

Centre Number						Candidate Number				
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Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Pages	Mark
3	
4 – 5	
6 – 7	
8 – 9	
10 – 11	
12 – 13	
14 – 15	
16 – 17	
18 – 19	
20 – 21	
22 – 23	
24 – 25	
<b>TOTAL</b>	



Level 2 Certificate in Further Mathematics  
June 2013

# Further Mathematics

**8360/2**

## Level 2

**Paper 2 Calculator**

**Friday 21 June 2013 9.00 am to 11.00 am**

<p><b>For this paper you must have:</b></p> <ul style="list-style-type: none"> <li>• a calculator</li> <li>• mathematical instruments.</li> </ul>	
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**Time allowed**

- 2 hours

- Instructions**
- Use black ink or black ball-point pen. Draw diagrams in pencil.
  - Fill in the boxes at the top of this page.
  - Answer **all** questions.
  - You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
  - Do all rough work in this book. Cross through any work that you do not want to be marked.
  - In all calculations, show clearly how you work out your answer.
  - If your calculator does not have a  $\pi$  button, take the value of  $\pi$  to be 3.14 unless another value is given in the question.

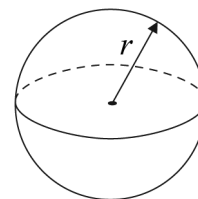
- Information**
- The marks for questions are shown in brackets.
  - The maximum mark for this paper is 105.
  - You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.
  - The use of a calculator is expected but calculators with a facility for symbolic algebra must **not** be used.



### Formulae Sheet

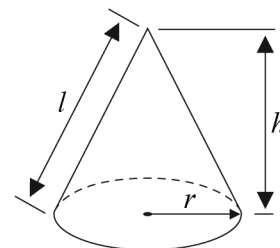
**Volume of sphere**  $= \frac{4}{3}\pi r^3$

**Surface area of sphere**  $= 4\pi r^2$



**Volume of cone**  $= \frac{1}{3}\pi r^2 h$

**Curved surface area of cone**  $= \pi r l$



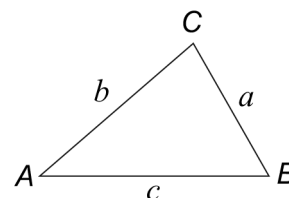
**In any triangle ABC**

**Area of triangle**  $= \frac{1}{2}ab \sin C$

**Sine rule**  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

**Cosine rule**  $a^2 = b^2 + c^2 - 2bc \cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$



### The Quadratic Equation

The solutions of  $ax^2 + bx + c = 0$ , where  $a \neq 0$ , are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

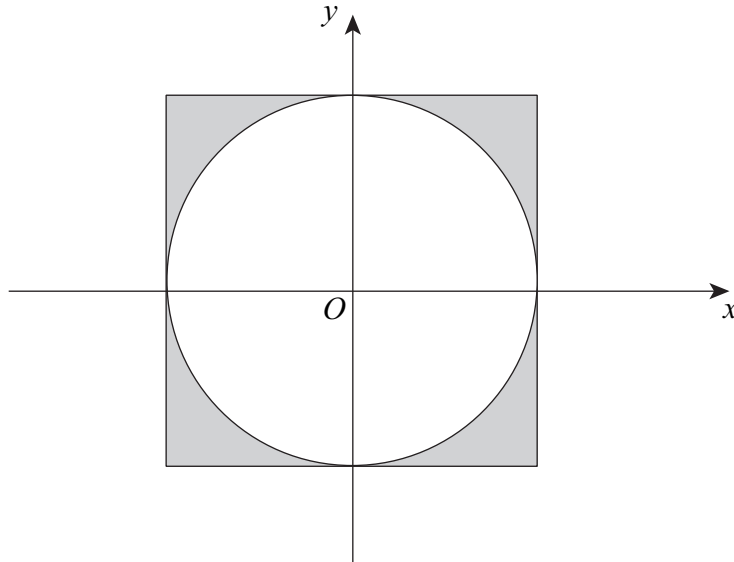
### Trigonometric Identities

$$\tan \theta \equiv \frac{\sin \theta}{\cos \theta} \quad \sin^2 \theta + \cos^2 \theta \equiv 1$$



Answer **all** questions in the spaces provided.

- 1 The circle  $x^2 + y^2 = 25$  touches each side of the square as shown.



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Work out the total shaded area.

