AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

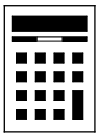
Level 2 Certificate
FURTHER MATHEMATICS

Paper 2 Calculator

Monday 17 June 2019 Afternoon Time allowed: 2 hours

Materials

For this paper you must have:
- a calculator
- mathematical instruments.



Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 105.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.
- The use of a calculator is expected but calculators with a facility for symbolic algebra must **not** be used.

For Examiner's Use	
Pages	Mark
3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
26–27	
28–29	
30	
TOTAL	

J U N 1 9 8 3 6 0 2 0 1

IB/M/Jun19/E6

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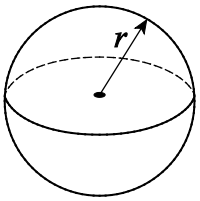
Formulae Sheet

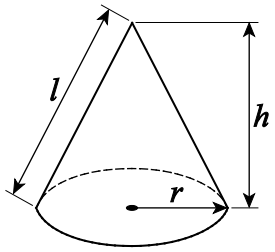
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



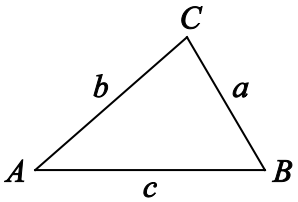
Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$



In any triangle ABC

Area of triangle $= \frac{1}{2}ab \sin C$



Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Trigonometric Identities

$\tan \theta \equiv \frac{\sin \theta}{\cos \theta}$ $\sin^2 \theta + \cos^2 \theta \equiv 1$

Answer **all** questions in the spaces provided.

1 (a) $a\begin{pmatrix}3\\5\end{pmatrix} = 4\begin{pmatrix}2a+3\\b\end{pmatrix}$

Work out the values of a and b.

[3 marks]

$a =$ _____ $b =$ _____

1 (b) $\begin{pmatrix}m & -1\\1 & 1\end{pmatrix}\begin{pmatrix}2 & 2\\-2 & -1\end{pmatrix} = \mathbf{I}$ where **I** is the identity matrix.

Work out the value of m.

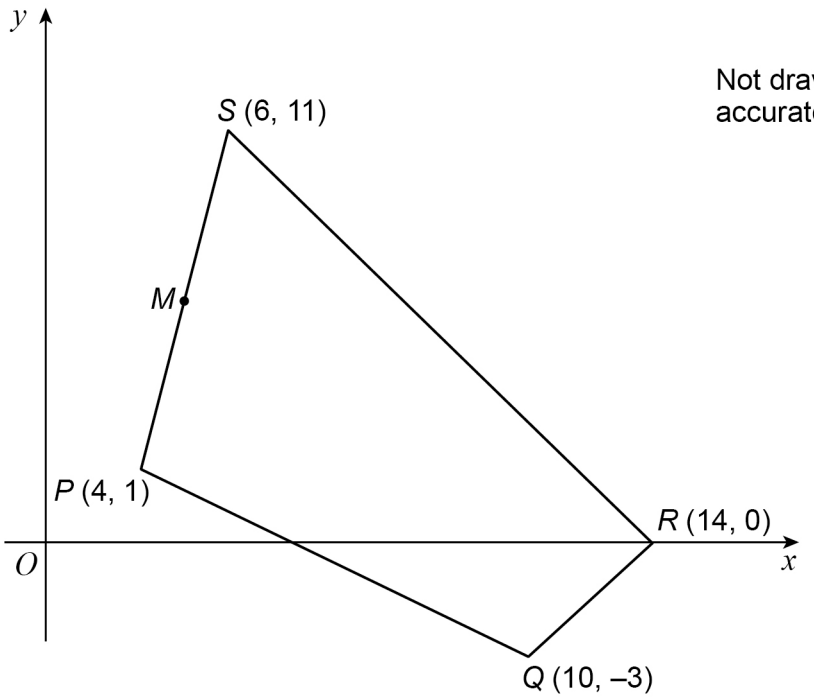
[2 marks]

Answer _____

5

2 Here is a sketch of quadrilateral *PQRS*.

M is the midpoint of *PS*.



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Use gradients to show that *MR* is parallel to *PQ*.

[3 marks]

3 $-2 < a < 0$ and $-1 < b < 1$

Tick the correct box for each statement.

[4 marks]

	Always true	**Sometimes true**	**Never true**
$a^2 < 0$	☐	☐	☐
$-1 < b^3 < 1$	☐	☐	☐
$\frac{b}{a} < 0$	☐	☐	☐
$a - b > 0$	☐	☐	☐

Turn over for the next question

7

4 *P* is a point on a curve.

