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Level 2 Certificate in Further Mathematics

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Paper 2 8360/2

Final

Mark Scheme

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Glossary for Mark Schemes

These examinations are marked in such a way as to award positive achievement wherever possible. Thus, for these papers, marks are awarded under various categories.

M	Method marks are awarded for a correct method which could lead to a correct answer.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
M dep	A method mark dependent on a previous method mark being awarded.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
ft	Follow through marks. Marks awarded following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
oe	Or equivalent. Accept answers that are equivalent. eg, accept 0.5 as well as $\frac{1}{2}$

Q	Answer	Mark	Comments
1	$r = 5$ **or** $r^2 = 25$ **or** $r = \sqrt{25}$ **or** $d = 10$	B1	May be seen on diagram
	$(2 \times \text{their } r)^2 - \pi \times \text{their } r^2$	M1	
	[21.45, 21.5] **or** $100 - 25\pi$	A1ft	ft from B0 M1 Allow 21 with working (uses $25\pi = 79$) Ignore any units seen
2 (a)	$\frac{6}{3} \le w < \frac{18}{3}$ **or** $2 \le w$ ……. **or** …….. $w < 6$	M1	
	$2 \le w < 6$ **or** $2 \le w \le 5$	A1	
	2 3 4 5	A1ft	ft M1 A0 and inequality of form $a \le w < b$ or $a \le w \le b$ SC2 Answer 2 3 4 5 6 **or** 3 4 5 with M0 SC1 Answer 6 9 12 15 with M0 SC1 $\frac{6}{3} < w \le \frac{18}{3}$
2 (b)	16	B1	
2 (c)	their min from (a) – 3	M1	
	– 1	A1ft	ft their min from (a)
3 (a)	(5, 0)	B1	$(5x, 0y)$ is B0 Check diagram for answer written next to P if answer line is blank
3 (b)	Correct elimination of a letter eg $2x = 15 - 3x$	M1	oe eg $y = 15 - \frac{3}{2}y$
	Correctly collects terms eg $2x + 3x = 15$	M1dep	oe eg $y + \frac{3}{2}y = 15$
	(3, 6)	A1	Allow $x = 3$ **and** $y = 6$ if not contradicted on answer line

3 (c)	$\frac{1}{2}$ × their 5 × their 6	M1	oe eg $\frac{2\times 6}{2}+\frac{3\times 6}{2}$ their 5 from (a) and their 6 from (b)
	15	A1ft	ft their 5 from (a) and their 6 from (b)

4 (a)	$\frac{2}{5}n$ **or** $0.4n$	B1	oe
4 (b)	$(10m =)$ 10 × their $\frac{2}{5}n$ $(= 4n)$	M1	10 × 2 (= 20) **and** 3 × 5 (= 15)
	4 : 3	A1ft	oe numerical ratio of integers ft their $\frac{2}{5}n$ if used

5	$25x^2 - 15x - 15x + 9$	M1	4 terms with 3 correct including a term in x^2
	$25x^2 - 15x - 15x + 9$ **or** $25x^2 - 30x + 9$	A1	Fully correct
	Correctly differentiates their quadratic $50x - 15 - 15$ **or** $50x - 30$	M1	ft their $25x^2 - 15x - 15x + 9$
	$10(5x - 3)$ **or** $5(10x - 6)$ **or** $2(25x - 15)$	A1ft	ft M1 A0 M1 if their $50x - 30$ factorises to $a(bx - c)$ where a, b and c are integers > 1
	Alternative		
	$2(5x - 3) \times 5$	M2	
	$10(5x - 3)$ **or** $5(10x - 6)$ **or** $2(25x - 15)$	A2	

6 (a)	$(c + 4)(c + 1)$ **or** $3(c + 1)$	M1	Correct factorisation
	$\frac{(c+4)(c+1)}{3(c+1)} = \frac{c+4}{3}$	A1	Must be a fraction and completed to $\frac{c+4}{3}$
	Correctly converts to a common denominator eg 1 $\frac{2(c+4)}{6} + \frac{3-2c}{6}$ eg 2 $\frac{6(c+4)}{18} + \frac{3(3-2c)}{18}$	M1	M2 $\frac{2c}{6}+\frac{8}{6}+\frac{3}{6}-\frac{2c}{6}$