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Answer..... (3 marks)



**2**  $w$  is an integer such that  $6 \leq 3w < 18$   
 $x$  is an integer such that  $-4 \leq x \leq 3$

**2 (a)** Work out **all** the possible integer values of  $w$ .

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Answer..... (3 marks)

**2 (b)** Write down the **highest** possible value of  $x^2$

Answer..... (1 mark)

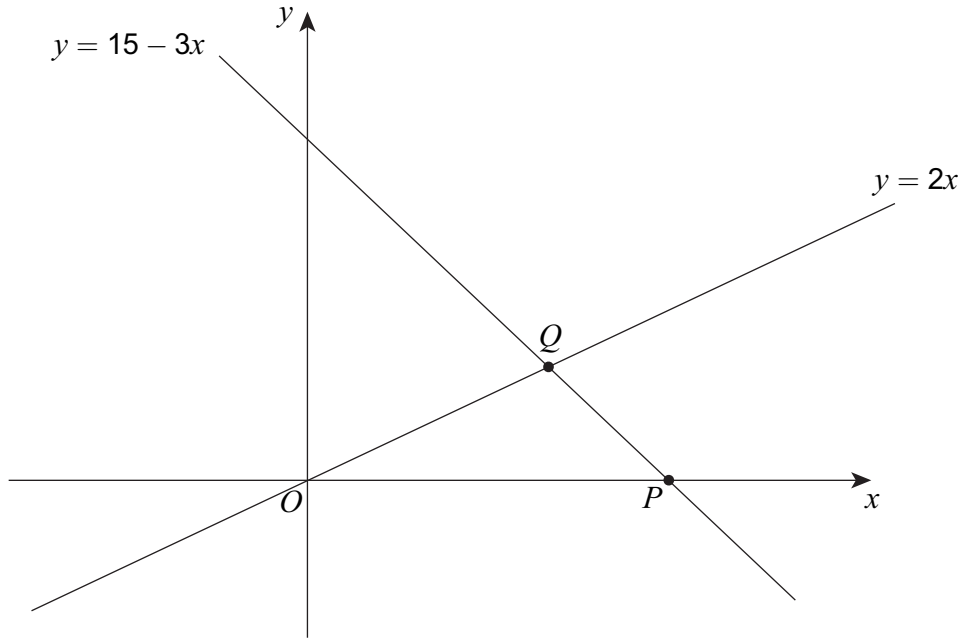
**2 (c)** Work out the **lowest** possible value of  $w - x$

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Answer..... (2 marks)



3 The sketch graphs of two straight lines are shown.



3 (a) Work out the coordinates of  $P$ .

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Answer ( ..... , ..... ) (1 mark)

3 (b) Work out the coordinates of  $Q$ .

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Answer ( ..... , ..... ) (3 marks)

3 (c) Use your answers to parts (a) and (b) to work out the area of triangle  $OPQ$ .

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Answer..... (2 marks)



4 You are given that  $m : n = 2 : 5$

4 (a) Write  $m$  in terms of  $n$ .

$m = \dots\dots\dots$  (1 mark)

4 (b) You are also given that  $a : b = 10m : 3n$

Work out  $a : b$  where  $a$  and  $b$  are integers.

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Answer ..... : ..... (2 marks)



5

$$y = (5x - 3)^2$$

Work out  $\frac{dy}{dx}$

Give your answer in the form  $a(bx - c)$  where  $a$ ,  $b$  and  $c$  are integers  $> 1$

$$\frac{dy}{dx} = \dots\dots\dots (4 \text{ marks})$$

**Turn over for the next question**



6 (a) Show that  $\frac{c^2 + 5c + 4}{3c + 3}$  simplifies to  $\frac{c + 4}{3}$

(2 marks)

6 (b) Hence, or otherwise, simplify fully  $\frac{c^2 + 5c + 4}{3c + 3} + \frac{3 - 2c}{6}$

