AQA

LEVEL 2 CERTIFICATE
Further Mathematics

Paper 2 8360/2 Calculator
Mark scheme

8360
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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

M	Method marks are awarded for a correct method which could lead to a correct answer.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
ft	Follow through marks. Marks awarded for correct working following a mistake in an earlier step.
SC	Special case. Marks awarded for a common misinterpretation which has some mathematical worth.
M dep	A method mark dependent on a previous method mark being awarded.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
oe	Or equivalent. Accept answers that are equivalent. eg accept 0.5 as well as $\frac{1}{2}$
[a, b]	Accept values between a and b inclusive.
[a, b)	Accept values a ≤ value < b
3.14…	Accept answers which begin 3.14 eg 3.14, 3.142, 3.1416
Use of brackets	It is not necessary to see the bracketed work to award the marks.

Examiners should consistently apply the following principles

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a student has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the student. In cases where there is no doubt that the answer has come from incorrect working then the student should be penalised.

Questions which ask students to show working

Instructions on marking will be given but usually marks are not awarded to students who show no working.

Questions which do not ask students to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Students often copy values from a question incorrectly. If the examiner thinks that the student has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Continental notation

Accept a comma used instead of a decimal point (for example, in measurements or currency), provided that it is clear to the examiner that the student intended it to be a decimal point.

Q	Answer	Mark	Comments
1(a)	$\frac{3-5\times20}{2}$ or $\frac{3-100}{2}$ or $(–)\frac{97}{2}$ or $(–)48.5$ or $\frac{3-5\times8}{2}$ or $\frac{3-40}{2}$ or $(–)\frac{37}{2}$ or $(–)18.5$ or $12\times(–)\frac{5}{2}$	M1	oe
	(–)30	A1	Accept if both 30 and –30 are seen
	Additional Guidance		

Q	Answer	Mark	Comments
1(b)	$-\frac{3}{2}$ or $-1\frac{1}{2}$ or -1.5	B1	oe
	Additional Guidance		
	Condone $\frac{3}{-2}$ or $n\to-1.5$ or $-1\frac{1}{2}\to\infty$		B1
	$-\frac{3\cancel{n}}{2\cancel{n}}$		B1
	$-\frac{3n}{2n}$ not processed		B0
	$\frac{3}{0-2}$ not processed		B0
	$-1.5n$		B0

2(a)	$\begin{pmatrix} 13 & -2 \\ 6 & 1 \end{pmatrix}$	B2	B1 13 or –2 or 6 or 1 in correct position in a 2 by 2 matrix
	Additional Guidance		
	Condone missing brackets for B2 or B1 if numbers in a 2 by 2 array		
	Brackets may be square or curly etc		
	Ignore commas and fraction lines		
	$\begin{pmatrix} 13 & -2 \\ 6 & 1 \end{pmatrix}$ followed by further work		B1

2(b)	$5k = 11 - 3k$ or $2k = 11 - 6k$ or $11 - 3k = \frac{5}{2}(11 - 6k)$ or $8k = 11$	M1	oe Any one correct equation
	$\frac{11}{8}$ or $1\frac{3}{8}$ or 1.375 with no incorrect equation seen	A1	oe
	Additional Guidance		
	$\begin{pmatrix} 5k \\ 2k \end{pmatrix} = \begin{pmatrix} 11-3k \\ 11-6k \end{pmatrix}$ with no further correct work		M0
	Ignore subsequent attempt to convert $\frac{11}{8}$ to a mixed fraction or decimal		M1A1
	Ignore subsequent attempt to convert $1\frac{3}{8}$ to an improper fraction or decimal		M1A1
	Ignore subsequent rounding or truncation of 1.375		M1A1
	Answer only 1.37 or 1.38 or 1.4		M0
	T & I is 2 or zero		

Q	Answer	Mark	Comments
2(c)	Valid explanation	B1	eg Number of columns of **B** does not equal number of rows of **A**
	Additional Guidance		
	‘2 by 1’ (or ‘2 × 1’) matrix means 2 rows and 1 column		
	‘First matrix’ means B and ‘second matrix’ means A		
	B columns ≠ A rows		B1
	B rows ≠ A columns		B0
	B columns ≠ A columns		B0
	B rows ≠ A rows		B0
	Number of rows in second matrix cannot be more than number of columns in first matrix		B1
	B has 1 column, A has 2 rows		B1
	A should only have 1 row		B1
	A has too many rows		B1
	B only has one column		B1
	B needs another column		B1
	It is a 2 x 1 multiplied by a 2 × 2		B1
	There’s nothing to multiply the 3 by		B1
	It is a 2 x 2 multiplied by 2 × 1		B0
	It is a 1 by 2 multiplied by a 2 by 2		B0
	B values can’t multiply with all the A values		B0
	They are not compatible		B0
	Because the dimensions of A and B are different		B0
	Can’t work it out this way round		B0
	Can work out AB but not BA		B0
	B has to be a 2 by 2 matrix		B0

3(a)	3 (×) 455 or 5 (×) 273 or 7 (×) 195 or 13 (×) 105 or 15 (×) 91 or 21 (×) 65 or 35 (×) 39 or 3 (×) 5 (×) 7 (×) 13	M1	oe eg 1365 ÷ 5 = 273 Any order Must be integers May be seen in a factor tree or repeated division
	3 5 91 or 3 7 65 or 3 13 35 or 5 7 39 or 5 13 21 or 7 13 15	A1	Any order Must be integers
	Additional Guidance		
	If using division the correct answer must be seen for M1		
	Correct answer can be implied by working lines eg 3 (×) 5 (×) 91 with blank answer line	M1A1	
	Answer line correct	M1A1	
	Allow inclusion of 1 for M1 eg 1 (×) 3 (×) 455	M1	

3(b)	$b(a - 11)$ or $-b(11 - a)$	M1	Implied by square numbers > 1 used eg1 4(36 – 11) eg2 9(16 – 11)
	a = 36 and b = square number > 1 with working for M1 seen	A1	Must be in correct order Allow unprocessed squares eg $a = 6^2$ and $b = 5^2$ SC1 a = 36 and b = square number > 1 without working for M1 seen
	Additional Guidance		
	$b(a - 11) = 0$ or $b(a - 11)$ with further work	M1	
	Answer line takes precedence over working lines		
	Embedded answer eg 81(36 – 11)	M1A0	

4	$\left(\frac{56}{4}\right)^3$ or 14^3 or $4^3x = 56^3$ or $64x = 175\ 616$ or $\frac{56^3}{x} = 4^3$	M1	oe oe equation in $x^{(1)}$ or $\frac{1}{x^{(1)}}$
	2744	A1	
	Additional Guidance		
	$\sqrt[3]{x} = \frac{56}{4}$ or $\sqrt[3]{x} = 14$ with no correct further work		M0
	$56\,x^{-\frac{1}{3}} = 4$		M0
	Solving $\frac{56}{3x} = 4$		M0
	Answer 14^3 with 2744 not seen in working		M1A0
	Embedded solution		M1A0

	Alternative method 1		
	$\frac{a+4}{2} = 3a$ or $3a - a = 4 - 3a$ or $a + \frac{4-a}{2} = 3a$ or $4 - \frac{4-a}{2} = 3a$ or $4 - a = 2(3a - a)$	M1	oe
	$6a - a = 4$ or $3a - a + 3a = 4$ or $2a - a - 6a = -4$ or $8 - 4 = 6a - a$ or $4 = 4a + a$ or $5a = 4$	M1dep	oe Allow eg $3a \times 2$ for $6a$ Terms collected
5	$\frac{4}{5}$ or 0.8	A1	oe
	Alternative method 2		
	$\frac{8-6}{3a-a} = \frac{10-6}{4-a}$ or $\frac{8-6}{3a-a} = \frac{10-8}{4-3a}$ or $\frac{10-6}{4-a} = \frac{10-8}{4-3a}$	M1	oe eg fractions inverted
	$8a + 2a = 8$ or $6a + 4a = 8$ or $-12a + 2a = 8 - 16$ or $5a = 4$	M1dep	oe Allow eg $2a \times 4$ for $8a$ Terms collected
	$\frac{4}{5}$ or 0.8	A1	oe

