

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										



Level 2 Certificate in Further Mathematics  
June 2015

# Further Mathematics

# 8360/2

## Level 2

Paper 2 Calculator

Friday 19 June 2015 9.00 am to 11.00 am

<p><b>For this paper you must have:</b></p> <ul style="list-style-type: none"> <li>• a calculator</li> <li>• mathematical instruments.</li> </ul>	
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**Time allowed**

- 2 hours

**Instructions**

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

**Information**

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 105.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.
- The use of a calculator is expected but calculators with a facility for symbolic algebra must **not** be used.

For Examiner's Use	
Examiner's Initials	
Pages	Mark
3	
4 – 5	
6 – 7	
8 – 9	
10 – 11	
12 – 13	
14 – 15	
16 – 17	
18 – 19	
20 – 21	
22 – 23	
24 – 25	
26 – 27	
28	
<b>TOTAL</b>	

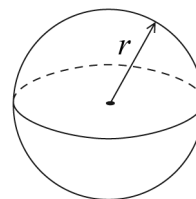


J U N 1 5 8 3 6 0 2 0 1

### Formulae Sheet

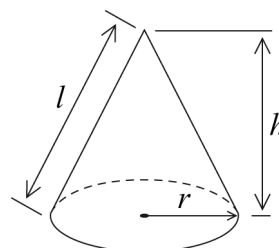
**Volume of sphere**  $= \frac{4}{3}\pi r^3$

**Surface area of sphere**  $= 4\pi r^2$



**Volume of cone**  $= \frac{1}{3}\pi r^2 h$

**Curved surface area of cone**  $= \pi r l$



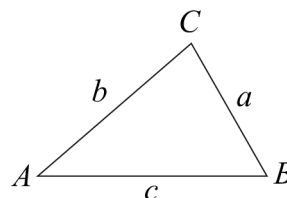
**In any triangle ABC**

**Area of triangle**  $= \frac{1}{2}ab \sin C$

**Sine rule**  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

**Cosine rule**  $a^2 = b^2 + c^2 - 2bc \cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$



### The Quadratic Equation

The solutions of  $ax^2 + bx + c = 0$ , where  $a \neq 0$ , are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

### Trigonometric Identities

$$\tan \theta \equiv \frac{\sin \theta}{\cos \theta} \quad \sin^2 \theta + \cos^2 \theta \equiv 1$$



Answer **all** questions in the spaces provided.

**1** A circle, centre (0, 0), has circumference  $12\pi$

Work out the equation of the circle.

**[2 marks]**

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Answer .....

**2**  $a : b : c = 5 : 3 : 2$

Work out  $4a - c : 3b$   
Give your answer in its simplest form.

**[2 marks]**

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Answer ..... : .....



3 The distance between the points  $(2, 5p)$  and  $(2, -10)$  is 30 units.  
Work out the **two** possible values of  $p$ .

[3 marks]

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Answer..... and .....



4 The first term of a sequence is  $1 - a$

The term-to-term rule of a sequence is

add  $2a$  then multiply by 3

4 (a) Show that the second term is  $3 + 3a$

[1 mark]

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4 (b) The third term is 16

Work out the value of  $a$ .

[3 marks]

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Answer .....