6 (b)	Correctly expands their brackets (must have common denominator) $\frac{2c+8+3-2c}{6}$ **or** $\frac{2c+8}{6}+\frac{3-2c}{6}$	M1	Allow M1 if their first line of working is $\frac{2c+4+3-2c}{6}$ **or** $\frac{2c+4}{6}+\frac{3-2c}{6}$
	$\frac{11}{6}$ **or** $1\frac{5}{6}$ **or** 1.833(…..)	A1	$\frac{33}{18}$ A0 $\frac{5.5}{3}$ A0 $\frac{8+3}{6}$ A0
	Alternative method		
	Correctly converts to a common denominator eg $\frac{6(c^2+5c+4)}{6(3c+3)}+\frac{(3-2c)(3c+3)}{6(3c+3)}$	M1	oe May also expand the denominator
	Correctly expands their brackets (must have common denominator) $\frac{6c^2+30c+24+9c+9-6c^2-6c}{6(3c+3)}$ **or** $\frac{6c^2+30c+24}{6(3c+3)}+\frac{9c+9-6c^2-6c}{6(3c+3)}$	M1	oe May also expand the denominator
	$\frac{11}{6}$ **or** $1\frac{5}{6}$ **or** 1.833(….).	A1	$\frac{33}{18}$ A0 $\frac{5.5}{3}$ A0 $\frac{8+3}{6}$ A0
7	Scale on the y-axis identified correctly eg Intercept of line *A* with y-axis identified as 2	B1	oe Must be unambiguous identification
	Scale on the x-axis identified correctly eg Intercept of line *A* with x-axis identified as 2	B1	oe Must be unambiguous identification
	Correct attempt at gradient eg $\frac{\text{their } 5}{\text{their } 6}$	M1	ft their scales
	$y=\frac{5}{6}x-5$ **or** $6y=5x-30$	A1ft	ft B0 B1 M1 or B1 B0 M1 oe $\frac{5}{6}x-5$ is B2 M1 A0

8 (a)	$y = -3$ **or** $y + 3 = 0$	B1	Allow $y = 0x - 3$
8 (b)	$x = 1$ **or** $x - 1 = 0$	B1	
8 (c)	$-2 < x < 1$	B1	Unambiguously selected

9	(horizontal =) 8 cos 42 (= [5.9, 6]) **or** (horizontal =) 8 sin 48 (= [5.9, 6])	M2	M1 $\cos 42 = \frac{x}{8}$ **or** $\sin 48 = \frac{x}{8}$ (x is the horizontal)
	(vertical =) 8 sin 42 (= [5.35, 5.4]) **or** (vertical =) 8 cos 48 (= [5.35, 5.4]) **or** (vertical =) $\sqrt{8^2 - \text{their}\,[5.9,6]^2}$ (= [5.35, 5.4])	M2	M1 $\sin 42 = \frac{y}{8}$ **or** $\cos 48 = \frac{y}{8}$ (y is the vertical) **or** 8^2 – their $[5.9, 6]^2$
	[35.4, 35.5]	A1	
	Alternative		
	(vertical =) 8 sin 42 (= [5.35, 5.4]) (vertical =) 8 cos 48 (= [5.35, 5.4])	M2	M1 $\sin 42 = \frac{y}{8}$ **or** $\cos 48 = \frac{y}{8}$ (y is the vertical)
	(horizontal =) 8 cos 42 (= [5.9, 6]) **or** (horizontal=) 8 sin 48 (= [5.9, 6]) **or** (horizontal =) $\sqrt{8^2 - \text{their}\,[5.35, 5.4]^2}$ (= [5.9, 6])	M2	M1 $\cos 42 = \frac{x}{8}$ **or** $\sin 48 = \frac{y}{8}$ (x is the horizontal) **or** 8^2 – their $[5.35, 5.4]^2$
	[35.4, 35.5]	A1	SC2 [31.8, 31.9] **or**

10	Straight line through (–3, 0) and (0, 3)	B1	Lines must be ruled Only penalise (by 1 mark) extended lines if B1 B1 B1 SC2 Any graph that passes through (–3, 0) **and** (0, 3) **and** (1, 3) **and** (2, 1)
	Straight line through (0, 3) and (1, 3)	B1	
	Straight line through (1, 3) and (2,1)	B1	

11 (a)	$\begin{pmatrix} -a & 2b - c \\ 0 & \frac{1}{3}b \end{pmatrix}$	B2	B1 2 or 3 correct elements
11 (b)	$a = -1$	B1ft	ft their matrix in (a) or if (a) correct ft their b when finding c
	$b = 3$	B1ft	