Alternative method 3 and Additional Guidance continue on the next page

Q	Answer	Mark	Comments
5 cont	**Alternative method 3**		
	$(8-6)^2 + (3a-a)^2 = (10-8)^2 + (4-3a)^2$ or $5a^2 - 24a + 16 \quad (= 0)$ or $(10-6)^2 + (4-a)^2 = 2^2((8-6)^2 + (3a-a)^2)$ or $15a^2 + 8a - 16 \quad (= 0)$ or $(10-6)^2 + (4-a)^2 = 2^2((10-8)^2 + (4-3a)^2)$ or $35a^2 - 88a + 48 \quad (= 0)$	M1	oe Using $PM^2 = MQ^2$ or $PQ^2 = 4PM^2$ or $PQ^2 = 4MQ^2$
	$(5a-4)(a-4) \quad (= 0)$ or $\dfrac{--24 \pm \sqrt{(-24)^2 - 4 \times 5 \times 16}}{2 \times 5}$ or $(5a-4)(3a+4) \quad (= 0)$ or $\dfrac{-8 \pm \sqrt{8^2 - 4 \times 15 \times -16}}{2 \times 15}$ or $(5a-4)(7a-12) \quad (= 0)$ or $\dfrac{--88 \pm \sqrt{(-88)^2 - 4 \times 35 \times 48}}{2 \times 35}$	M1dep	oe eg $\dfrac{12}{5} \pm \sqrt{\dfrac{64}{25}}$ or $-\dfrac{4}{15} \pm \sqrt{\dfrac{256}{225}}$ or $\dfrac{44}{35} \pm \sqrt{\dfrac{256}{1225}}$ Correct attempt to solve their 3-term quadratic Allow recovery of brackets in formula Allow eg 24^2 for $(-24)^2$ Implied by correct solutions to their 3-term quadratic seen
	$\dfrac{4}{5}$ or 0.8	A1	oe Must reject solution 4 or $-\dfrac{4}{3}$ or $\dfrac{12}{7}$
	Additional Guidance		
	Terms must be collected but do not have to be processed for M1dep eg (Alt 1) $a + 4 = 6a$ needs terms collecting to $4 = 6a - a$ for M1dep		
	Rejection of solution is implied by only $\dfrac{4}{5}$ on answer line		

6	$\sin 38 = \frac{r}{20}$ or $\cos(90 - 38) = \frac{r}{20}$ or $\frac{r}{\sin 38} = \frac{20}{\sin 90}$ or $\frac{\sin 38}{r} = \frac{\sin 90}{20}$	M1	oe Any letter
	$20 \times \sin 38$ or $20 \times \cos(90 - 38)$ or $\frac{20}{\sin 90} \times \sin 38$	M1dep	oe M2 $\sqrt{20^2 - (20\cos 38)^2}$ M2 $\frac{\sqrt{20^2 + 20^2 - 2 \times 20 \times 20 \times \cos(38 \times 2)}}{2}$
	12.3(...)	A1	SC2 Angle *VAC* = 38 seen on diagram **and** answer 15.76(…) or 15.8

Additional Guidance

If trigonometry and Pythagoras are used it must be a fully correct method that would lead to the correct value of *r* for M2	
If cosine rule with angle (38 × 2) used it must be a fully correct method that would lead to the correct value of *r* for M2	
Answer 15.76(…) or 15.8 but angle *VAC* = 38 not seen	Zero
12.3(…) seen and angle 38 in correct place with further work eg 20 sin 38 = 12.3 $\sqrt{20^2 - 12.3^2}$ = 15.8	Zero
sin 38 × 20 (even if subsequently evaluates sin 760)	M2
Throughout, accept opp or o for *r* eg $\sin 38 = \frac{\text{opp}}{20}$	M1
$\sin = \frac{r}{20}$ or $\sin\theta = \frac{r}{20}$ (unless recovered)	M0
Answer 12.3(…) coming from scale drawing	M2A1
Answer 12 coming from scale drawing	Zero
12.3(…) seen with no further work followed by answer 12	M2A1

7	**Alternative method 1**		
	(x-coordinate of A =) 10 **and** (y-coordinate of B =) 8	B1	May be implied on diagram eg 10 written next to A **and** 8 written next to B
	(x-coordinate of P =) $\frac{2}{2+3}$ × their 10 or $\frac{2 \times \text{their } 10 + 3 \times 0}{2+3}$ or 4	M1	oe their 10 must be their x-coordinate of A May be seen on diagram
	(area of triangle OBP =) $\frac{1}{2}$ × their 8 × their 4	M1dep	oe their 8 must be their y-coordinate of B
	16	A1ft	ft B0M2
	Alternative method 2		
	(x-coordinate of A =) 10 **and** (y-coordinate of B =) 8	B1	May be implied on diagram eg 10 written next to A **and** 8 written next to B
	(area of triangle OAB =) $\frac{1}{2}$ × their 10 × their 8 or 40	M1	oe
	(area of triangle OBP =) $\frac{2}{2+3}$ × their 40	M1dep	oe eg their 40 – $\frac{3}{2+3}$ × their 40
	16	A1ft	ft B0M2

Alternative methods 3 and 4 and Additional Guidance continue on the next two pages

7 cont	**Alternative method 3**		
	(x-coordinate of *A* =) 10 **and** (y-coordinate of *B* =) 8	B1	May be implied on diagram eg 10 written next to *A* **and** 8 written next to *B*
	(area of triangle *OAB* =) $\frac{1}{2} \times$ their 10 $\times$ their 8 or 40	M1	oe
	(y-coordinate of *P* =) $\frac{3}{2+3} \times$ their 8 or 4.8 **and** (area of triangle *OPA* =) $\frac{1}{2} \times$ their 10 $\times$ their 4.8 or 24 **and** (area of triangle *OBP* =) their 40 – their 24	M1dep	oe their 8 must be their y-coordinate of *B* y-coordinate of *P* may be seen on diagram
	16	A1ft	ft B0M2

Alternative method 4 and Additional Guidance continue on the next page

7 cont	**Alternative method 4**		
	(x-coordinate of A =) 10 **and** (y-coordinate of B =) 8	B1	May be implied on diagram eg 10 written next to A **and** 8 written next to B
	(AB =) $\sqrt{\text{their } 10^2 + \text{their } 8^2}$ or $\sqrt{100+64}$ or $\sqrt{164}$ or $2\sqrt{41}$ or 12.8(…) **and** (BP =) $\frac{2}{2+3}$ × their 12.8(…) or 5.12(…) **and** (angle OBP =) $\tan^{-1}\frac{\text{their } 10}{\text{their } 8}$ or 51.3(…)	M1	oe their 10 must be their x-coordinate of A their 8 must be their y-coordinate of B
	(area of triangle OBP =) $\frac{1}{2}$ × their 8 × their 5.12 × sin their 51.3	M1dep	oe their 8 must be their y-coordinate of B
	16	A1ft	ft B0M2

7 cont	**Additional Guidance**	
	A = 10 and B = 8	B1
	A (8, 0) and B (0, 10) is B0 but can subsequently score up to M2A1ft (answer 16)	
	A (0, 10) and B (8, 0) is B0 but can score up to M2A1ft if uses x-coordinate of A as 10 and y-coordinate of B as 8 (answer 16)	
	A (0, 8) and B (10, 0) is B0 but can score up to M2A1ft if uses x-coordinate of A as 8 and y-coordinate of B as 10 (answer 16)	
	Area triangle OBP may be seen as the sum of two right-angled triangles	
	Area triangle OBP may be seen as area trapezium $OBPX$ – area triangle OPX X is on the x-axis with PX perpendicular to the x-axis	
	Allow marks for valid working seen even if not subsequently used	
	15.9(…) → answer 16 Answer 15.9(…)	4 marks B1M2A0