The curve has **gradient function** $\frac{x^5 - 17}{10}$

The tangent to the curve at *P* is parallel to the line $3x - 2y = 9$

Work out the x-coordinate of *P*.

[4 marks]

Answer _____

5 (a) Write $\sqrt[4]{a \times a^{-9}}$ as an integer power of a.

[2 marks]

Answer _____

5 (b) Simplify fully $\dfrac{(4cd^2)^3}{2cd^4}$

[3 marks]

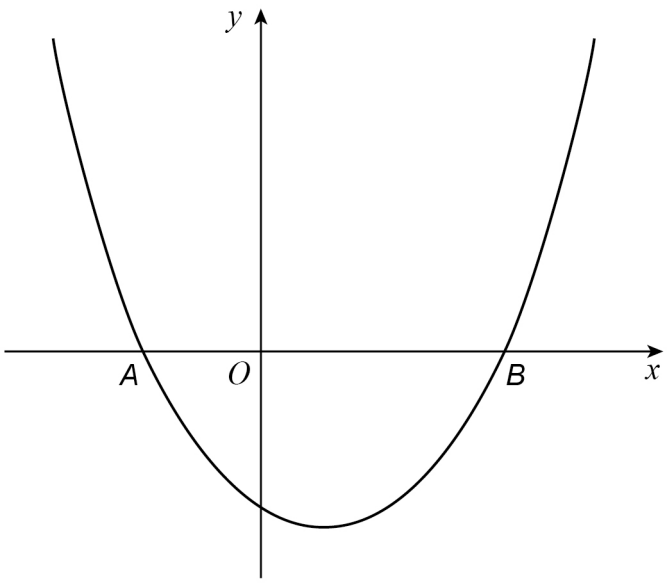
Answer _____

Turn over for the next question

9

6 Here is a sketch of the curve $y = (2x + 3)(x - 2)$

The curve intersects the x-axis at A and B.



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6 (a) Complete the coordinates of A and B.

[2 marks]

A (_____ , 0) B (_____ , 0)

6 (b) Write down the range of values for x for which $(2x + 3)(x - 2) < 0$

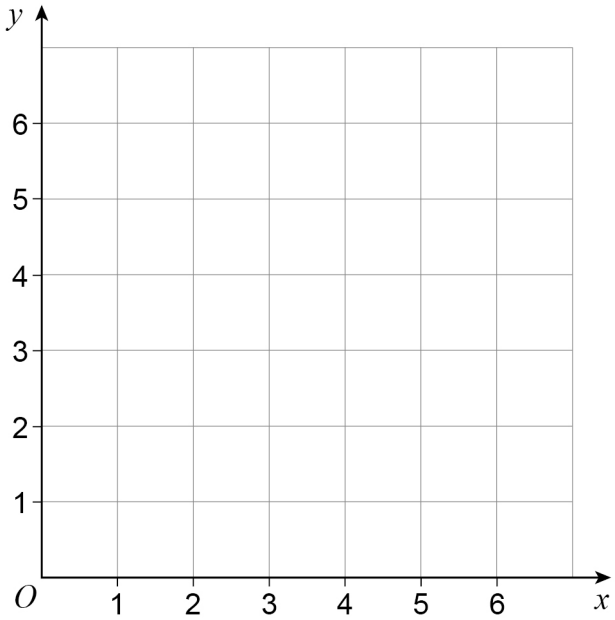
[1 mark]

Answer _____

7 (a) On the grid, sketch a graph for which

the rate of change of y with respect to x is always zero.

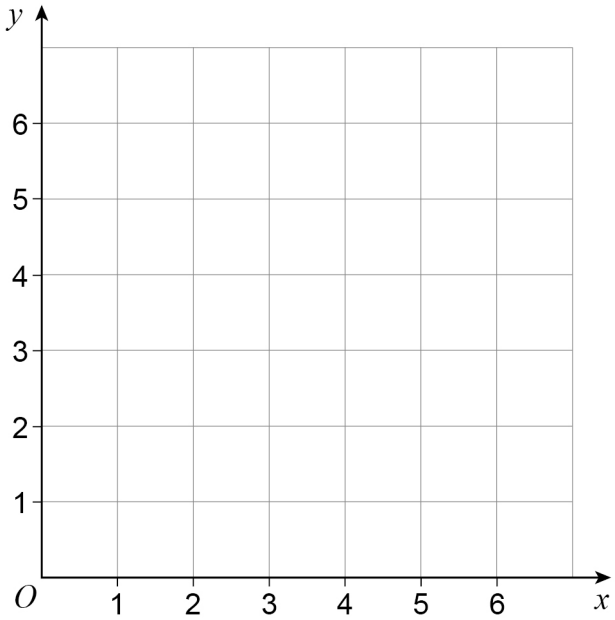
[1 mark]



7 (b) On the grid, sketch a graph for which

the rate of change of y with respect to x is always a positive constant.

