
ADDITIONAL MATHEMATICS**4037/22**

Paper 2

May/June 2019

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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This document consists of **9** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘dep’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1	Either: Quotient rule: $\frac{d}{dx}(\sin x) = \cos x$	B1	
	$\frac{d}{dx}(\ln x^2) = \frac{2}{x}$	B1	
	$\frac{(\ln x^2)(\text{their } \cos x) - (\sin x)\left(\text{their } \frac{2}{x}\right)}{(\ln x^2)^2}$ oe	M1	
	$\frac{(\ln x^2)\cos x - (\sin x)\left(\frac{2}{x}\right)}{(\ln x^2)^2}$ oe isw	A1	
	Or: Product rule on $y = (\sin x)(\ln x^2)^{-1}$ $\frac{d}{dx}(\sin x) = \cos x$	B1	
	$\frac{d}{dx}((\ln x^2)^{-1}) = -(\ln x^2)^{-2} \times \frac{2}{x}$	B1	
	$(\sin x) \times \text{their} \left(-(\ln x^2)^{-2} \times \frac{2}{x}\right) + (\text{their } \cos x)(\ln x^2)^{-1}$ oe	M1	
	$(\sin x) \times \left(-(\ln x^2)^{-2} \times \frac{2}{x}\right) + (\cos x)(\ln x^2)^{-1}$ oe isw	A1	
2	$k^2 - 4(k-1)(-k)$ oe	B1	
	$k(5k - 4)$	M1	
	Correct critical values 0, 0.8 oe	A1	
	$k < 0, k > 0.8$ oe	A1	FT <i>their</i> critical values provided <i>their</i> $ak^2 + bk + c > 0$ has positive a and there are 2 values; mark final answer If B1 M0 allow SC1 for a final answer of $k > 0.8$ oe

Question	Answer	Marks	Partial Marks
3(i)	Uses $x = 2$ as a root: $a(2^3) - 12(2^2) + 5(2) + 6 = 0$	M1	or $2 \begin{array}{r rrrr} a & -12 & 5 & 6 \\ \downarrow & 2a & -24+4a & -38+8a \\ \hline a & -12+2a & -19+4a & -32+8a = 0 \end{array}$
	Solves $8a - 48 + 10 + 6 = 0$ to find $a = 4$	A1	or solves $-32 + 8a = 0$ to find $a = 4$ If M0 then SC1 for $4(2^3) - 12(2^2) + 5(2) + 6 = 0$ or showing that the synthetic division with $a = 4$ results in a remainder of 0
3(ii)	$(x - 2)(4x^2 - 4x - 3)$	B2	B1 for any two terms correct in quadratic factor
	Product of three correct linear factors: $(x - 2)(2x - 3)(2x + 1)$	B1	
	$x = 2, x = 1.5, x = -0.5$ oe	B1	dep on all previous marks having been earned If B2 B0 then award SC1 for correct factorisation of correct quadratic factor leading to 3 correct roots
4	Either: $A = \frac{1}{4}\pi x^2$ oe, soi	B1	
	$\frac{dA}{dx} = \frac{2}{4}\pi x$ oe, soi	B1	
	$\frac{dx}{dt} = 0.01$ soi	B1	
	$\frac{2}{4}\pi(6) \times 0.01$	M1	FT their $\frac{dA}{dx}$ when $x = 6$
	0.03π or exact equivalent	A1	mark final answer
	Or: $A = \pi r^2$ and $r = \frac{x}{2}$ soi	B1	
	$\frac{dA}{dr} = 2\pi r$ oe soi	B1	
	$\frac{dr}{dt} = 0.005$ oe soi	B1	
	$2\pi(3) \times 0.005$	M1	
0.03π or exact equivalent	A1		