	Alternative method 1		
	(BC =) 12	B1	Allow as two 6s labelled on BC after perpendicular drawn from A
	their $12^2 = 7^2 + 8^2$ $- 2 \times 7 \times 8 \times \cos A$ or $144 = 49 + 64 - 112 \cos A$ or $144 = 113 - 112 \cos A$ or $\frac{7^2 + 8^2 - \text{their } 12^2}{2 \times 7 \times 8}$ or $\frac{49 + 64 - 144}{112}$ or $-\frac{31}{112}$ or [–0.277, –0.27] or –0.28	M1	oe Do not allow if their 12 comes from use of Pythagoras' theorem ie (BC =) $\sqrt{7^2 + 8^2}$ or $\sqrt{113}$ or 10.6(…) is B0M0
	$\cos^{-1}\left(\frac{7^2 + 8^2 - \text{their } 12^2}{2 \times 7 \times 8}\right)$	M1dep	oe May be implied by final answer
8	[106, 106.1]	A1ft	Only ft B0M2
	Alternative method 2		
	(BC =) 12	B1	Allow as two 6s labelled on BC after perpendicular drawn from A
	(angle ABC =) $\cos^{-1}\left(\frac{7^2 + \text{their } 12^2 - 8^2}{2 \times 7 \times \text{their} 12}\right)$ or 39.8… **and** $\sin A = \frac{\sin \text{their } 39.8}{8} \times$ their 12 or $\sin A = 0.96$…	M1	oe eg works out angle ACB (= 34.09(…) or 34.1) and uses sine rule Do not allow if their 12 comes from use of Pythagoras' theorem ie (BC =) $\sqrt{7^2 + 8^2}$ or $\sqrt{113}$ or 10.6(…) is B0M0
	$180 - \sin^{-1}$ (their 0.96…)	M1dep	oe May be implied by final answer
	[106, 106.1]	A1ft	Only ft B0M2

Additional Guidance continues on the next page

	Additional Guidance	
8 cont	$\cos^{-1}$ or $\cos^{-1}$ans does not score M1dep unless recovered	
	For the M1dep must have correct rearrangement but allow arithmetic errors	
	Answer outside range is A0 eg 106.2(…) from $\cos^{-1}(-0.28)$	

9	**Alternative method 1**		
	$-\frac{11}{5} < x \leqslant \frac{5}{5}$ or $-2.2 < x \leqslant \frac{5}{5}$	M1	oe eg $x \leqslant \frac{5}{5}$ and $x > -\frac{11}{5}$
	$-\frac{11}{5} < x \leqslant 1$ or $-2.2 < x \leqslant 1$ or $-2 \leqslant x \leqslant 1$ or $-2, -1, 0, 1$	A1	oe eg $x \leqslant 1$ and $x > -\frac{11}{5}$
	$6x - 4x \leqslant 4 - 7$ or $2x \leqslant -3$	M1	oe Collects terms
	$x \leqslant -\frac{3}{2}$ or $x \leqslant -1.5$ or $x < -\frac{3}{2}$ or $x < -1.5$ or $x \leqslant -2$ or $-2, -3$ (, -4, ...)	A1	$-2.2 < x \leqslant -1.5$ or $-2 \leqslant x \leqslant -1.5$ implies M1A1M1A1
	-2 with no other values given	A1	Must have gained M1A1M1A1
	Alternative method 2		
	Shows that -2 satisfies either $-11 < 5x \leqslant 5$ or $6x + 7 \leqslant 4x + 4$	M1	eg $-11 < -10 \leqslant 5$ or $5x = -10$ and yes
	Shows that -2 satisfies both $-11 < 5x \leqslant 5$ and $6x + 7 \leqslant 4x + 4$	A1	
	Shows that -1 does not satisfy $6x + 7 \leqslant 4x + 4$ **or** shows that -3 does not satisfy $-11 < 5x \leqslant 5$	M1	eg $-6 + 7 > -4 + 4$
	Shows that -1 does not satisfy $6x + 7 \leqslant 4x + 4$ **and** shows that -3 does not satisfy $-11 < 5x \leqslant 5$	A1	
	-2 with no other values given	A1	Must have gained M1A1M1A1

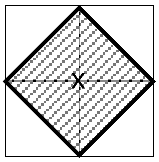
Alternative methods 3 and 4 and Additional Guidance continue on the next two pages

9	**Alternative method 3**		
	$-\frac{11}{5} < x \leqslant \frac{5}{5}$ or $-2.2 < x \leqslant \frac{5}{5}$	M1	oe eg $x \leqslant \frac{5}{5}$ and $x > -\frac{11}{5}$
	$-\frac{11}{5} < x \leqslant 1$ or $-2.2 < x \leqslant 1$ or $-2 \leqslant x \leqslant 1$ or $-2, -1, 0, 1$	A1	oe eg $x \leqslant 1$ and $x > -\frac{11}{5}$
	Shows that -2 satisfies $6x + 7 \leqslant 4x + 4$ **or** shows that -1 does not satisfy $6x + 7 \leqslant 4x + 4$	M1	eg $6 \times -2 + 7 = -5$ and $4 \times -2 + 4 = -4$ ✓
	Shows that -2 satisfies $6x + 7 \leqslant 4x + 4$ **and** shows that -1 does not satisfy $6x + 7 \leqslant 4x + 4$	A1	
	-2 with no other values given	A1	Must have gained M1A1M1A1

Alternative method 4 and Additional Guidance continue on the next page

Q	Answer	Mark	Comments
9	**Alternative method 4**		
	$6x - 4x \leqslant 4 - 7$ or $2x \leqslant -3$	M1	oe Collects terms
	$x \leqslant -\frac{3}{2}$ or $x \leqslant -1.5$ or $x < -\frac{3}{2}$ or $x < -1.5$ or $x \leqslant -2$ or –2, –3 (, –4, ...)	A1	
	Shows that –2 satisfies $-11 < 5x \leqslant 5$ **or** shows that –3 does not satisfy $-11 < 5x \leqslant 5$	M1	eg $-11 < -10 \leqslant 5$ or $5x = -10$ and yes
	Shows that –2 satisfies $-11 < 5x \leqslant 5$ **and** shows that –3 does not satisfy $-11 < 5x \leqslant 5$	A1	
	–2 with no other values given	A1	Must have gained M1A1M1A1
	Additional Guidance		
	Allow eg max 1 and min –2.2 for $-2.2 < x \leqslant 1$, unless contradicted by a list of values		
	Condone omission of non-critical values from lists eg –2, –1, 1		
	Using = signs when solving inequalities can score M marks only unless recovered		
	Incorrect notation eg $\leqslant$ for $<$ can score M marks only		
	If answers to trials evaluated they must be correct		
	Choose the scheme that favours the student		
	–2 identified as the only integer with no valid working		Zero

10	**Alternative method 1**		
	$\frac{1}{2} \times x \times x \times \sin 150$ or $\frac{1}{4}x^2$ or $\frac{1}{2} \times b \times c \times \sin 150 = 57.76$ or $\frac{1}{4} \times b \times c = 57.76$	M1	oe Any letter(s)
	$x^2 = \frac{57.76 \times 2}{\sin 150}$ or $x^2 = 57.76 \times 4$ or $x^2 = 231(.04)$ or $\frac{1}{2}x = \sqrt{57.76}$ or $\sqrt{231(.04)}$ or $2\sqrt{57.76}$	M1dep	oe eg $x^2 = \frac{57.76}{\frac{1}{2}\sin 150}$ Must have either $x^2 =$ or $\frac{1}{2}x = \sqrt{57.76}$ or $\sqrt{231(.04)}$ or $2\sqrt{57.76}$ Any letter
	15.2	A1	
	Alternative method 2		
	$\frac{1}{2} \times x \times x \cos\frac{150}{2} \times \sin\frac{150}{2}$ $= \frac{57.76}{2}$	M1	oe Any letter
	$x^2 = \frac{57.76}{\cos\frac{150}{2}\sin\frac{150}{2}}$ or $x^2 = 231(.04)$ or $\sqrt{231(.04)}$ or $2\sqrt{57.76}$	M1dep	oe Must have either $x^2 =$ or $\sqrt{231(.04)}$ or $2\sqrt{57.76}$ Any letter
	15.2	A1	
	Additional Guidance		
	Do not allow 15 as a misread of 150		
	x can be b or AB or AC etc		
	b and c can be a and b or AB and AC etc		