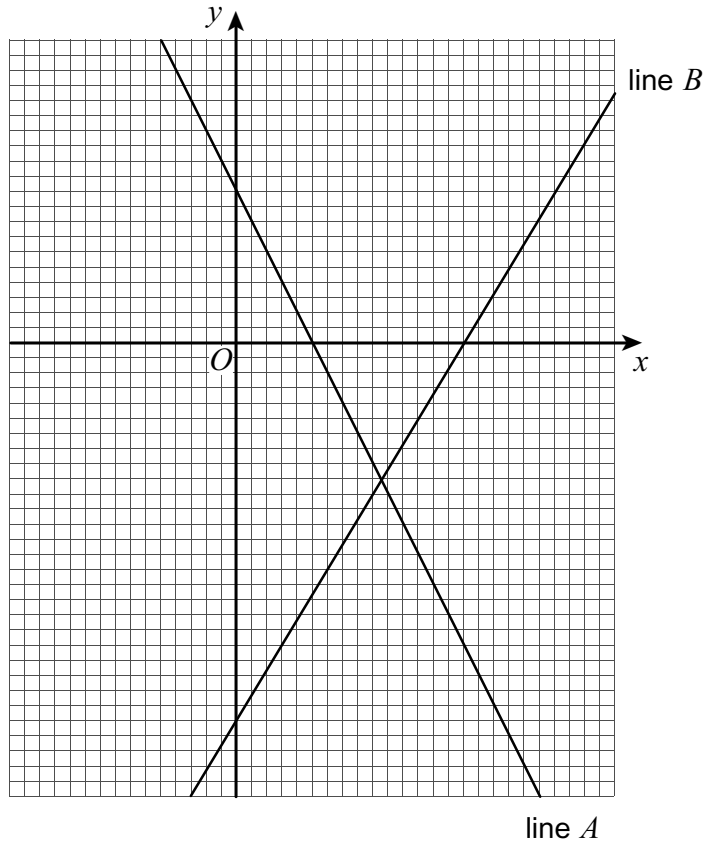
Answer..... (3 marks)





7

The graph shows two straight lines.



The equation of line  $A$  is  $y = 2 - x$

Work out the equation of line  $B$ .

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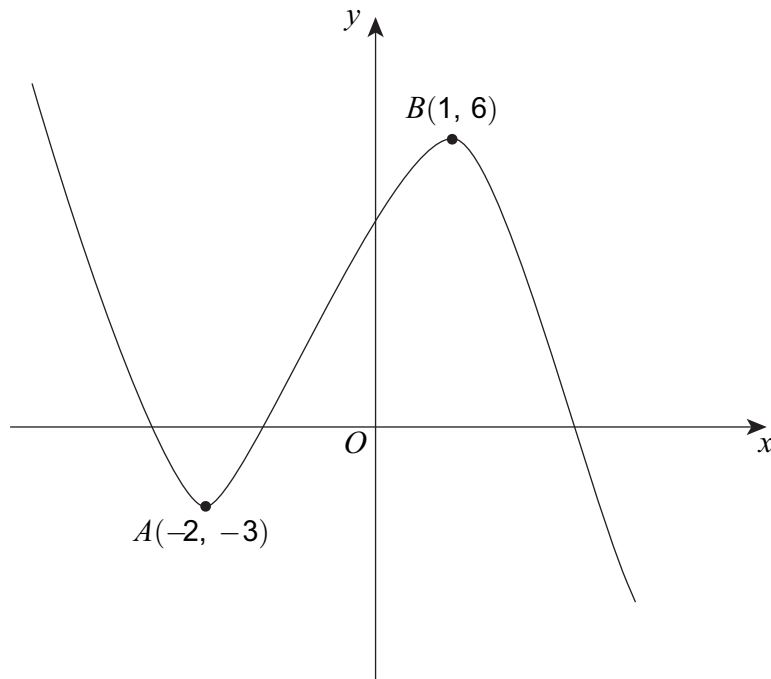
Answer..... (4 marks)

9
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Turn over ►



- 8** A sketch of  $y = f(x)$  is shown.  
There are stationary points at  $A$  and  $B$ .



- 8 (a)** Write down the equation of the tangent to the curve at  $A$ .

Answer..... (1 mark)

- 8 (b)** Write down the equation of the normal to the curve at  $B$ .

Answer..... (1 mark)

- 8 (c)** Circle the range of values of  $x$  for which  $f(x)$  is an increasing function.