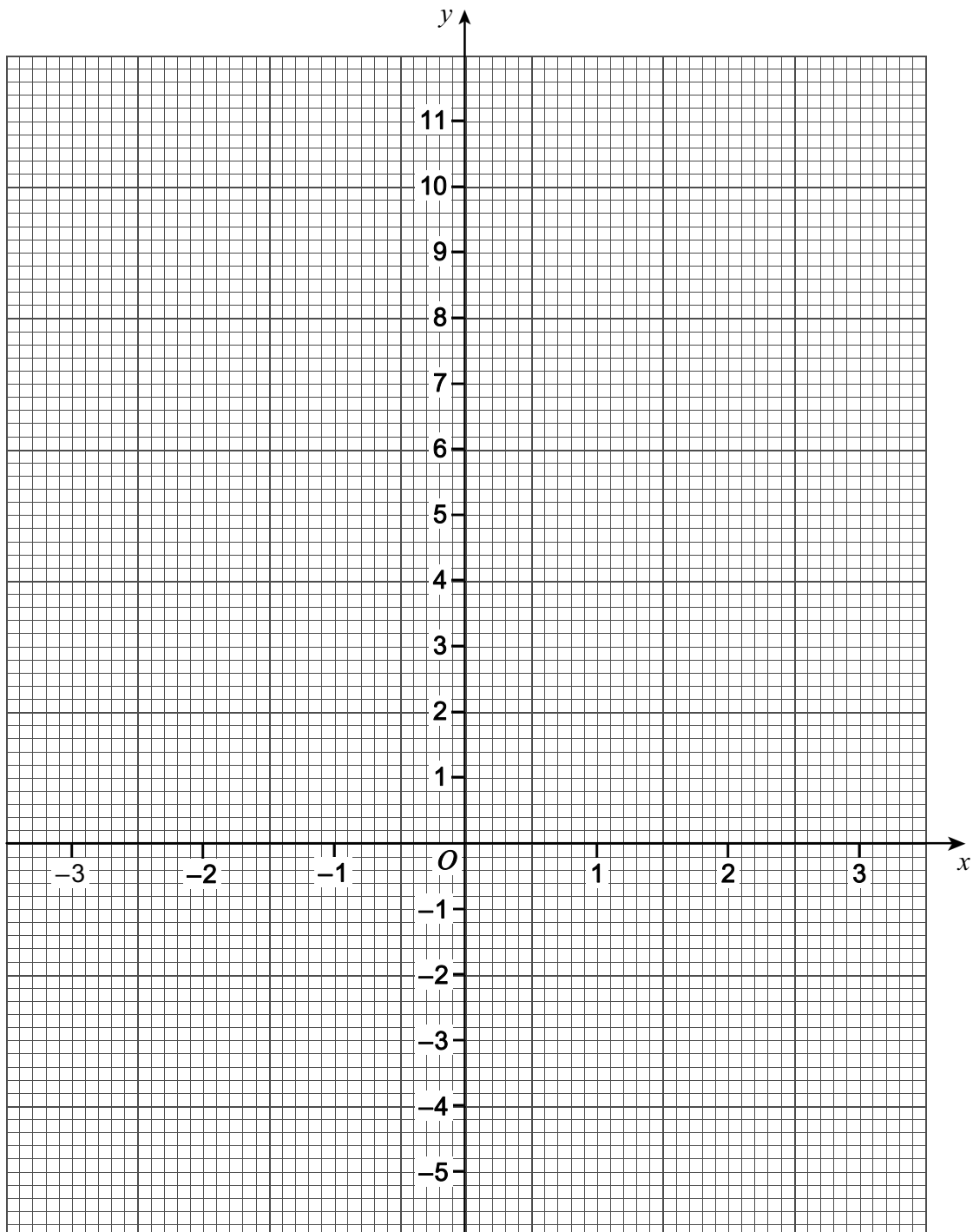


5

A straight line L

is parallel to the straight line  $y = 1 - 2x$   
passes through  $(3, -1)$

On the grid below, draw the straight line L for values of  $x$  from  $-3$  to  $3$ .

**[4 marks]**

6 Write  $\frac{15x^8 - 18x^7}{3x^2}$  in the form  $ax^n - nx^a$  where  $a$  and  $n$  are integers.

[2 marks]

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7  $y = \frac{2}{3}x^6 - 8x^3$

Work out the rate of change of  $y$  with respect to  $x$  when  $x = -1$

[3 marks]

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Answer .....



**8 (a)**  $f(x) = x^4$   
The domain of  $f(x)$  is  $x \geq 2$

Work out the range of  $f(x)$ .

[1 mark]

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Answer .....

**8 (b)**  $g(x) = x^2 - 1$   
The domain of  $g(x)$  is  $-2 \leq x \leq 3$

Work out the range of  $g(x)$ .

[2 marks]

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Answer .....

**8 (c)**  $h(x) = 5x - 3$   
The **range** of  $h(x)$  is  $-2 < h(x) < 1$

Work out the domain of  $h(x)$ .