11(a)	Straight line between (–2, 7) and (0, 3)	B1	Tolerance of ±1 small square Allow line to be extended
	Points (0, 3) (1, 4) (2, 3) (3, 0) (4, –5)	M1	Tolerance of ±1 small square May be plotted or seen in a table Points can be implied
	Correct smooth parabolic curve with maximum at (1, 4)	A1	Tolerance of ±1 small square Allow (ruled) straight line between (3, 0) and (4, –5) Curve passing through all correct points within tolerance scores M1A1
	Straight line between (4, –5) and (5, 0)	B1	Tolerance of ±1 small square Allow line to be extended
	Additional Guidance		
	Ignore extra points plotted		
	Tolerance of ±1 small square means it is on the edges of or within the shaded area 		
	Points only can score a maximum of M1		
	Ruled straight lines for curve apart from between (3, 0) and (4, –5)		A0
	If all 4 marks would be awarded but either (i) graph has a line or a curve that extends beyond the individual domains or (ii) the curve does not meet a line at a cusp		3 marks

11(b)	$-5 \leqslant f(x) \leqslant 7$ or $7 \geqslant f(x) \geqslant -5$ or [–5, 7]	B2ft	Correct or ft their graph in (a) for B2 ft their graph in (a) for B1 B1ft $-5 \leqslant f(x)$ or $f(x) \leqslant 7$ on their own or embedded within an interval for $f(x)$ or only –5 and 7 chosen eg $-5 < f(x) < 7$

Additional Guidance

Allow $f(x)$ to be y or f or fx eg1 $-5 \leqslant y \leqslant 7$ eg2 $f \leqslant 7$	 B2 B1
Allow as two inequalities $f(x) \geqslant -5$ (and/or) $f(x) \leqslant 7$	B2
ft their graph if incomplete eg no graph drawn for $-2 \leqslant x < 0$ but otherwise correct and answer $-5 \leqslant f(x) \leqslant 4$	B2ft
ft their graph if drawn for x values beyond [–2, 5] eg1 straight line from (–3, 8) to (6, –1) and answer $-1 \leqslant y \leqslant 8$ eg2 straight line from (–3, 8) to (6, –1) and answer $f(x) \leqslant 8$	 B2ft B1ft
Straight line from (–2, 9) to (6, –7) and answer $-7 \leqslant y \leqslant 9$	B2ft
Straight line from (0, 9) to (5, –4) and answer $-4 \leqslant f(x) \leqslant 9$	B2ft
B2ft (or B1ft) can be awarded for a range beyond [–7, 9] if it is clear from working (eg a table of values) where the answer is from	
–5 to 7 inclusive is B2 whereas –5 to 7 is B1	
B1 for a correct inequality embedded eg1 $-5 < f(x) \leqslant 7$ eg2 $-5 \leqslant f(x) \leqslant 0$ eg3 $-2 \leqslant y \leqslant 7$	 B1 B1 B1
For B1 ignore incorrect notation if only –5 and 7 chosen eg1 $-5 \leqslant x \leqslant 7$ eg2 $-5 < x \leqslant 7$ eg3 $-5 \geqslant f(x) \geqslant 7$ eg4 –5, 7	 B1 B1 B1 B1
{–5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, 7}	B0
Working out a statistical range eg –5 to 7 = 12	B0

<table>
<tr><td rowspan="7">12(a)</td><td>$3(25 - x^2)$ or $-3(x^2 - 25)$
or $(15 + 3x)(5 - x)$
or $(x + 5)(15 - 3x)$</td><td>M1</td><td colspan="2">oe partial factorisation
eg $-(3x + 15)(x - 5)$
Brackets in either order
Do not allow $-(3x^2 - 75)$</td></tr>
<tr><td>$3(5 + x)(5 - x)$
or $3(-x - 5)(x - 5)$
or $-3(x + 5)(x - 5)$
or $-3(5 - x)(-x - 5)$</td><td>A1</td><td colspan="2"></td></tr>
<tr><td colspan="4">Additional Guidance</td></tr>
<tr><td colspan="3">$(-x + 5)$ is equivalent to $(5 - x)$ etc</td><td></td></tr>
<tr><td colspan="3">Do not allow A1 for incorrect notation in final answer
eg $(5 + x)3(5 - x)$</td><td>M1A0</td></tr>
<tr><td colspan="3">Do not allow A1 for use of multiplication signs in final answer
eg $3 \times (5 + x) \times (5 - x)$</td><td>M1A0</td></tr>
<tr><td colspan="3">Correct answer followed by incorrect further work</td><td>M1A0</td></tr>
</table>

12(b)	**Alternative method 1**		
	$9n^2 + 3n + 3n + 1$ or $9n^2 + 6n + 1$ or $9n^2 – 3n – 3n + 1$ or $9n^2 – 6n + 1$	M1	oe Terms may be seen in a grid
	$12n$ with no incorrect working	A1	Brackets can be recovered
	Alternative method 2		
	$(3n + 1 + 3n – 1)(3n + 1 – (3n – 1))$ or $(3n + 1 + 3n – 1)(3n + 1 – 3n + 1)$	M1	oe Brackets around $3n – 1$ can be recovered
	$12n$	A1	
	Additional Guidance		
	Alt 1 $12n$ may come from incorrect working eg1 $3n^2 + 6n + 1 – (3n^2 – 6n + 1) = 12n$ eg2 $9n^2 + 3n + 1 – (9n^2 – 3n + 1) = 12n$		 M0A0 M0A0
	Alt 1 Recovery of brackets eg1 $9n^2 + 6n + 1 – 9n^2 – 6n + 1 = 12n$ eg2 $9n^2 + 6n + 1 – 9n^2 – 6n + 1 = 2$		 M1A1 M1A0
	Alt 2 Recovery of brackets eg1 $(3n + 1 + 3n – 1)(3n + 1 – 3n – 1) = 12n$ eg2 $(3n + 1 + 3n – 1)(3n + 1 – 3n – 1) = 0$		 M1A1 M0A0
	Do not allow A1 for use of multiplication signs in final answer eg $12 \times n$ with no incorrect working		 M1A0

Q	Answer	Mark	Comments
	Single correct fraction with terms processed	M1	eg1 $\dfrac{600a^5 + 1200a^4}{36a^3 + 72a^2}$ eg2 $\dfrac{50a^3 + 100a^2}{3a + 6}$ Only bracket allowed is (a + 2) eg $\dfrac{50a^4(a + 2)}{3a^3 + 6a^2}$ (scores M2)
13	Factorises correctly using (a + 2)	M1	Only needs to be seen once eg1 $\dfrac{8a}{3a+6} \times \dfrac{5(a+2)}{3a^2} \div \dfrac{4}{15a^3}$ eg2 $\dfrac{8a}{3(a+2)} \times \dfrac{5a+10}{3a^2} \times \dfrac{15a^3}{4}$ Award M2 for fully correct unprocessed expression with full cancelling seen eg $\dfrac{{}^{2}\cancel{8}a}{3\cancel{(a+2)}} \times \dfrac{5\cancel{(a+2)}}{\cancel{3}\,\cancel{a^2}} \times \dfrac{{}^{5}\cancel{15}a\cancel{{}^{3}}{}^{1}}{\cancel{4}}$ or $\dfrac{2a}{3} \times 5 \times 5a$ oe
	$\dfrac{50a^2}{3}$ or $16\dfrac{2}{3}a^2$ or $16.\dot{6}a^2$	A1	