Question	Answer	Marks	Partial Marks
5(i)	$5(x-1.5)^2 - 10.25$ isw	B3	B1 for each of p, q, r correct in correctly formatted expression; allow correct equivalent values If B0 then SC2 for $5(x-1.5) - 10.25$ or SC1 for correct values but other incorrect format
5(ii)	$\frac{\text{their} - 10.25}{5}$ is least value when $x = \text{their}1.5$	B2	STRICT FT <i>their</i> part (i); B1 STRICT FT for each
6(a)	2×4 or 2 by 4	B1	
6(b)(i)	$(\mathbf{A}^{-1} =) \frac{1}{2} \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$ isw	B2	B1 for $\frac{1}{2} \times \text{their} \left(\begin{pmatrix} & \\ & \end{pmatrix} \right)$ or for $k \times \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$, where k is not 0 or 1
6(b)(ii)	$\mathbf{B} = \frac{1}{4} \begin{pmatrix} 13 & 20 \\ 5 & 8 \end{pmatrix}$ oe isw	B3	FT <i>their</i> \mathbf{A}^{-1} provided B1 earned in (b)(i) B1 for the strategy of using \mathbf{A}^{-1} : $\mathbf{B} = \mathbf{A}^{-1} \mathbf{A}^{-1}$ soi or $\mathbf{AB} = \mathbf{A}^{-1}$ soi or $\mathbf{BA} = \mathbf{A}^{-1}$ soi or $\mathbf{B} = (\mathbf{A}^{-1})^2$ and B1 for two or three elements of \mathbf{B} correct or correct FT
7(a)	$\lg(x^2 - 3) = \lg 1$ soi	M1	
	-2 and 2	A1	Implies M1
7(b)(i)	Two separate terms in numerator: $(\sin(2x+5)) \ln a$ or $\log_a a^{\sin(2x+5)} = (\sin(2x+5)) \log_a a$	B1	Combines terms in numerator: or $\ln \left(\frac{a^{\sin(2x+5)}}{a} \right)$
	$-\ln a$ or $\log_a a^{-1} = -\log_a a$	B1	or $(\sin(2x+5) - 1) \ln a$ or $\frac{\ln \left(\frac{a^{\sin(2x+5)}}{a} \right)}{\ln a} = \log_a (a^{\sin(2x+5)-1})$
	$\sin(2x+5) - 1$	B1	dep all previous marks awarded;
7(b)(ii)	$-\frac{1}{2} \cos(2x+5) + (\text{their} - 1)x + c$	B3	FT <i>their</i> numerical k B2 for $-\frac{1}{2} \cos(2x+5)$ seen or B1 for $a \cos(2x+5)$, $a < 0$ or for $\frac{1}{2} \cos(2x+5)$ or for $-\frac{1}{2} \cos 2x + 5$ seen

Question	Answer	Marks	Partial Marks
8(a)	$-\frac{20}{8}a^3[x^3]$ and $-\frac{6}{32}a[x^5]$ oe soi or $\frac{20}{8}a^3[x^3]$ and $\frac{6}{32}a[x^5]$ oe soi	B2	B1 for either $-\frac{20}{8}a^3[x^3]$ or $-\frac{6}{32}a[x^5]$ oe or for $-\frac{{}^6C_3}{8}a^3[x^3]$ and $-\frac{{}^6C_5}{32}a[x^5]$ oe, or for $\frac{{}^6C_3}{8}a^3[x^3]$ and $\frac{{}^6C_5}{32}a[x^5]$ oe or for $\frac{20}{8}a^3[x^3]$ and $ka[x^5]$ oe where $k > 0$ or for $ka^3[x^3]$ and $\frac{6}{32}a[x^5]$ oe where $k > 0$
	<i>their</i> $\frac{20}{8}a^3 = 120 \times$ <i>their</i> $\frac{6}{32}a$ oe soi	M1	
	$[a =] \pm 3$	A1	
8(b)(i)	$1 + 40x + 760x^2 + 9120x^3$	B2	B1 for three out of the four terms correct If B0 then SC1 for 1, 40x, 760x ² , 9120x ³ seen but not summed
8(b)(ii)	$1 + 40(-0.01) + 760(-0.01)^2 + 9120(-0.01)^3$ or $1 - 0.4 + 0.076 - 0.00912$ oe leading to 0.66688 cao	B2	or M1 for use of $x = -0.01$ oe in <i>their</i> expansion seen or implied by e.g. 0.66688 without working or $1 - 0.4 + 0.076 - 0.00912$
9(a)	$6(1 - \cos^2 x) - 13\cos x = 1$ oe	B1	
	Solves or factorises <i>their</i> 3-term quadratic	M1	
	70.5 and 289.5	A2	with no extras in range A1 for either, ignoring extras in range
9(b)(i)	Numerator: Substitution of $\tan y = \frac{\sin y}{\cos y}$	M1	
	Denominator: Substitution of $1 + \tan^2 y = \sec^2 y$ or substitution of $1 + \tan^2 y = 1 + \frac{\sin^2 y}{\cos^2 y}$ and correct rearrangement to $\frac{1}{\cos^2 y}$ oe	M1	
	Correct completion to $4\sin y$ cao	A1	