[2 marks]

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Answer .....





**9 (a)** Solve  $6(2y - 3) - 10 = 2y$

[3 marks]

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$$y = \text{.....}$$

**9 (b)** Solve  $\frac{\sqrt{w+4}}{2} = 6$

[3 marks]

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$$w = \text{.....}$$

**9 (c)** Solve  $3m^{\frac{1}{5}} + 9 = 0$

[2 marks]

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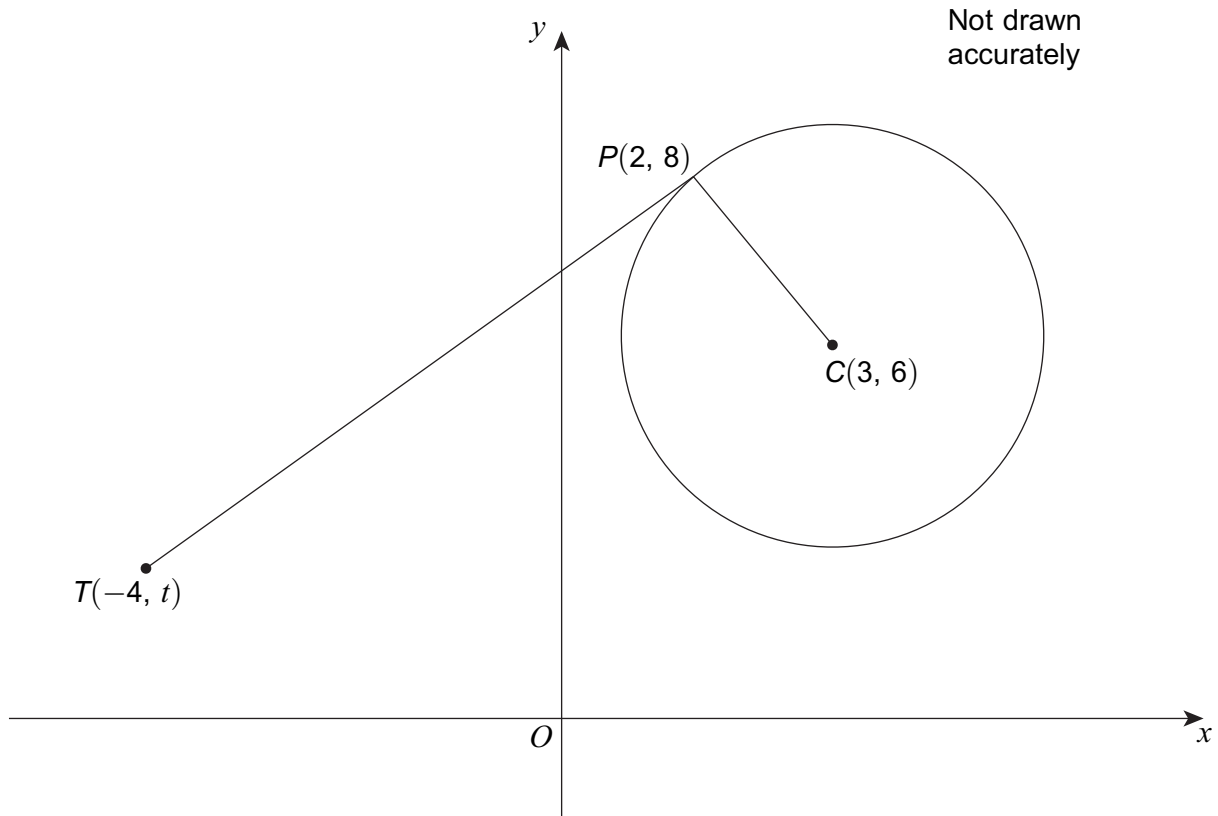
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$$m = \text{.....}$$



10

The diagram shows a circle, centre  $C$ .  
 $TP$  is a tangent to the circle at  $P$ .



Work out the value of  $t$ .

[4 marks]

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Answer .....



11 (a) Expand and simplify  $(3w + 2y)(w - 4y)$

[3 marks]

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Answer .....