Additional Guidance	
$\dfrac{50 \times a \times a}{3}$	M2A0
A correct single fraction with (a + 2) cancelled will be M2 eg1 $\dfrac{250a^2}{15}$ eg2 $\dfrac{50a^4}{3a^2}$	M2A0
$\dfrac{8a}{3} \times \dfrac{5(a+2)}{3a^2} \times \dfrac{15a^3}{4}$	M0M1A0
$3a + 6 = 3(a + 2)$ with no other valid working	M0M1A0
Brackets other than (a + 2) may be seen $\dfrac{10a^2(5a+10)}{3a+6}$	M0M0
Correct answer followed by incorrect further work	M2A0
Allow one miscopy for up to M2A0	

14	**Alternative method 1**		
	$-\frac{1}{4}$ or –0.25	B1	gradient of $x + 4y = 74$ Do not allow embedded May be implied
	(gradient =) $\frac{-1}{\text{their } -\frac{1}{4}}$ or 4	M1	ft their $-\frac{1}{4}$ Only ft a non-zero numerical value Implied by $y = 4x + b$ or $a = 4$ (B1M1)
	$(y =)\ \frac{74-2}{4}$ or $\frac{72}{4}$ or 18	M1	oe May be seen on diagram
	their 18 = their 4 × 2 + b or y – their 18 = their 4(x – 2)	M1dep	oe dep on M2
	$b = 10$	A1ft	ft 18 – their 4 × 2 if B0M3
	Alternative method 2		
	$-\frac{1}{4}$ or –0.25	B1	gradient of $x + 4y = 74$ Do not allow embedded May be implied
	(gradient =) $\frac{-1}{\text{their } -\frac{1}{4}}$ or 4	M1	ft their $-\frac{1}{4}$ Only ft a non-zero numerical value Implied by $y = 4x + b$ or $a = 4$ (B1M1)
	Correct method for elimination of y from $x + 4y = 74$ and y = their $4x + b$	M1dep	eg $x + 4(4x + b) = 74$ or $17x + 4b = 74$
	Substitutes $x = 2$ into their equation	M1dep	eg $34 + 4b = 74$
	$b = 10$	A1ft	ft 18 – their 4 × 2 if B0M3

Alternative method 3 and Additional Guidance continue on the next page

14 cont	**Alternative method 3**		
	$-\frac{1}{4}$ or -0.25	B1	gradient of $x + 4y = 74$ Do not allow embedded May be implied
	(gradient =) $\frac{-1}{\text{their } -\frac{1}{4}}$ or 4	M1	ft their $-\frac{1}{4}$ Only ft a non-zero numerical value Implied by $y = 4x + b$ or $a = 4$ (B1M1)
	$(y =)$ $\frac{74-2}{4}$ or $\frac{72}{4}$ or 18	M1	oe May be seen on diagram
	Correct method for elimination of x from $x + 4y = 74$ and $y =$ their $4x + b$ **and** substitutes $y =$ their 18	M1dep	eg $y = 4(74 - 4y) + b$ or $17y = 296 + b$ **and** $306 = 296 + b$ dep on M2
	$b = 10$	A1ft	ft 18 – their 4 × 2 if B0M3
	Additional Guidance		
	$y = 4x + 10$ will gain full marks unless contradicted		
	If an error is made in the constant term when rearranging $x + 4y = 74$ the B1 can still be awarded for gradient = $-\frac{1}{4}$ eg $y = -\frac{1}{4}x + 19$ and gradient = $-\frac{1}{4}$ is B1 (all other marks are possible)		
	In alt 1 and alt 3 the mark for $y = 18$ will sometimes be the only mark awarded		

Q	Answer		Mark	Comments
15	**Alternative method 1**			
	$wy = 8x - y$	$w = \frac{8x}{y} - 1$ or $w + 1 = \frac{8x}{y}$	M1	
	$wy + y = 8x$ or $y(w + 1) = 8x$	$\frac{w+1}{8x} = \frac{1}{y}$	M1dep	oe y term(s) collected eg $-wy - y = -8x$ M2 $\frac{8x}{w+1}$ or $\frac{-8x}{-w-1}$ or $\frac{-8x}{-(w+1)}$
	$y = \frac{8x}{w+1}$ or $y = \frac{-8x}{-w-1}$ or $y = \frac{-8x}{-(w+1)}$		A1	oe eg $y = \frac{4x}{0.5w+0.5}$ Must have $y =$
	Alternative method 2			
	$y = \frac{8x}{w} - \frac{y}{w}$		M1	
	$y + \frac{y}{w} = \frac{8x}{w}$ or $y(1 + \frac{1}{w}) = \frac{8x}{w}$		M1dep	oe y term(s) collected M2 $\frac{\frac{8x}{w}}{1+\frac{1}{w}}$
	$y = \frac{\frac{8x}{w}}{1+\frac{1}{w}}$		A1	oe Must have $y =$
	Additional Guidance			
	$y = \frac{8x}{w+1}$ in working with $\frac{8x}{w+1}$ on answer line etc			M2A1
	Allow multiplications signs and 1s throughout			
	$w = \frac{8x}{y} - \frac{y}{y}$ with no further simplification			M0
	Correct answer followed by incorrect further work			M2A0

Q	Answer	Mark	Comments
16(a)	3^{-2b}	B1	
	Additional Guidance		

Q	Answer	Mark	Comments
16(b)	5^{x+2}	B1	
	Additional Guidance		

Q	Answer	Mark	Comments
16(c)	2^{3m}	B1	
	Additional Guidance		

Q	Answer	Mark	Comments
17(a)	$3x^2$ or (–)$12x$	M1	Attempt at $\frac{dy}{dx}$
	their $(3x^2 - 12x) = 0$	M1dep	Must have at least 2 terms for their $\frac{dy}{dx}$ The $= 0$ can be implied by sight of a correct non-zero solution to their $(3x^2 - 12x) = 0$
	$x = 4$ (and $x = 0$)	A1ft	ft M2 if their $\frac{dy}{dx}$ is a 2-term quadratic
	(4, –25) with correct expression for $\frac{dy}{dx}$ seen	A1	
	Additional Guidance		
	Condone $y = 3x^2 - 12x$ etc		M1
	Ignore working for second derivative or testing for minimum point		
	Stating $\frac{dy}{dx} = 0$ is not sufficient for second M mark but may be implied by correct solution(s) seen		

17(b)	**Alternative method 1**		
	$(-1)^3 - 6(-1)^2 + 7 = 0$ with no incorrect evaluations seen or $-1 - 6 + 7 = 0$	B1	Must have = 0
	Alternative method 2		
	$(x + 1)(x^2 - 7x + 7) = 0$ **and** $(x + 1) = 0$ **and** $x = -1$	B1	
	Additional Guidance		
	$(-1)^3 - 6(-1)^2 + 7$ or $-1 - 6 + 7$		B0
	Allow -1^3 or (-1^3) for $(-1)^3$		
	Allow recovery of brackets for $(-1)^2$ eg1 $-1^3 - 6 \times -1^2 + 7 = 0$ eg2 $-1^3 - 6 \times -1^2 + 7 = -1 - 6 + 7 = 0$		 B0 B1