	$c = 6$	B1ft	

12	$5n^2 - 5n + 3n - 3$	M1	oe 4 terms with 3 correct including a term in n^2
	$5n^2 - 5n + 3n - 3$	A1	Fully correct oe eg $5n^2 - 2n - 3$
	$6n^2 - 3$	A1	
	$3(2n^2 - 1)$ **or** states that both terms are multiples of 3	A1	oe

13	Identifies (1, 3) **or** (5, 11)	B1	May be implied by M1 or seen in a table of values or on a graph or as a mapping (eg 1→ 3)
	$\frac{\text{their } 11 - \text{their } 3}{\text{their } 5 - \text{their } 1}$ (= 2)	M1	oe
	y – their 3 = their 2(x – their 1) **or** y – their 11 = their 2(x – their 5)	M1	y = their $2x + c$ **and** substitutes their (1, 3) or their (5, 11)
	(y =) $2x + 1$	A1	
	Alternative 1		
	Identifies (1, 11) **or** (5, 3)	B1	May be implied by M1 or seen in a table of values or on a graph or as a mapping (eg 3→ 1)
	$\frac{\text{their } 11 - \text{their } 3}{\text{their } 1 - \text{their } 5}$ (= –2)	M1	oe
	y – their 11 = their –2(x – their 1) **or** y – their 3 = their –2(x – their 5)	M1	y = their – $2x + c$ **and** substitutes their (1, 11) or their (5, 3)
	(y =) – $2x + 13$	A1	
	Alternative 2		
	$m + c = 3$ **or** $5m + c = 11$	B1	$m + c = 11$ **or** $5m + c = 3$
	Eliminates a letter from their 2 equations eg $5m - m = 11 - 3$	M1	Eliminates a letter from their 2 equations eg $5m - m = 3 - 11$
	$m = 2$ **or** $c = 1$	A1	$m = -2$ **or** $c = 13$
	(y =) $2x + 1$	A1	(y =) – $2x + 13$

14	First **and** second differences correct ie 4 6 8 (10) 2 2 (2)	M1	
	Correctly subtracts their $\frac{2}{2}n^2$ from given sequence ie 10 11 12 (13 14)	M1	
	$(1)n$	M1dep	dep on M2
	$n^2 + n + 9$	A1	oe eg $n^2 + n + 10 - 1$
	Alternative method		
	Any three of $a + b + c = 11$ $4a + 2b + c = 15$ $9a + 3b + c = 21$ $16a + 4b + c = 29$ $25a + 5b + c = 39$	M1	Allow one error but each of their three equations must have a, b and c
	Eliminates one variable to obtain a pair of equations in two variables eg $3a + b = 4$ **and** $5a + b = 6$	M1	Allow one error
	Eliminates one variable correctly eg $2a = 2$	M1dep	dep on M2
	$n^2 + n + 9$	A1	oe eg $n^2 + n + 10 - 1$

15 (a)	$\frac{a^9(\times)b^{10}}{a^{11}(\times)b^6}$ **or** $a^{9-11}(\times)\ b^{10-6}$	M1	
	$a^{-2}(\times)\ b^4$ **or** $\frac{b^4}{a^2}$	A2	A1 a^{-2} **or** b^4 (M1 is implied) **or** $\left(\frac{b^2}{a}\right)^2$ **or** $(a^{-1}(\times)\ b^2)^2$ SC1 $a^2(\times)\ b^{-4}(\times\ c)$

15 (b)	$q^{-3}\,(\times)\,r^{-2}$ **or** $\dfrac{1}{q^{3}(\times)r^{2}}$	B2	B1 q^{-3} **or** r^{-2} **or** $(q^{6}(\times)r^{4})^{-\frac{1}{2}}$ **or** $(q^{-6}(\times)r^{-4})^{\frac{1}{2}}$ **or** $\dfrac{1}{\sqrt{q^{6}(\times)r^{4}}}$ **or** $\sqrt{\dfrac{1}{q^{6}(\times)r^{4}}}$ **or** $(q^{3}(\times)\,r^{2})^{-1}$ **or** $p^{-1} = q^{3}\,(\times)\,r^{2}$ **or** $\dfrac{1}{p} = q^{3}\,(\times)\,r^{2}$ **or** $p^{2} = q^{-6}\,(\times)\,r^{-4}$ **or** $p^{2} = \dfrac{1}{q^{6}(\times)r^{4}}$