11 (b) Expand and simplify  $\frac{3}{x^2} \left( \frac{x}{3} + 3x^2 - 1 \right)$

[3 marks]

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Answer .....

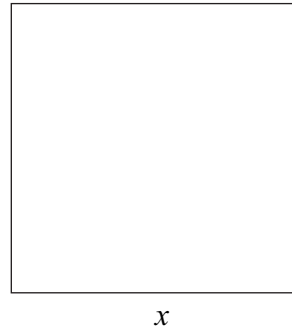
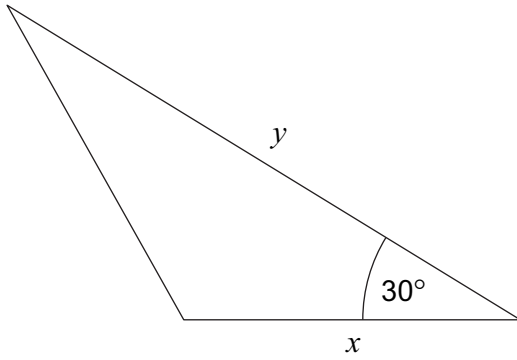
Turn over ►



12

The area of the triangle is equal to the area of the square.  
All dimensions are in centimetres.

Not drawn accurately



Write  $y$  in terms of  $x$ .

[2 marks]

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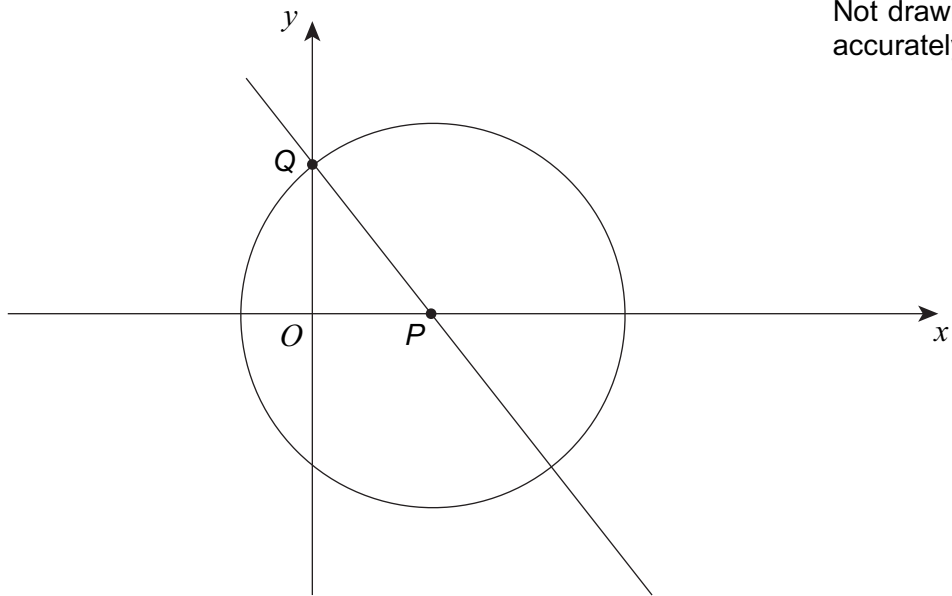
Answer .....



13

The diagram shows a circle, centre  $P$ , and a straight line passing through points  $P$  and  $Q$ .  $Q$  lies on the  $y$ -axis and on the circumference of the circle.

The equation of the circle is  $(x - 3)^2 + y^2 = 25$



Work out the equation of the straight line through  $P$  and  $Q$ .  
Give your answer in the form  $ax + by + c = 0$  where  $a$ ,  $b$  and  $c$  are integers.