[1 mark]



5

8 (a) A linear sequence has first term $7 + 12\sqrt{5}$

The term-to-term rule is

add $9 - 2\sqrt{5}$

One term of the sequence is an integer.

Work out the value of this integer.

[2 marks]

Answer _____

8 (b) The nth term of a different sequence is $\dfrac{3n^2 - 1}{n^2 + 1}$

Work out the sum of the first three terms.

[2 marks]

Answer _____

8 (c) The first four terms of a quadratic sequence are

–3 3 13 27

Work out an expression for the nth term.

[3 marks]

Answer _____

Turn over for the next question

7

9 Factorise fully $(p + 6)^{11} - (p + 6)^{10}$

[2 marks]

Answer _____

10 (a) $f(x) = x^3 - 2$

The domain of $f(x)$ is $x \leqslant 3$

Work out the range of $f(x)$.

[2 marks]

Answer _____

10 (b) $g(x) = 5 - x^2$

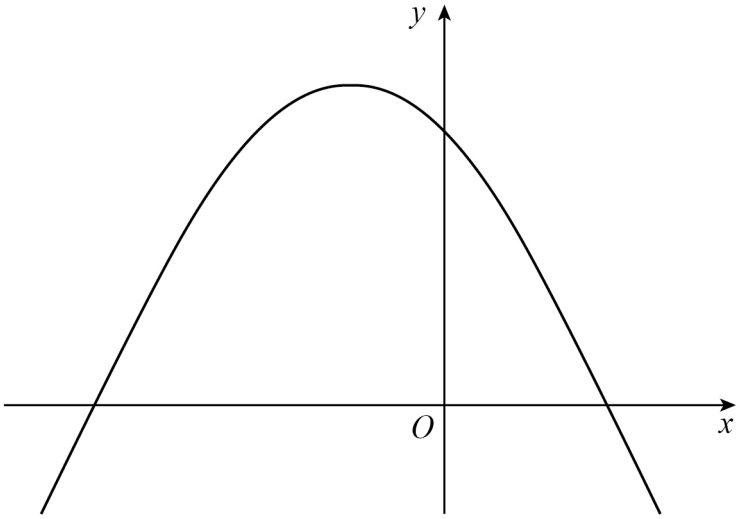
The domain of $g(x)$ is $-2 \leqslant x \leqslant 1$

Work out the range of $g(x)$.

[2 marks]

Answer _____

11 Here is a sketch of a quadratic curve which has a maximum point at (–2, 5)



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What is the equation of the normal to the curve at the maximum point?

Circle your answer.

[1 mark]

$x = -2$ $y = 5$ $x = 5$ $y = -2$

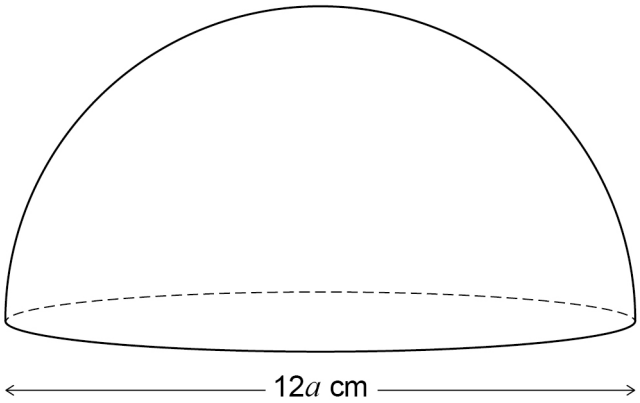
Turn over for the next question

7

12 The diagram shows a solid hemisphere.

The diameter is $12a$ cm

The volume is 486π cm^3



Work out the value of a.

[3 marks]

Answer _____

13 Simplify fully $\dfrac{x - x^3}{2x + 2x^2}$

You **must** show your working.