$x < -2$        $-2 < x < 1$        $-3 < x < 6$        $x > 1$

(1 mark)

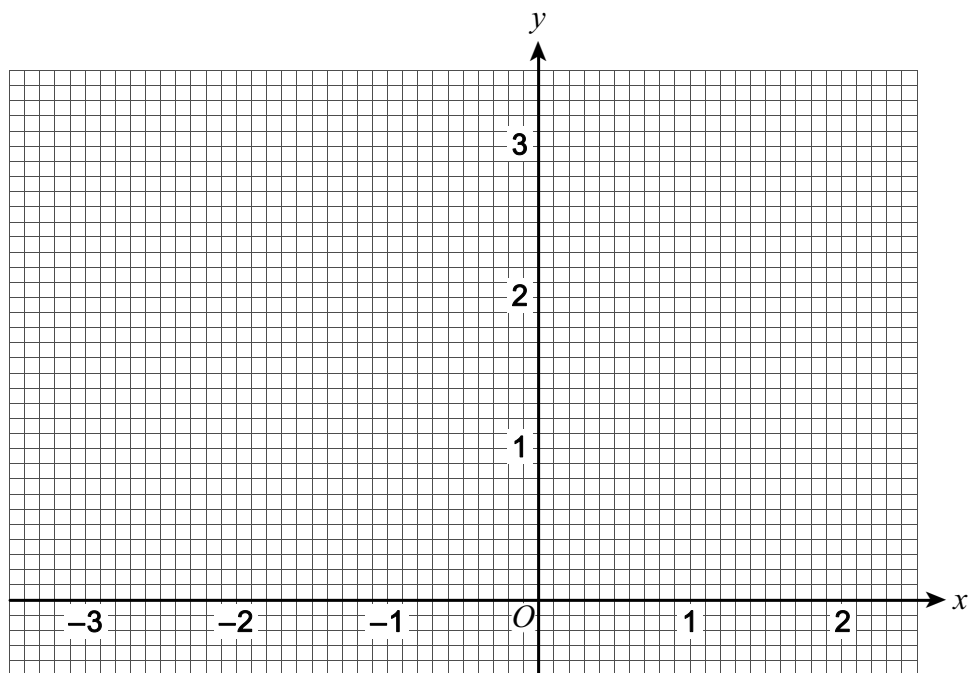




10

A function  $f(x)$  is defined as

$$\begin{aligned} f(x) &= x + 3 & -3 \leq x < 0 \\ &= 3 & 0 \leq x < 1 \\ &= 5 - 2x & 1 \leq x \leq 2 \end{aligned}$$

Draw the graph of  $y = f(x)$  for  $-3 \leq x \leq 2$ 

(3 marks)



11 (a) Work out  $\begin{pmatrix} 2 & -1 \\ \frac{1}{3} & 0 \end{pmatrix} \begin{pmatrix} 0 & b \\ a & c \end{pmatrix}$

Give your answer in terms of  $a$ ,  $b$  and  $c$ .

Answer  $\begin{pmatrix} \dots\dots\dots & \dots\dots\dots \\ \dots\dots\dots & \dots\dots\dots \end{pmatrix}$  (2 marks)

11 (b) You are given that  $\begin{pmatrix} 2 & -1 \\ \frac{1}{3} & 0 \end{pmatrix} \begin{pmatrix} 0 & b \\ a & c \end{pmatrix} = \mathbf{I}$  where  $\mathbf{I}$  is the identity matrix.

Work out the values of  $a$ ,  $b$  and  $c$ .

$a = \dots\dots\dots$ ,  $b = \dots\dots\dots$ ,  $c = \dots\dots\dots$  (3 marks)



12 Prove that  $(5n + 3)(n - 1) + n(n + 2)$  is a multiple of 3 for all integer values of  $n$ .

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(4 marks)

13 The graph of  $y = f(x)$  is a straight line.

The domain of  $f(x)$  is  $1 \leq x \leq 5$

The range of  $f(x)$  is  $3 \leq f(x) \leq 11$

Work out **one** possible expression for  $f(x)$ .

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$f(x) = \dots\dots\dots$  (4 marks)





**15 (a)**  $a^{11} \times b^6 \times c = a^9 \times b^{10}$

Write  $c$  in terms of  $a$  and  $b$ .  
Give your answer in its simplest form.

$c = \dots\dots\dots$  (3 marks)

**15 (b)**  $p^{-2} = q^6 \times r^4$

Write  $p$  in terms of  $q$  and  $r$ .  
Give your answer in its simplest form.