[4 marks]

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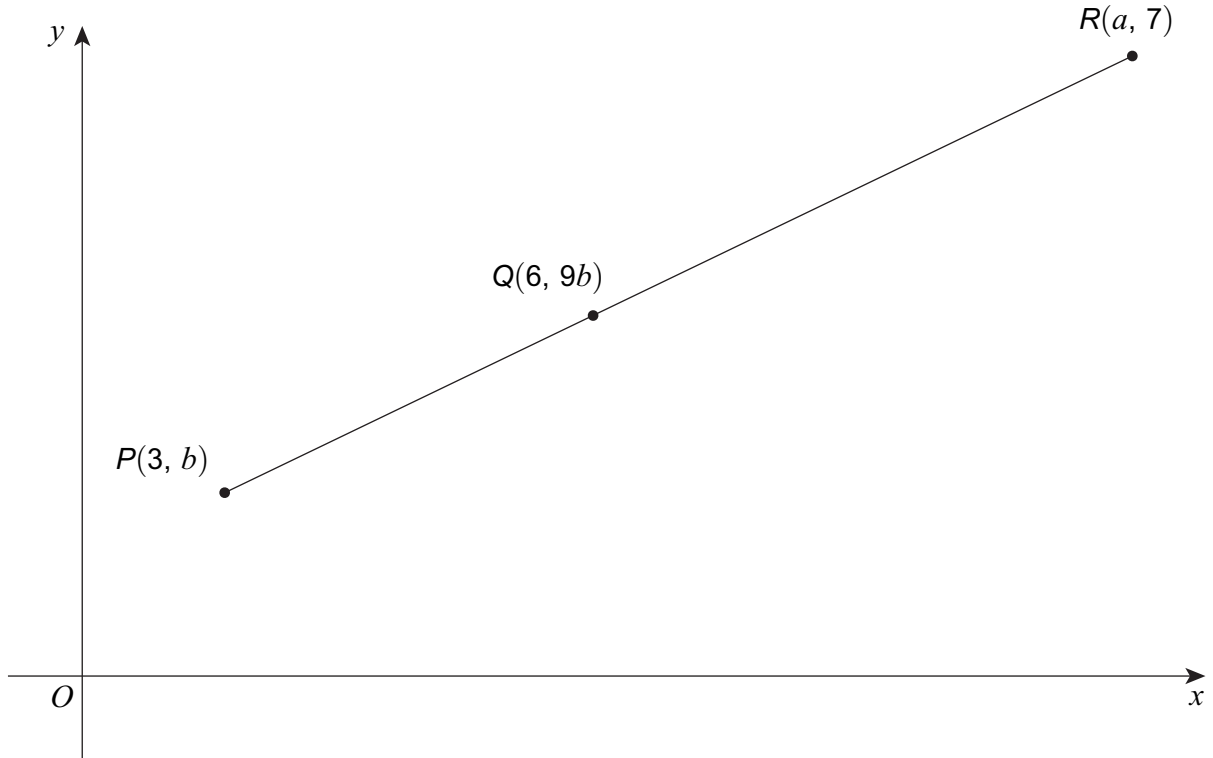
Answer .....



14

$PQR$  is a straight line.  
 $PQ : QR$  is  $2 : 3$

Not drawn accurately



14 (a)

Show that  $a = 10.5$

[2 marks]

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14 (b)

Work out the value of  $b$ .

[3 marks]

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Answer .....

15

Use algebra to prove that the value of  
values of  $c$ .

$$\frac{8c^2 + 16}{3c^2 + 6} + \frac{1}{3}$$

is an integer for all

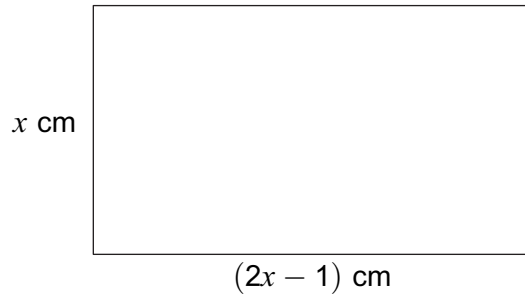
[3 marks]

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16

The diagram shows a rectangle with area  $9 \text{ cm}^2$



Not drawn accurately

Set up and solve an equation to work out the value of  $x$ .  
Give your answer to 3 significant figures.