Question	Answer	Marks	Partial Marks
9(b)(ii)	$-0.848[06\dots]$ rot to 3 or more figures	B1	with no extras in range
10(a)	$\sqrt{5^2 + (-15)^2}$ seen	M1	
	$\frac{1}{5\sqrt{10}}(5\mathbf{i} - 15\mathbf{j})$ oe, isw	A1	
10(b)(i)	$\begin{pmatrix} 9 \\ 12 \end{pmatrix}$ oe, soi	B1	
	$\begin{pmatrix} 3 \\ -5 \end{pmatrix} + \frac{2}{3} \left(\text{their} \begin{pmatrix} 9 \\ 12 \end{pmatrix} \right)$ oe, soi or $\begin{pmatrix} 12 \\ 7 \end{pmatrix} - \frac{1}{3} \left(\text{their} \begin{pmatrix} 9 \\ 12 \end{pmatrix} \right)$ oe, soi	M1	
	$\begin{pmatrix} 9 \\ 3 \end{pmatrix}$	A1	If B1 M0, award SC1 for a final answer of $\begin{pmatrix} -9 \\ -3 \end{pmatrix}$ oe
10(b)(ii)	Forms a valid vector relationship using \overline{DC} and e.g. \overline{OD} or \overline{DB} e.g. $\text{their} \begin{pmatrix} 9 \\ 3 \end{pmatrix} = \overline{OD} + \begin{pmatrix} 6 \\ 1.25 \end{pmatrix}$ oe or $\overline{DB} = \begin{pmatrix} 6 \\ 1.25 \end{pmatrix} + \frac{1}{3} \times \text{their} \begin{pmatrix} 9 \\ 12 \end{pmatrix}$ oe	M1	or $\overline{DC} = \overline{OC} - \overline{OD}$ $\begin{pmatrix} 6 \\ 1.25 \end{pmatrix} = \text{their} \begin{pmatrix} 9 \\ 3 \end{pmatrix} - \frac{1}{\lambda} \begin{pmatrix} 12 \\ 7 \end{pmatrix}$ soi or $\overline{DC} = \overline{DB} - \overline{CB}$ $\begin{pmatrix} 6 \\ 1.25 \end{pmatrix} = \frac{\lambda - 1}{\lambda} \begin{pmatrix} 12 \\ 7 \end{pmatrix} - \frac{1}{3} \times \text{their} \begin{pmatrix} 9 \\ 12 \end{pmatrix}$ soi
	Finds a correct proportion e.g. $\overline{OB} = 4\overline{OD}$ oe soi or $3\overline{OB} = 4\overline{DB}$ oe soi	A1	or solves a correct equation in λ e.g. $6 = 9 - \frac{1}{\lambda} \times 12$
	$\lambda = 4$	A1	from a fully correct method
11(i)	$v \neq 0$ or $v > 0$ oe	B1	
11(ii)	Differentiates : $4 \times -3(t+1)^{-4}$ oe, isw	B2	B1 for $k(t+1)^{-4}$ with $k \neq -12$
	$-\frac{1}{108}$ oe or -0.00926	B1	
11(iii)	Integrates: $[s =] -2(t+1)^{-2} + 2$ oe, isw	B3	B2 for $-2(t+1)^{-2} [+c]$ or B1 for $k(t+1)^{-2} [+c]$ with $k \neq -2$

Question	Answer	Marks	Partial Marks
11(iv)	Finds <i>their s</i> from (iii) when $t = 4$ or when $t = 3$ or finds $\left[\text{their} \left(-2(t+1)^{-2} [+2] \right) \right]_3^4$ $= \text{their} \left(-2(4+1)^{-2} - \left(-2(3+1)^{-2} \right) \right)$	M1	
	$\frac{9}{200}$ or 0.045	A1	
12(a)(i)	$g > -9$	B1	
12(a)(ii)	$x > 1$	B1	
12(a)(iii)	$[\text{gf}(x) =] 4(5x-2)^2 - 9$	B1	
	$100x^2 - 80x - 38 = 0$ or $(5x-2)^2 = \frac{45+9}{4}$	M1	
	$[x =] \frac{-(-80) \pm \sqrt{(-80)^2 - 4(100)(-38)}}{2(100)}$ leading to $\frac{4+3\sqrt{6}}{10}$ oe only or $\frac{1}{5} \left(2 + \sqrt{\frac{54}{4}} \right)$ or better only	A1	
12(b)(i)	(They are) reflections (of each other) in (the line) $y = x$ oe	B1	
12(b)(ii)	$x^2 = y^2 + 1$ or $y^2 = x^2 + 1$	M1	
	$x = [\pm] \sqrt{y^2 + 1}$ or $y = [\pm] \sqrt{x^2 + 1}$	A1	
	$-\sqrt{x^2 + 1}$ nfww	A1	