$p = \dots\dots\dots$  (2 marks)



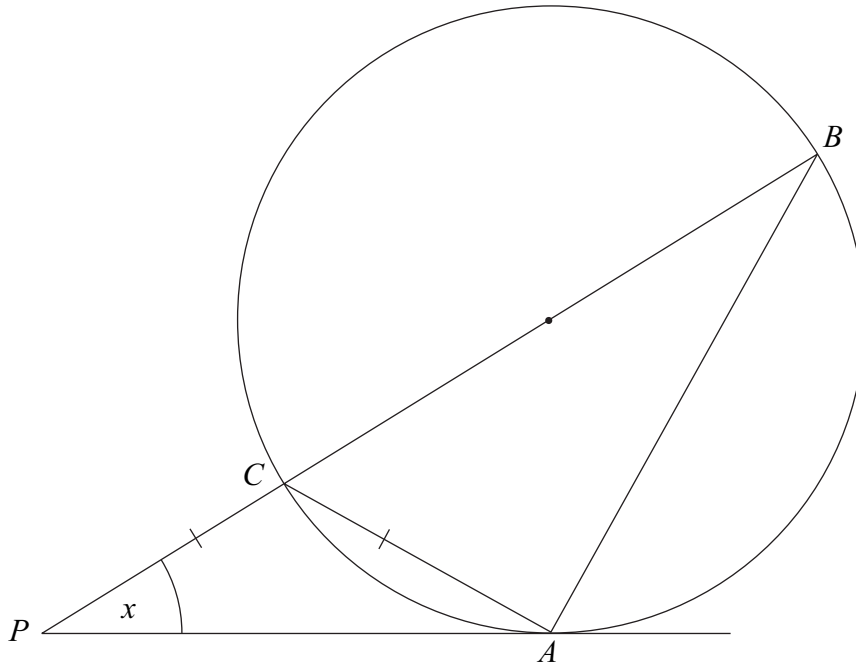


16

$A$ ,  $B$  and  $C$  are points on the circumference of a circle.

- $BC$  is a diameter
- $BCP$  is a straight line
- $AP$  is a tangent to the circle
- $PC = CA$

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Work out the value of angle  $CPA$ , marked  $x$  on the diagram.

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$x =$  ..... degrees (5 marks)

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Turn over ►



17

Solve  $\frac{4}{x-2} + \frac{1}{x+3} = 5$ 

Answer..... (7 marks)



18 The curve  $y = x^3 + bx + c$  has a stationary point at  $(-2, 20)$ .

Work out the values of  $b$  and  $c$ .

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$b =$  .....

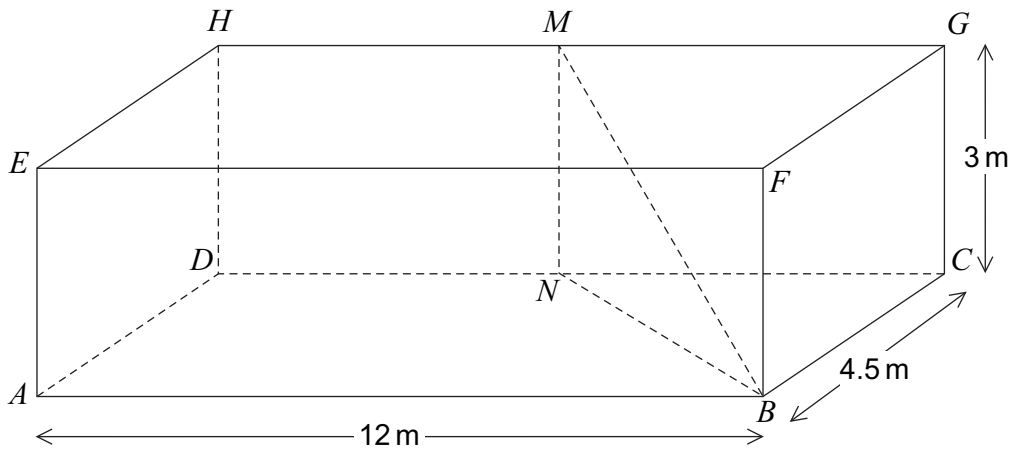
$c =$  ..... (5 marks)

Turn over for the next question



19

$ABCDEFGH$  is a cuboid.  
 $M$  is the midpoint of  $HG$ .  
 $N$  is the midpoint of  $DC$ .