[5 marks]

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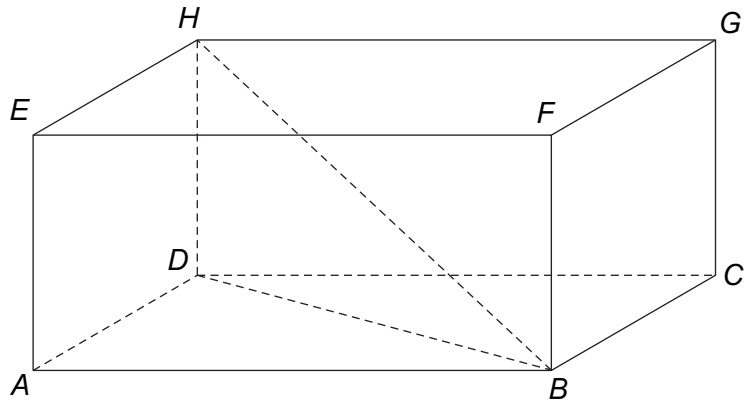
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$x =$  .....





17 *ABCDEFGH* is a cuboid.



$HB = 34$  cm  
 $HD = 16$  cm  
 $AD = 18$  cm

17 (a) Work out the length of  $AB$ .

[3 marks]

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Answer..... cm

17 (b) Work out the angle between  $HB$  and  $ABCD$ .

[2 marks]

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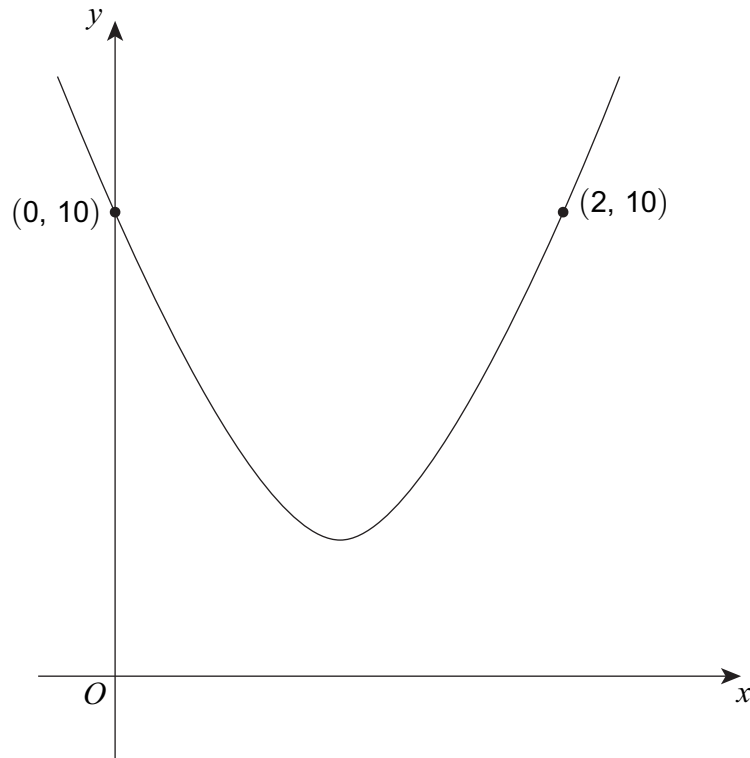
Answer..... degrees



18

The sketch shows the quadratic curve  $y = 4(x - a)^2 + b$   
 The curve passes through  $(0, 10)$  and  $(2, 10)$

Not drawn  
 accurately



18 (a)

Give reasons why the value of  $a$  is 1.

[2 marks]

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**18 (b)** Work out the value of  $b$ .

[2 marks]

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Answer .....

**18 (c)** Write the equation of the curve in the form  $y = px^2 + qx + r$

[2 marks]

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Answer .....

**19** Use the factor theorem to show that  $(x - 3)$  is **not** a factor of  $x^3 - 10x - 3$

[2 marks]

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**20 (a)** The transformation matrix **P** represents a  $90^\circ$  anti-clockwise rotation about the origin.

Describe fully the **single** transformation represented by the matrix  $\mathbf{P}^3$

[2 marks]

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**20 (b)** The transformation matrix **Q** is  $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

The transformation matrix **R** is  $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

Describe fully the **single** transformation represented by the matrix **QR**.