[4 marks]

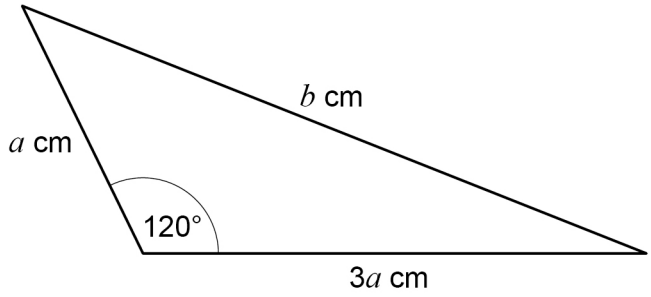
Answer _____

Turn over for the next question

7

14 Here is a triangle.

Not drawn accurately



Use the cosine rule to work out the ratio $b^2 : a^2$

[3 marks]

Answer _____ : _____

15 Rearrange $m = \frac{2p+1}{p} + \frac{p+5}{3p}$ to make p the subject.

[4 marks]

Answer _____

16 The curve $y = 2\sqrt{x-a} + 5$ passes through the point (1, 8)

Work out the value of a.

[3 marks]

Answer _____

10

17 Show that $(x + 1)(x + 3)(x + 4) - x(x^2 + 7x + 11)$

can be written in the form $(x + a)(x + b)$ where a and b are positive integers.

[5 marks]

18 Solve $4(x-5)^2 = k^2$ where k is a constant.

Give your answers in their simplest form in terms of k.

[3 marks]

Answer _____

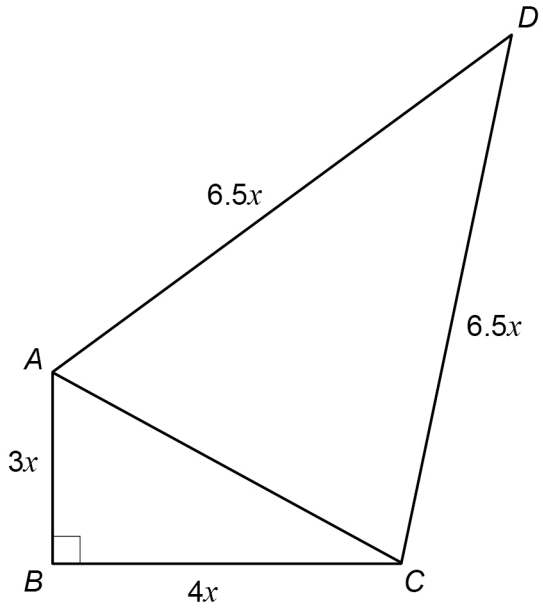
Turn over for the next question

8

19 *ABC* is a right-angled triangle.

ACD is an isosceles triangle.

All dimensions are in centimetres.



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19 (a) Show that $AC = 5x$

[1 mark]

19 (b) Work out an expression, in cm^2, for the area of quadrilateral *ABCD*.

Give your answer in the form px^2 where p is an integer.

[5 marks]

Answer _____ cm^2

Turn over for the next question

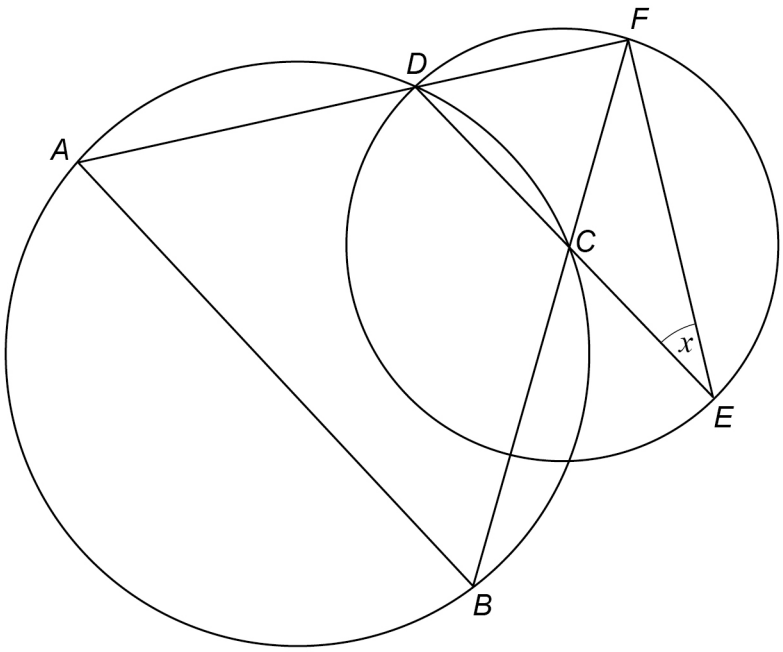
6

20 *A*, *B*, *C* and *D* are points on a circle.

D, *E* and *F* are points on a different circle, centre *C*.

DCE, *ADF* and *BCF* are straight lines.

angle *DEF* = x



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20 (a) Prove that angle *BAD* = $2x$

[3 marks]

20 (b) In the case when *AB* is parallel to *DE*, work out the size of angle x.