19 (a) Show that  $BN = 7.5\text{ m}$

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(2 marks)



**19 (b)** Work out the angle between the line  $MB$  and the plane  $ABCD$ .

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Answer ..... degrees (2 marks)

**19 (c)** Work out the **obtuse** angle between the planes  $MNB$  and  $CDHG$ .

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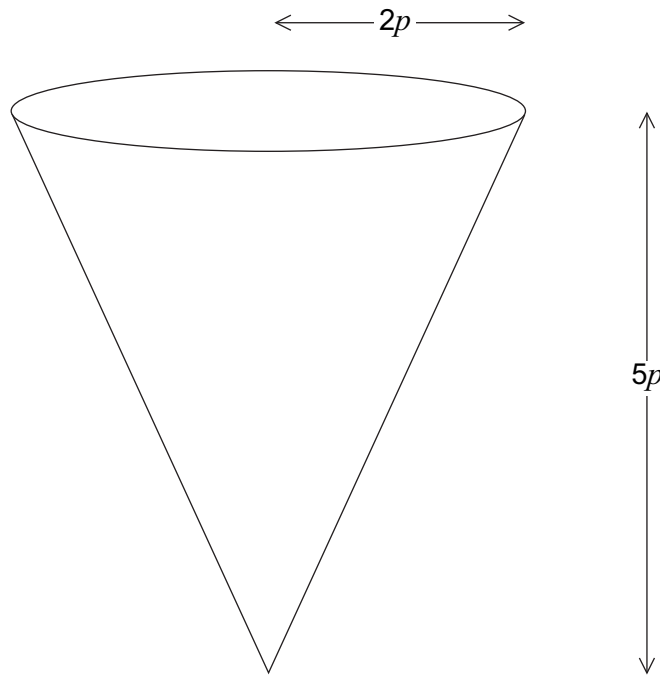
Answer ..... degrees (2 marks)

Turn over ►



20

This right circular cone has radius  $2p$  and height  $5p$ .  
The dimensions are in centimetres.



The volume of the cone is  $22\,500\pi \text{ cm}^3$ .

Work out the value of  $p$ .

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$p = \dots\dots\dots \text{ cm}$  (4 marks)



21

$(x - a)$  is a factor of  $2x^3 - 7ax + 3a$

Work out the **largest** possible value of  $a$ .

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Answer..... (4 marks)

22

Solve  $\tan^2 \theta + 3 \tan \theta = 0$  for  $0^\circ < \theta < 360^\circ$

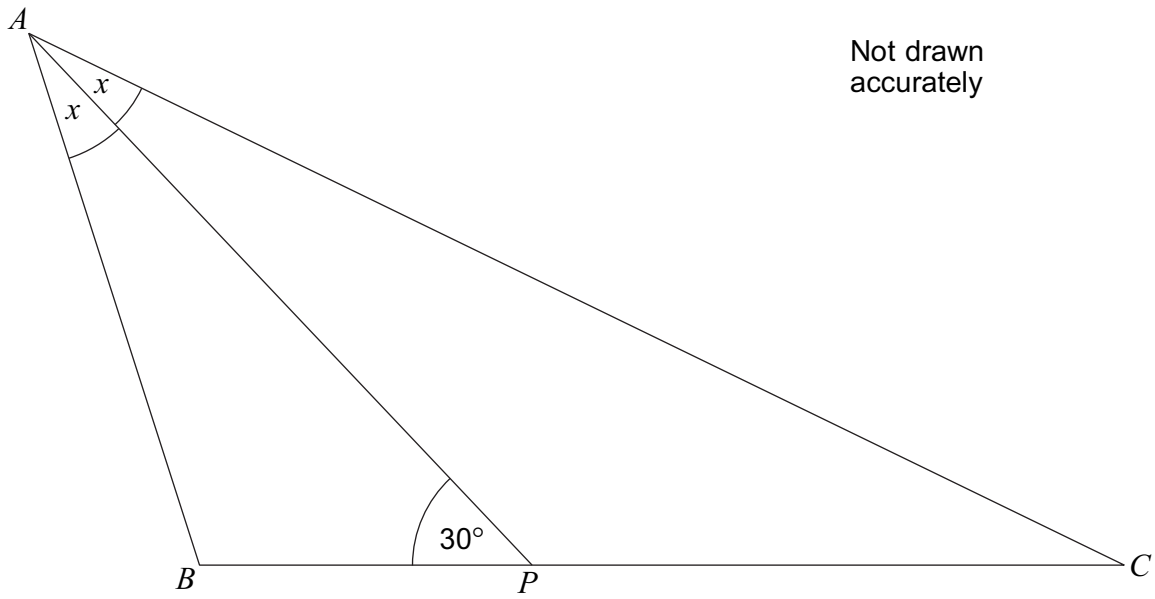
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Answer..... (5 marks)



23

In triangle  $ABC$ ,  $AP$  bisects angle  $BAC$ .



Use the sine rule in triangles  $ABP$  and  $ACP$  to prove that  $\frac{AB}{AC} = \frac{BP}{PC}$

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