[2 marks]

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21

A cubic curve has

a maximum point at  $A (-4, 10)$

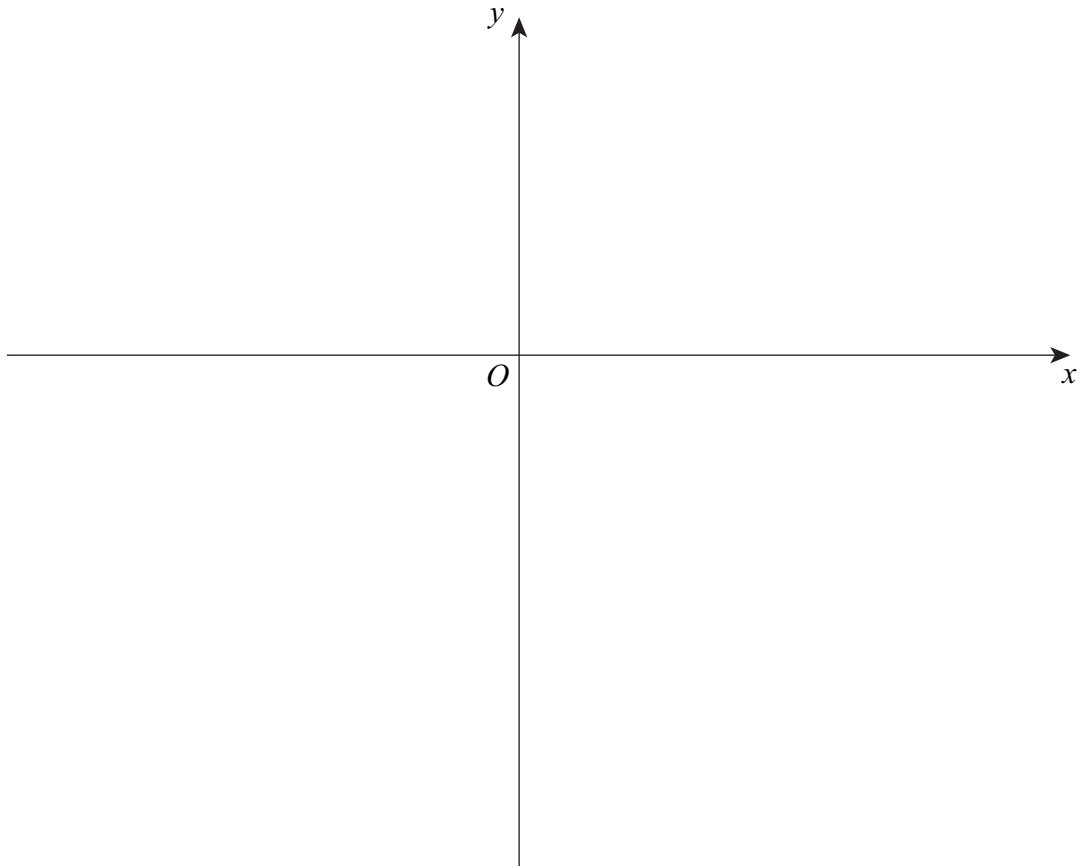
a minimum point at  $B (2, -26)$

The tangent to the curve at  $A$  and the normal to the curve at  $B$  intersect at point  $C$ .

Work out the area of triangle  $ABC$ .

You may sketch a diagram to help you.

[3 marks]



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Answer.....square units

7
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Turn over ►



**22** A quadratic sequence starts

302                  600                  894                  1184                  .....

**22 (a)** Work out an expression for the  $n$ th term.

**[3 marks]**

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Answer .....

**22 (b)** A term in the sequence has value 0

Find the position of this term.

**[2 marks]**

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Answer .....



23

The continuous curve  $y = f(x)$  has exactly **two** stationary points.

$P$  is a maximum point when  $x = a$

$Q$  is a stationary point of inflection when  $x = b$

$a < b$

Which of these is correct?

Tick **one** box only.