16	Correct expressions or value for any three of these angles angle $PAC = x$ angle $CAB = 90$ angle $PBA = x$ angle $PCA = 180 - 2x$ **or** $90 + x$ angle $ACB = 90 - x$ **or** $2x$ angle $COA = 2x$ **or** $90 - x$ angle $PAO = 90$ angle $CAO = 90 - x$ **or** $2x$ angle $BAD = 2x$ **or** $90 - x$ angle $AOB = 180 - 2x$ **or** $90 + x$ angle $OAB = x$	B3	O is the centre of the circle D is the point at the end of PA extended B2 Any 2 correct B1 Any 1 correct
	Writes a correct equation that has solution 30 eg 1 $PAC + CAB + x + PBA = 180$ eg 2 $PCA + ACB = 180$ eg 3 $ACB + CAB + CBA = 180$ eg 4 $PAO + APC + POA = 180$	M1	oe
	30	A1	

Q	Answer	Mark	Comments
17	$4(x + 3) + x – 2$ **or** $\frac{4(x+3)}{(x-2)(x+3)} + \frac{x-2}{(x-2)(x+3)}$	M1	Must be correct
	$4x + 12 + x – 2$ $(= 5x + 10)$ **or** $\frac{4x+12}{(x-2)(x+3)} + \frac{x-2}{(x-2)(x+3)}$	A1	
	$5(x – 2)(x + 3)$	M1	Must have 5 and be correct Must be in an equation **and** not a denominator oe eg $(5x – 10)(x + 3)$
	$(5)(x^2 + 3x – 2x – 6)$	M1	5 may be missing Must be in an equation **and** not a denominator 4 terms including term in x^2 with 3 correct oe eg 1 $x^2 + x – 6$ eg 2 $5x^2 + 15x – 10x – 6$ (1 error)
	$5x^2 = 40$	A1	oe eg $5x^2 – 40 = 0$ Must collect all terms and have an equation
	Correct attempt at solution of their quadratic eg $x = \sqrt{\frac{40}{5}}$	M1dep	dep on M3 Quadratic formula must have no errors in substitution If completing square must have no errors up to $p(x – q)^2 = r$ $p(x – q)^2 – r = 0$
	[2.8, 2.83] **and** [–2.83, –2.8]	A1ft	oe eg $(+)\sqrt{8}$ **and** $-\sqrt{8}$ **or** $\pm\sqrt{8}$ ft their quadratic equation if M4 SC7 Both solutions correct (no valid method) SC3 One solution correct (no valid method)

18	$3x^2 + b$	M1	At least one term correct
	Substitutes –2 into their $\frac{dy}{dx}$ **and** equates to zero $3 \times (–2)^2 + b = 0$	M1dep	Must have a term in x $12 + b = 0$
	$b = –12$	A1	
	$(–2)^3$ + their $b(–2) + c = 20$	M1dep	dep on M2 and having a value for b
	$c = 4$	A1ft	ft their b and M2 A0 M1dep with no errors in their final M1

Q	Answer	Mark	Comments
19 (a)	$(12 \div 2)^2 + 4.5^2$ **or** $36 + 20.25$ **or** $7.5^2 - 4.5^2$ **or** $7.5^2 - 6^2$	M1	$4.5 \div 3$ (= 1.5) **and** $6 \div 4$ (= 1.5)
	$\sqrt{56.25} = 7.5$ **or** $\sqrt{36 + 20.25} = 7.5$ **or** $\sqrt{6^2 + 4.5^2} = 7.5$ **or** $6^2 + 4.5^2 = 56.25$ **and** $7.5^2 = 56.25$ **or** $\sqrt{20.25} = 4.5$ **or** $\sqrt{36} = 6$ $\sqrt{7.5^2 - 4.5^2} = 6$ **or** $\sqrt{7.5^2 - 6^2} = 4.5$ **or**	A1	$5 \times 1.5 = 7.5$
19 (b)	$\tan MBN = \frac{3}{7.5}$ **or** $\sin MBN = \frac{3}{\sqrt{3^2 + 7.5^2}}$ **or** $\cos MBN = \frac{7.5}{\sqrt{3^2 + 7.5^2}}$	M1	Must be correct oe eg $\tan^{-1} \frac{3}{7.5}$
	[21.8, 21.80141]	A1	
19 (c)	$\sin BNC = \frac{4.5}{7.5}$ **or** $\cos BNC = \frac{12 \div 2}{7.5}$ **or** $\tan BNC = \frac{4.5}{12 \div 2}$	M1	oe eg1 $\sin^{-1} \frac{4.5}{7.5}$ eg 2 $\cos BNC = \frac{7.5^2 + 6^2 - 4.5^2}{2 \times 7.5 \times 6}$
	[143, 143.1301024]	A1	SC1 [36.8698976, 37]
	Alternative 1		
	$BD = \sqrt{12^2 + 4.5^2}$ **or** $BD^2 = 12^2 + 4.5^2$ **and** $\cos BND = \frac{7.5^2 + 6^2 - \text{their } BD^2}{2 \times 7.5 \times 6}$	M1	
	[143, 143.1301024]	A1	SC1 [36.8698976, 37]
	Alternative 2		

