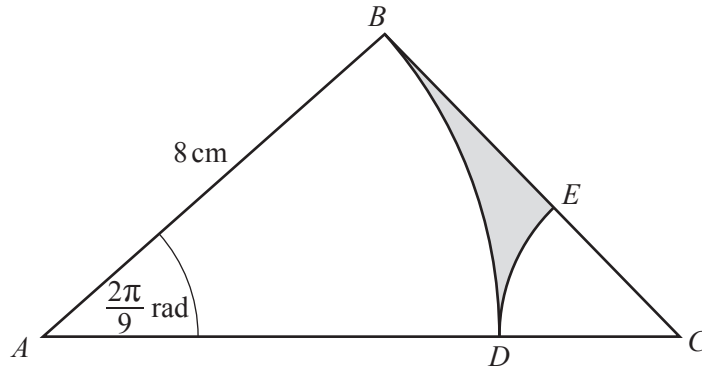
(ii) The matrix $\mathbf{BC} = \begin{pmatrix} 9 & -12 & 15 \\ 3 & -8 & -3 \\ 6 & -3 & 20 \end{pmatrix}$. Explain why $\mathbf{CB} \neq \mathbf{BC}$.

[2]

7 The variables x , y and u are such that $y = \tan u$ and $x = u^3 + 1$.

(i) State the rate of change of y with respect to u . [1]

(ii) Hence find the rate of change of y with respect to x , giving your answer in terms of x . [4]



The diagram shows a right-angled triangle ABC with $AB = 8$ cm and angle $ABC = \frac{\pi}{2}$ radians. The points D and E lie on AC and BC respectively. BAD and ECD are sectors of the circles with centres A and C respectively. Angle $BAD = \frac{2\pi}{9}$ radians.

- (i) Find the area of the shaded region. [6]

- (ii) Find the perimeter of the shaded region.

9 (a) Eleven different television sets are to be displayed in a line in a large shop.

(i) Find the number of different ways the televisions can be arranged. [1]

Of these television sets, 6 are made by company A and 5 are made by company B .

(ii) Find the number of different ways the televisions can be arranged so that no two sets made by company A are next to each other. [2]

(b) A group of people is to be selected from 5 women and 3 men.

(i) Calculate the number of different groups of 4 people that have exactly 3 women. [2]

(ii) Calculate the number of different groups of at most 4 people where the number of women is the same as the number of men. [2]

10 Solutions to this question by accurate drawing will not be accepted.

The points A and B have coordinates $(p, 3)$ and $(1, 4)$ respectively and the line L has equation $3x + y = 2$.

(i) Given that the gradient of AB is $\frac{1}{3}$, find the value of p . [2]

(ii) Show that L is the perpendicular bisector of AB . [3]

(iii) Given that $C(q, -10)$ lies on L , find the value of q . [1]

(iv) Find the area of triangle ABC . [2]

11 (a) (i) Show that $\frac{\operatorname{cosec} \theta - \cot \theta}{\sin \theta} = \frac{1}{1 + \cos \theta}$.

[4]

(ii) Hence solve $\frac{\operatorname{cosec} \theta - \cot \theta}{\sin \theta} = \frac{5}{2}$ for $180^\circ < \theta < 360^\circ$.

[2]

(b) Solve $\tan(3\phi - 4) = -\frac{1}{2}$ for $0 \leq \phi \leq \frac{\pi}{2}$ radians.

[3]

- 12 (a) Given that $\int_0^a e^{2x} dx = 50$, find the exact value of a . You must show all your working.

[4]

(b) A curve is such that $\frac{dy}{dx} = 3 - 2 \cos 5x$. The curve passes through the point $\left(\frac{\pi}{5}, \frac{8\pi}{5}\right)$.

(i) Find the equation of the curve.

[4]

(ii) Find $\int y dx$ and hence evaluate $\int_{\frac{\pi}{2}}^{\pi} y dx$.

[5]

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