17(c)	**Alternative method 1**		
	(x – –1) or (x + 1) seen	M1	
	$(x + 1)(x^2 - 7x + c)$	M1dep	c can be any non-zero value Implied by $(x + 1)(x^2 + bx + c)$ **and** $b + 1 = -6$ or $b = -7$
	$x^2 - 7x + 7$ (= 0)	A1	
	$\dfrac{--7 \pm \sqrt{(-7)^2 - 4 \times 1 \times 7}}{2 \times 1}$ or $\dfrac{7 \pm \sqrt{21}}{2}$	M1	oe eg $\dfrac{7}{2} \pm \sqrt{\dfrac{21}{4}}$ Correct attempt to solve their 3-term quadratic Allow recovery of brackets Allow 7^2 for $(-7)^2$ Implied by correct solutions to their 3-term quadratic seen
	5.79 and 1.21 with $x^2 - 7x + 7$ (= 0) seen	A1	Must both be to 2 dp
	Alternative method 2		
	(x – –1) or (x + 1) seen	M1	
	$x + 1 \overline{) x^3 - 6x^2 (+0x) + 7}$ with quotient $x^2 - 7x$ …	M1dep	
	$x^2 - 7x + 7$ (= 0)	A1	
	$\dfrac{--7 \pm \sqrt{(-7)^2 - 4 \times 1 \times 7}}{2 \times 1}$ or $\dfrac{7 \pm \sqrt{21}}{2}$	M1	oe eg $\dfrac{7}{2} \pm \sqrt{\dfrac{21}{4}}$ Correct attempt to solve their 3-term quadratic Allow recovery of brackets Allow 7^2 for $(-7)^2$ Implied by correct solutions to their 3-term quadratic seen
	5.79 and 1.21 with $x^2 - 7x + 7$ (= 0) seen	A1	Must both be to 2 dp

Additional Guidance is on the next page

17(c) cont	**Additional Guidance**	
	Final A1 mark can be awarded if both answers seen in working with $x^2 - 7x + 7$ (= 0) seen but only one answer is written on answer line	
	(x + 1) followed by 5.79 and 1.21 without $x^2 - 7x + 7$ (= 0) seen	M1M0A0 M0A0
	(x – 1) instead of (x + 1) can score a maximum of M0M0A0M1A0	
	T & I on the cubic equation	Zero

18	$x^2 + (2x)^2 = (4y)^2$ or $x^2 + 4x^2 = 16y^2$	M1	oe eg $5x^2 = (4y)^2$ Missing brackets may be recovered
	$x^2 = \frac{16}{5}y^2$ or $5x^2 = 16y^2$ or $x = \sqrt{\frac{16y^2}{5}}$ or $x\sqrt{5} = 4y$	M1	oe equation of the form $ax^2 = by^2$ or $cx = dy$ or $x = \sqrt{ky^2}$ eg $x^2 = 16y^2 \div 5$ ft if Pythagoras used with only error being missing brackets
	$2 \times$ their $\frac{16}{5}y^2$ or their $16y^2 \div \frac{\text{their } 5}{2}$	M1dep	oe $2 \times$ their x^2 or $2 \times (\text{their } x)^2$ dep on at least one M
	$\frac{32}{5}y^2$ or $6\frac{2}{5}y^2$ or $6.4y^2$	A1	
	Additional Guidance		
	$5x^2 = (4y)^2$ with no further work		M1M0M0
	$x^2 + 4x^2 = 4y^2$ with answer $\frac{8}{5}y^2$		M0M1M1A0
	$x^2 + 2x^2 = 16y^2$ with answer $\frac{32}{3}y^2$		M0M1M1A0
	$x^2 + 2x^2 = 4y^2$ with answer $\frac{8}{3}y^2$		M0M1M1A0
	$(2x)^2 = (4y)^2 - x^2$ $x^2 = 16y^2$ and $2 \times 16y^2 = 32y^2$		M1 M0M1A0
	$\frac{32}{5}y^2$ followed by further work		M3A0

19(a)	k	B1	
	Additional Guidance		
	$k = 0$ or $k = 1$ etc		B0

19(b)	$-k$	B1	
	Additional Guidance		
	$-k = 0$ or $-k = 1$ etc		B0

19(c)	$k^2 + \cos^2 \alpha = 1$ or $1 - k^2$	M1	oe eg $(1 + k)(1 - k)$
	$\sqrt{1-k^2}$ or $\sqrt{(1+k)(1-k)}$	A1	
	Additional Guidance		
	Answer $-\sqrt{1-k^2}$ or $\pm\sqrt{1-k^2}$		M1A0
	Correct answer followed by incorrect further work		M1A0
	Answer $1 - k^2$		M1A0
	Allow $\cos^2 x$ or $\cos^2 \theta$ etc or $\cos^2$ or c^2 or $(\cos \alpha)^2$ for $\cos^2 \alpha$		
	Condone $\cos \alpha^2$ for $\cos^2 \alpha$		
	$\cos(\sin^{-1}k)$		M0A0

Q	Answer	Mark	Comments
20(a)	Angle in a semicircle (is a right angle) or Angle at centre is 180°, angle at circumference is half the angle at the centre (= 90°) or Angle at centre is 180°, angle at centre is twice the angle at the circumference or Angle subtended at circumference by a diameter	B1	
	Additional Guidance		
	Do not allow half a circle to mean a semicircle		
	Allow extra words if not contradictory eg1 Angle at the circumference in a semicircle eg2 Angle inscribed in a semicircle		 B1 B1
	Angle subtended by a diameter (no mention of at circumference)		B0
	Angle in a hemisphere is 90		B0
	Angle at centre is 180		B0
	Angle at circumference is half the angle at the centre		B0
	2 chords on diameter meet at 90		B0
	Triangle in a semicircle always has a right angle		B0
	Angle in a semicircle is 180		B0
	Angle on a diameter is a right angle		B0
	Because *AB* is a diameter		B0

Q	Answer		Mark	Comments
20(b)	angle *ABE* $= 90 - x$ **or** angle *CBE* $= 90 + x$	angle *DEB* = 90 **and** angle *DCB* = 90	B1	
	angle *CDE* = $90 - x$		B1dep	
	angle *CED* = $90 - x$		B1dep	
	angle *DCE* = $2x$ **and** all reasons given for their proof		B1dep	See guidance for acceptable wording for reasons
	Additional Guidance			
	To award a particular mark, all previous marks must have been awarded			
	First three B marks can be awarded with no or incorrect reasons			
	Do not mark any working on the diagram – statements are needed			
	Incorrect angles score B0 eg1 angle *ABE* = $90 - x$ angle *DEC* = $90 + x$ eg2 angle *ABE* = $90 - x$ angle *CDE* = $90 - x$ angle *DCE* = $90 + x$			 B1B0B0B0 B1B1B0B0
	Angle *CDE* and angle *CDA* are the same angle etc			
	Angle *EBA* and angle *ABE* are the same angle etc			
	Condone *ABE* for angle *ABE* etc			
	Do not allow angle *C* for angle *DCE* etc			
	CE must be proven to be a tangent if used in a response			
	Reasons angle sum of triangle (is 180°) or angles in a triangle (add to 180°) or 180° in a triangle (adjacent) angles on a (straight) line (add to 180°) or 180° on a (straight) line exterior angle of triangle (= sum of opposite interior angles) (equal angles in an) isosceles (triangle) or *CD* = *CE* (opposite angles in a) cyclic quadrilateral (add to 180°) exterior angle of cyclic quadrilateral (= opposite interior angle)			Degrees symbol may be omitted Abbreviations are allowed eg quad for quadrilateral

21(a)	(0, 8)	B1	
	Additional Guidance		
	Answer line takes precedence over working lines and diagram		
	Answer line blank with *C* labelled (0, 8) on diagram		B1
	Answer line blank with 8 written next to *C* on diagram		B0
	(8, 0)		B0
	Answer 8		B0