[1 mark]

When  $a < x < b$ ,  $\frac{dy}{dx}$  is positive

**and**

when  $x > b$ ,  $\frac{dy}{dx}$  is positive

When  $a < x < b$ ,  $\frac{dy}{dx}$  is positive

**and**

when  $x > b$ ,  $\frac{dy}{dx}$  is negative

When  $a < x < b$ ,  $\frac{dy}{dx}$  is negative

**and**

when  $x > b$ ,  $\frac{dy}{dx}$  is positive

When  $a < x < b$ ,  $\frac{dy}{dx}$  is negative

**and**

when  $x > b$ ,  $\frac{dy}{dx}$  is negative



24

$a^2 < 4$       and       $a + 2b = 8$

Work out the range of possible values of  $b$ .  
Give your answer as an inequality.

**[4 marks]**

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Answer .....





25

Work out the values of  $x$  between  $0^\circ$  and  $360^\circ$  for which

$$25 \cos^2 x = 9$$

Give your answers to 1 decimal place.

[4 marks]

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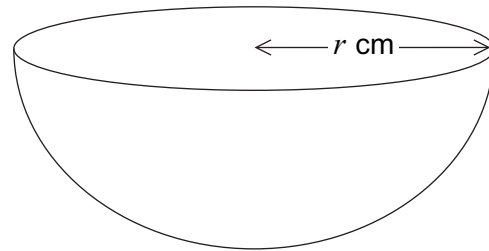
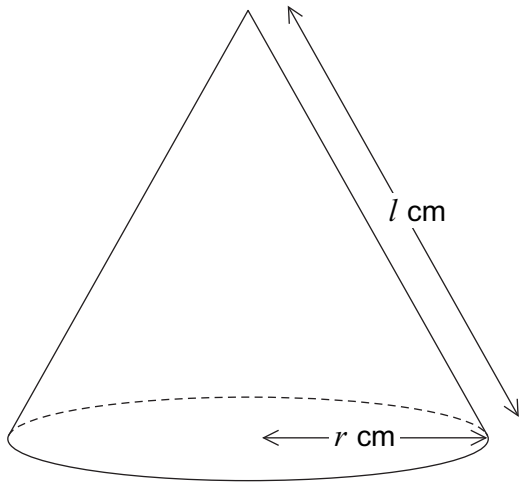
Answer .....



26

A cone has base radius  $r$  cm and slant height  $l$  cm

A hemisphere has radius  $r$  cm



26 (a)

The curved surface area of the cone equals the curved surface area of the hemisphere.

Show that  $l = 2r$

[1 mark]

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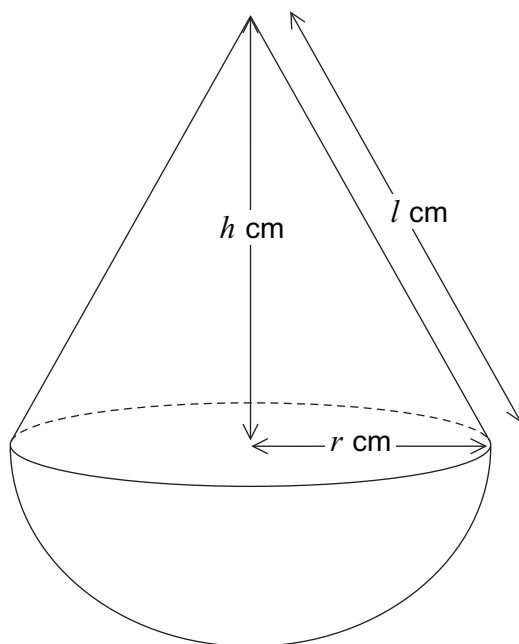
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26 (b)

The cone has vertical height  $h$  cm

The cone and hemisphere are joined to make the shape shown below.



Show that the volume of the shape can be written as

$$\frac{1}{3}\pi r^3(a + \sqrt{b}) \text{ cm}^3 \quad \text{where } a \text{ and } b \text{ are integers.}$$

[4 marks]

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Turn over ►



27

Work out the values of  $a$  when

$$2^{a^2} = 8^a \times 16$$

Do **not** use trial and improvement.  
You **must** show your working.

[4 marks]

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Answer .....

**END OF QUESTIONS**