	$\sin XNB = \frac{12 \div 2}{7.5}$ (= [53.1, 53.13]) **or** $\cos XNB = \frac{4.5}{7.5}$ (= [53.1, 53.13]) **or** $\tan XNB = \frac{12 \div 2}{4.5}$ (= [53.1, 53.13])	M1	X is midpoint of AB
	[143, 143.1301024]	A1	SC1 [53, 53.1301024]

20	$\frac{1}{3}$ (×) π (×) $(2p)^2$ (×) $5p$ $\left(= \frac{20\pi}{3} p^3\right)$	B1	oe Missing brackets B0 unless recovered May be implied by working for M1
	their $\frac{1}{3}$ (×) π (×) $(2p)^2$ (×) $5p$ = 22 500π	M1	oe eg $\frac{20\pi}{3} p^3 = 22\,500\pi$ π may already be cancelled or value for π may be substituted in Must be equating two volumes
	Correctly rearranges to p^3 = eg $p^3 = 22500\pi \div$ their $\frac{20\pi}{3}$	M1dep	oe eg $p = \sqrt[3]{3375}$
	15	A1	SC3 [18.8, 18.9]

21	$2a^3 - 7a^2 + 3a$	M1	Must be correct
	$2a^2 - 7a + 3$	M1dep	Must be correct May also see factor a
	$(2a - 1)(a - 3)$	A1	May also see factor a
	3	A1ft	ft M1 M1 A0 Other solutions may be seen but 3 must be selected as their answer
	Alternative method		
	$(x - a)(2x^2 + 2ax - 3)$	M1	Must be correct
	$-3(x) - 2a^2(x) = -7a(x)$	M1dep	Equating coefficients of x
	$2a^2 - 7a + 3$ **and** $(2a - 1)(a - 3)$	A1	
	3	A1ft	ft M1 M1 A0 Other solutions may be seen but 3 must be selected as their answer

Q	Answer	Mark	Comments
22	$\tan \Theta(\tan \Theta + 3)$ **or** $\tan \Theta = 0$ **or** $\sin \Theta(\sin \Theta + 3\cos \Theta)$ **or** $\sin \Theta = 0$	M1	oe eg $t(t + 3)$ Must be correct
	180	A1	
	$\tan \Theta = -3$	A1	
	[108, 108.44]	A1	
	[288, 288.44]	B1ft	ft 180 + any angle (other than 0 and 90) if in range

23	Appropriate and correct sine rule in triangle *ABP* eg $\frac{BP}{\sin x} = \frac{AB}{\sin 30}$	M1	oe eg $\frac{BP}{AB} = \frac{\sin x}{\sin 30}$
	Appropriate and correct sine rule in triangle *ACP* eg $\frac{PC}{\sin x} = \frac{AC}{\sin 150}$	M1	oe eg $PC = \frac{\sin x}{\sin 150} \times AC$
	Eliminates sin x eg $\frac{PC}{\frac{BP \sin 30}{AB}} = \frac{AC}{\sin 150}$	A1	Must have M1 M1 oe eg $\frac{BP}{AB}$ sin 30 = $\frac{PC}{AC}$ sin 150
	States sin 30 = sin 150	M1dep	dep on M1 M1 oe eg Substitutes sin 30 = $\frac{1}{2}$ **and** sin 150 = $\frac{1}{2}$
	Completes fully eg $\frac{PC}{AC} = \frac{BP}{AB}$ **and** $\frac{AB}{AC} = \frac{BP}{PC}$	A1	Must have all 4 previous marks SC1 Sine rule in triangle *ABP* using angle 150 – x **or** Sine rule in triangle *ACP* using angle 30 – x