21(b)	$-x^2 - 2x + 4x + 8$	M1	Allow one error but no omission Must have an x^2 term Terms may be seen in a grid Implied by $-x^2 + 2x + k$ $k \neq 0$ or $ax^2 + 2x + 8$ $a \neq 0$
	$-x^2 - 2x + 4x + 8$ or $-x^2 + 2x + 8$	A1	$-x^2 - 2x + 4x + 8$ but an error in any collection of terms is M1A0
	$-2x - 2 + 4$ or $-2x + 2$ or $-2(x - 1)$ or $2(1 - x)$	A1ft	oe ft their quadratic in x with M1 awarded
	Additional Guidance		
	$2 - 2x$ with final answer 2 (from substituting in $x = 0$)		M1A1A0
	Condone $y = 2 - 2x$ or $f(x) = 2 - 2x$ in working for M1A1 If $(\frac{dy}{dx}$ or $f'(x) =)$ $2 - 2x$ on answer line also award final A1		
	$y = 2 - 2x$ or $f(x) = 2 - 2x$ on answer line		M1A1A0
	When marking (b), a maximum of M1A1A0 can be awarded from an expansion seen on the previous page if not contradicted by an expansion in (b) The final A1 must be seen in (b) eg1 (b) no expansion seen with an answer of $2x + 2$ At top of previous page $-x^2 + 2x + 8$ eg2 (b) no expansion seen with an answer of $-2x + 6$ In (a) $-x^2 + 2x + 4x + 8 = -x^2 + 6x + 8$		M1A1A0 M1A0A1ft
	Correct use of product rule and gradient function $= -2x + 2$		3 marks

21(c)	**Alternative method 1**		
	(gradient of curve at C =) 2	B1ft	Correct or ft their (b) when $x = 0$ May be implied
	$-\frac{1}{\text{their } 2}$ or $-\frac{1}{2}$	M1	oe ft their 2 Only ft a non-zero numerical value (gradient of normal =) $-\frac{1}{2}$ is B1M1
	y = (their $-\frac{1}{2}$)x + their 8 or y – their 8 = their $-\frac{1}{2}$ $(x - 0)$	M1dep	Must have used gradient of normal not gradient of tangent Correct or ft their 8 from (a) in the form $(0, k)$
	0 = (their $-\frac{1}{2}$)x + their 8 or 0 – their 8 = their $-\frac{1}{2}$ $(x - 0)$ or $x = 16$	M1dep	Correct or ft their 8 from (a) in the form $(0, k)$
	$x = 16$ and $BD = 12$ and $AB = 6$ with correct method seen	A1	oe
	Alternative method 2		
	(gradient of curve at C =) 2	B1ft	Correct or ft their (b) when $x = 0$ May be implied
	$-\frac{1}{\text{their } 2}$ or $-\frac{1}{2}$	M1	oe ft their 2 Only ft a non-zero numerical value (gradient of normal =) $-\frac{1}{2}$ is B1M1
	$\frac{0 - \text{their } 8}{x - 0}$ = their $-\frac{1}{2}$ or x = –their 8 ÷ their $-\frac{1}{2}$ or $x = 16$	M2dep	oe Correct or ft their 8 from (a) in the form $(0, k)$
	$x = 16$ and $BD = 12$ and $AB = 6$ with correct method seen	A1	oe
	Additional Guidance		

22	**Alternative method 1**		
	$(x-2)^2 + (2x+1-1)^2 = 16$	M1	oe Eliminates y
	$x^2 - 2x - 2x + 4 + 4x^2 = 16$ or $5x^2 - 4x - 12$ (= 0)	M1dep	oe Expands both brackets correctly
	$(5x+6)(x-2)$ (= 0) or $\dfrac{--4 \pm \sqrt{(-4)^2 - 4 \times 5 \times -12}}{2 \times 5}$	M1	oe eg $\dfrac{2}{5} \pm \sqrt{\dfrac{64}{25}}$ Correct attempt to solve their 3-term quadratic Allow recovery of brackets in formula Allow 4^2 for $(-4)^2$ Implied by correct solutions to their 3-term quadratic seen
	(x =) –1.2 and (x =) 2 or (x =) –1.2 and (y =) –1.4 or (x =) 2 and (y =) 5 with $5x^2 - 4x - 12$ (= 0) seen	A1	oe eg (x =) $-\dfrac{6}{5}$ and (x =) 2 with $5x^2 - 4x - 12$ (= 0) seen
	(–1.2, –1.4) and (2, 5) with $5x^2 - 4x - 12$ (= 0) seen	A1	oe eg $\left(-\dfrac{6}{5}, -\dfrac{7}{5}\right)$ and (2, 5) with $5x^2 - 4x - 12$ (= 0) seen

22	**Alternative method 2**		
	$x^2 - 2x - 2x + 4 + y^2 - y - y + 1 = 16$	M1	oe Expands both brackets correctly
	$x^2 - 2x - 2x + 4 + (2x + 1)^2$ $- (2x + 1) - (2x + 1) + 1 = 16$ or $5x^2 - 4x - 12$ (= 0)	M1dep	oe Eliminates y
	$(5x + 6)(x - 2)$ (= 0) or $\frac{--4 \pm \sqrt{(-4)^2 - 4 \times 5 \times -12}}{2 \times 5}$	M1	oe eg $\frac{2}{5} \pm \sqrt{\frac{64}{25}}$ Correct attempt to solve their 3-term quadratic Allow recovery of brackets in formula Allow 4^2 for $(-4)^2$ Implied by correct solutions to their 3-term quadratic seen
	(x =) –1.2 and (x =) 2 or (x =) –1.2 and (y =) –1.4 or (x =) 2 and (y =) 5 with $5x^2 - 4x - 12$ (= 0) seen	A1	oe eg (x =) $-\frac{6}{5}$ and (x =) 2 with $5x^2 - 4x - 12$ (= 0) seen
	(–1.2, –1.4) and (2, 5) with $5x^2 - 4x - 12$ (= 0) seen	A1	oe eg $(-\frac{6}{5}, -\frac{7}{5})$ and (2, 5) with $5x^2 - 4x - 12$ (= 0) seen

Alternative methods 3 and 4 and Additional Guidance continue on the next two pages

22	**Alternative method 3**		
	$(\left(\frac{y-1}{2}\right) - 2)^2 + (y-1)^2 = 16$	M1	oe Eliminates x
	$\left(\frac{y-1}{2}\right)^2 - 2\left(\frac{y-1}{2}\right) - 2\left(\frac{y-1}{2}\right) + 4$ $+ y^2 - y - y + 1 = 16$ or $5y^2 - 18y - 35$ (= 0)	M1dep	oe Expands $(\left(\frac{y-1}{2}\right) - 2)^2$ and $(y-1)^2$ correctly
	$(5y + 7)(y - 5)$ (= 0) or $\frac{--18 \pm \sqrt{(-18)^2 - 4 \times 5 \times -35}}{2 \times 5}$	M1	oe eg $\frac{9}{5} \pm \sqrt{\frac{256}{25}}$ Correct attempt to solve their 3-term quadratic Allow recovery of brackets in formula Allow 18^2 for $(-18)^2$ Implied by correct solutions to their 3-term quadratic seen
	(y =) –1.4 and (y =) 5 or (x =) –1.2 and (y =) –1.4 or (x =) 2 and (y =) 5 with $5y^2 - 18y - 35$ (= 0) seen	A1	oe eg (y =) $-\frac{7}{5}$ and (y =) 5 with $5y^2 - 18y - 35$ (= 0) seen
	(–1.2, –1.4) and (2, 5) with $5y^2 - 18y - 35$ (= 0) seen	A1	oe eg $(-\frac{6}{5}, -\frac{7}{5})$ and (2, 5) with $5y^2 - 18y - 35$ (= 0) seen