[2 marks]

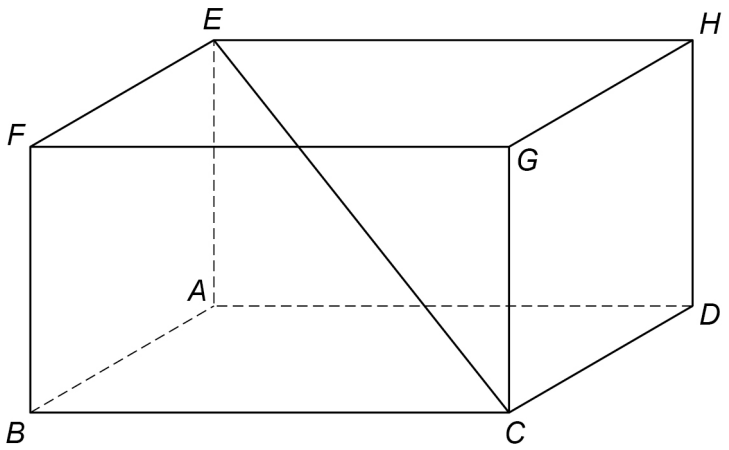
Answer _____ degrees

Turn over for the next question

5

21 *ABCDEFGH* is a cuboid.

$BC = 15$ cm $\quad CD = 12$ cm $\quad DH = 8$ cm



Work out the size of the angle between the line *CE* and the plane *CDHG*.

[4 marks]

Answer _____ degrees

22 (a) Show that $\dfrac{2\sin^2 x - 1 + \cos^2 x}{\sin x \cos x}$ is equivalent to $\tan x$

[3 marks]

22 (b) Hence solve $\dfrac{2\sin^2 x - 1 + \cos^2 x}{\sin x \cos x} = -1$ for $0° \leqslant x \leqslant 360°$

[2 marks]

Answer _____

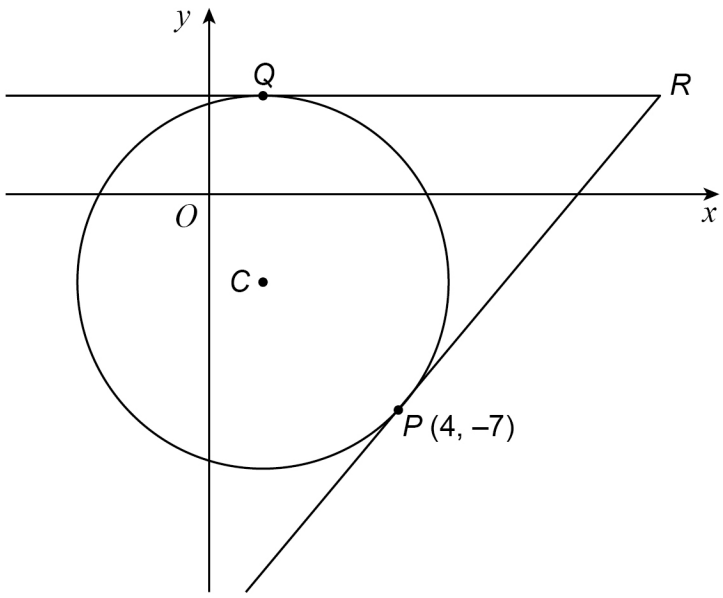
9

23 A circle has centre C and equation $(x-1)^2 + (y+3)^2 = 25$

P (4, –7) and Q are points on the circle.

The tangent at Q is parallel to the x-axis.

The tangents at P and Q intersect at point R.



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23 (a) Write down the coordinates of C.

[1 mark]

Answer _____

23 (b) Show that the equation of the tangent at Q is $y = 2$

[1 mark]

23 (c) Work out the x-coordinate of R.

[4 marks]

Answer _____

6

24 Show that the curve $y = \frac{3}{5}x^5 + x^4$ has **exactly two** stationary points.

[4 marks]

25 $f(x) = x^3 - 10x - c$ where c is a positive integer.

$(x + c)$ is a factor of $f(x)$.

Use the factor theorem to work out the value of c.

[3 marks]

Answer _____

26 f(x) is a function with domain all values of x.

$f(x) = \sqrt{x^2 + 6x - a}$ where a is a constant.

Work out the possible values of a.

Give your answer as an inequality.

[4 marks]

Answer _____

Turn over for the next question

11

27 The curve $y = f(x)$ has $\frac{dy}{dx} = (x + 2)^6 + (x + 2)^4$

The curve has exactly one stationary point at P where $x = -2$

Use the expression for $\frac{dy}{dx}$ to show that P is a point of inflection.

[3 marks]

END OF QUESTIONS

3

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