Alternative method 4 and Additional Guidance continue on the next page

22	**Alternative method 4**		
	$x^2 - 2x - 2x + 4 + y^2 - y - y + 1 = 16$	M1	oe Expands both brackets correctly
	$\left(\frac{y-1}{2}\right)^2 - 2\left(\frac{y-1}{2}\right) - 2\left(\frac{y-1}{2}\right) +$ $4 + y^2 - y - y + 1 = 16$ or $5y^2 - 18y - 35$ (= 0)	M1dep	oe Eliminates x
	$(5y + 7)(y - 5)$ (= 0) or $\frac{--18 \pm \sqrt{(-18)^2 - 4 \times 5 \times -35}}{2 \times 5}$	M1	oe eg $\frac{9}{5} \pm \sqrt{\frac{256}{25}}$ Correct attempt to solve their 3-term quadratic Allow recovery of brackets in formula Allow 18^2 for $(-18)^2$ Implied by correct solutions to their 3-term quadratic seen
	(y =) –1.4 and (y =) 5 or (x =) –1.2 and (y =) –1.4 or (x =) 2 and (y =) 5 with $5y^2 - 18y - 35$ (= 0) seen	A1	oe eg (y =) $-\frac{7}{5}$ and (y =) 5 with $5y^2 - 18y - 35$ (= 0) seen
	(–1.2, –1.4) and (2, 5) with $5y^2 - 18y - 35$ (= 0) seen	A1	oe eg $\left(-\frac{6}{5}, -\frac{7}{5}\right)$ and (2, 5) with $5y^2 - 18y - 35$ (= 0) seen
	Additional Guidance		
	Answers only (no valid working)		Zero
	Both solutions from scale drawing		5 marks
	(2, 5) is often seen without seeing any correct method		Zero
	Allow one miscopy for up to M3A0A0		

23	**Alternative method 1**		
	Replaces tan x with $\frac{\sin x}{\cos x}$ at least once in given expression	M1	eg $\dfrac{1}{\frac{\sin^2 x}{\cos^2 x}} - \dfrac{1}{\sin^2 x}$
	Correct steps leading to the single fraction $\dfrac{\cos^2 x - 1}{\sin^2 x}$ or $\dfrac{\cos^2 x - 1}{1 - \cos^2 x}$ or $\dfrac{1 - \sin^2 x - 1}{\sin^2 x}$ or $\dfrac{\cos^2 x - \cos^2 x - \sin^2 x}{\sin^2 x}$ or $\dfrac{-\sin^2 x}{\sin^2 x}$	M1dep	
	$\dfrac{\cos^2 x - 1}{\sin^2 x} = \dfrac{-\sin^2 x}{\sin^2 x} = -1$ or $\dfrac{\cos^2 x - 1}{1 - \cos^2 x} = -1$ or $\dfrac{1 - \sin^2 x - 1}{\sin^2 x} = -1$ or $\dfrac{-\sin^2 x}{\sin^2 x} = -1$	A1	Must see all steps leading to –1

Alternative method 2 and Additional Guidance continue on the next page

<table>
<tr><td rowspan="12">23 cont</td><td colspan="3">Alternative method 2</td></tr>
<tr><td>Replaces tan x with $\dfrac{\sin x}{\cos x}$ at least once in given expression</td><td>M1</td><td>eg $\dfrac{\sin^2 x - \dfrac{\sin^2 x}{\cos^2 x}}{\sin^2 x \dfrac{\sin^2 x}{\cos^2 x}}$</td></tr>
<tr><td>Correct steps leading to the single fraction
$\dfrac{\sin^2 x(\cos^2 x - 1)}{\sin^4 x}$
or $\dfrac{-\sin^4 x}{\sin^4 x}$</td><td>M1dep</td><td></td></tr>
<tr><td>$\dfrac{\sin^2 x(\cos^2 x - 1)}{\sin^4 x} = \dfrac{-\sin^4 x}{\sin^4 x} = -1$
or $\dfrac{-\sin^4 x}{\sin^4 x} = -1$</td><td>A1</td><td>Must see all steps leading to –1</td></tr>
<tr><td colspan="3">Additional Guidance</td></tr>
<tr><td colspan="2">Allow $\cos^2\theta$ etc or $\cos^2$ or c^2 or $(\cos x)^2$ for $\cos^2 x$ etc</td><td></td></tr>
<tr><td colspan="2">Condone $\cos x^2$ for $\cos^2 x$ etc</td><td></td></tr>
<tr><td colspan="2">Only substituting values for x</td><td>Zero</td></tr>
<tr><td colspan="2">$\dfrac{\cos^2 x - 1}{\sin^2 x}$ etc with no working</td><td>Zero</td></tr>
<tr><td colspan="2">Alt 2 $\dfrac{\sin^2 x\cos^2 x - \sin^2 x}{\sin^4 x}$ with no further working</td><td>M1M0A0</td></tr>
<tr><td colspan="2">Any fully correct response that shows how the given expression is equal to –1 is awarded 3 marks

eg $\dfrac{1}{\dfrac{\sin^2 x}{\cos^2 x}} - \dfrac{1}{\sin^2 x} = \dfrac{\cos^2 x}{\sin^2 x} - \dfrac{1}{\sin^2 x} = \dfrac{1 - \sin^2 x}{\sin^2 x} - \dfrac{1}{\sin^2 x}$
$= \dfrac{1}{\sin^2 x} - \dfrac{\sin^2 x}{\sin^2 x} - \dfrac{1}{\sin^2 x} = -1$</td><td>3 marks</td></tr>
<tr><td colspan="2">$\cot^2 x - \text{cosec}^2 x = -1$</td><td>3 marks</td></tr>
</table>

24	**Alternative method 1**		
	$12(x^2 - 5x)$ … or $12(x - 2.5)^2$ …	M1	oe eg $12\{(x^2 - 5x)$ …$\}$ or $12(x^2 - 5x$ …$)$
	$12\{(x - 2.5)^2 - 2.5^2\}$… or $12(x - 2.5)^2 - 75$ …	M1dep	oe eg $12\{(x - 2.5)^2 - 2.5^2$ …$\}$
	$12(x - 2.5)^2 - 12 \times 2.5^2 + 5$ or $12(x - 2.5)^2 - 70$	M1dep	oe eg $12(x - 2.5)^2 - 12 \times 2.5^2 + 12 \times \frac{5}{12}$
	$12\left(\frac{2x-5}{2}\right)^2 - 12 \times 2.5^2 + 5$	M1dep	oe eg $12\left(\frac{2x-5}{2}\right)^2 - 12 \times 2.5^2 + 12 \times \frac{5}{12}$
	$3(2x - 5)^2 - 70$ or $a = 3$ $b = 2$ $c = -5$ $d = -70$ or $3(5 - 2x)^2 - 70$ or $a = 3$ $b = -2$ $c = 5$ $d = -70$	A1	oe

Q	Answer	Mark	Comments
24	**Alternative method 2**		
	$3(4x^2 - 20x)$ … or $3(2x - 5)^2$ …	M1	oe eg $3\{(4x^2 - 20x)$ …$\}$ or $3(4x^2 - 20x$ …$)$
	$3\{(2x - 5)^2 - 5^2\}$ … or $3(2x - 5)^2 - 75$ …	M1dep	oe eg $3\{(2x - 5)^2 - 5^2$…$\}$
	$3\{(2x - 5)^2 - 5^2\} + 5$	M1dep	oe eg $3\{(2x - 5)^2 - 5^2 + \frac{5}{3}\}$
	$3(2x - 5)^2 - 3 \times 5^2 + 5$	M1dep	oe eg $3(2x - 5)^2 - 3 \times 5^2 + 3 \times \frac{5}{3}$
	$3(2x - 5)^2 - 70$ or $a = 3$ $b = 2$ $c = -5$ $d = -70$ or $3(5 - 2x)^2 - 70$ or $a = 3$ $b = -2$ $c = 5$ $d = -70$	A1	oe
	Additional Guidance		
	For M marks 2.5 may be seen as $\frac{5}{2}$		
	For M marks $(x - 2.5)^2$ may be replaced by $(2.5 - x)^2$ etc		
	Expansion of given form followed by trial and improvement eg1 $3(2x - 5)^2 - 70$ (or $a = 3$ $b = 2$ $c = -5$ $d = -70$) eg2 Not fully correct		 5 marks Zero