5 Basic Indices Rules/Laws

Multiplication

Rule 1: Add the powers when multiplying

$$x^a \times x^b = x^{a+b}$$

The bases must be the same to use this rule and notice how they do not change (base x stays x)

Why is this rule true?

$x^2 \times x^3 = (x \times x) \times (x \times x \times x) = x^5$

We have five x's in total. The power simply tells us how many of the base we have in total. Hence, we add the powers

> Simplify $5x^4 \times x^3$
>
> $5x^4 \times 1x^3 = (5 \times 1)x^{4+3} = 5x^7$

Rule 2: Multiply the powers with a bracket

$$(x^a)^b = x^{ab}$$

notice how the base does not change (base x stays x)

Why is this rule true?

$(x^2)^3$

$= x^2 \times x^2 \times x^2$ since we have x^2 three times

Now using rule 1 to add the powers gives x^6

Hence, we multiply the powers when we have a bracket

> Simplify $(4x^2y^3)^4$
>
> $(4x^2y^3)^4 = (4)^4(x^2)^4(y^3)^4 = 256x^2y^{12}$

Rule 3: Rule 2 can be extended when more than 1 term inside the bracket

$$(cx^ay^b)^d = (c)^d(x^a)^d(y^b)^d$$

Now applying rule 2 for each gives us $c^dx^{ad}y^{bd}$

Common Mistakes

Mistake 1: The base DOES NOT change

$2^3 \times 2^6$ doesn't equal 4^9

Instead, $2^3 \times 2^6 = 2^9$

Mistake 2: Don't ignore the power when it isn't written (it means power 1)

$2x^2 \times 3x$ doesn't equal $6x^2$.

Instead, $2x^2 \times 3x^1 = 6x^3$

Mistake 3: The power affects the first number term also

$(2x^2y^4)^3$ doesn't equal $2x^6y^{12}$

Instead, $(2x^2y^4)^3 = 8x^6y^{12}$

Mistake 4: We raise the first number to the power, we don't multiply it

$(5x)^3$ does not equal $15x^3$

Instead, $(5x)^3 = 5^3x^3 = 125x^3$

VERY COMMON Mistakes

Mistake 5: Don't mistake rule 3 when there_ is a sign ($+$ or $-$) in the middle.

$(2x)^2$ is not the same as $(2+x)^2$

$(2x)^2 = 4x^2$ whereas $(2+x)^2 = 4 + 4x + x^2$

The latter is expanding brackets

Mistake 6: Don't confuse addition/subtraction with multiplication. We can only add/subtract "like" terms (when we add/subtract the algebra part doesn't change)

- $2x + 3x$ is not the same as $2x \times 3x$
  $2x + 3x = 5x$ by collecting like terms
  $2x \times 3x = 6x^2$ using indices rule 1
- $2x^2 + 3x^2$ is not the same as $2x^2 \times 3x^2$
  $2x^2 + 3x^2 = 5x^2$ but $2x^2 \times 3x^2 = 6x^4$
- $2x^2 + 3x^3$ cannot be done/simplified
  but $2x^2 \times 3x^3 = 6x^5$

Division

Rule 1: Subtract the powers when dividing

$$x^a \div x^b \text{ or } \frac{x^a}{x^b} = x^{a-b}$$

The bases must be the same to use this rule and notice how they do not change (base x stays x)

Why is this rule true?

$$\frac{x^7}{x^4} = \frac{x \times x \times x \times x \times x \times x \times x}{x \times x \times x \times x}$$

$$= \frac{x \times x \times x \times \cancel{x \times x \times x \times x}}{\cancel{x \times x \times x \times x}}$$

$$= x^3$$

Hence, we subtract the powers

> Simplify $16x^2y^5 \div 4x^6y^3$
>
> $16x^2y^5 \div 4x^6y^3 = (16 \div 4)x^{2-6}y^{5-3} = 4x^{-4}y^2$

Common Mistakes

Mistake 1: The base DOES NOT change

$2^9 \div 2^6$ doesn't equal 1^3

Instead, $2^9 \div 2^6 = 2^3$

Mistake 2: Don't ignore the power when it isn't written (it means power 1)

$6x^2 \div 3x$ doesn't equal $2x^2$

Instead, $6x^2 \div 3x^1 = 2x$

Mistake 3: Don't let the fraction division notation confuse you

$$\frac{24x^6y^2}{32x^4y^3}$$

Deal with each part separately

$$\frac{24x^6y^2}{32x^4y^3} = \frac{3x^2}{4y}$$

How did we get this?

Think of it as simplifying $\frac{24}{32}$ which is $\frac{3}{4}$ and there are 6 x's and 2 y's in the numerator and 4 x's and 3 y's in the denominator

$$\frac{3xxxxxxyy}{4xxxxyyy}$$

We cross off the corresponding matching pairs

$$\frac{3xx\cancel{xxxx}\cancel{yy}}{4\cancel{xxxx}\cancel{yy}y}$$

We have 2 x's left in the numerator and 1 y left in the denominator

$$= \frac{3x^2}{4y}$$

OR:

Just think when we move the powers between numerator and denominators we subtract them

$$\frac{24x^6y^2}{32x^4y^3} = \frac{24x^{6-4}}{32y^{3-2}} = \frac{3x^2}{4y}$$

Raising Numbers to Powers

Rule 1: Raising to a power of zero:

Anything to the power of 0 is always 1

$$(\text{ANYTHING non zero})^0 = 1$$

> $2^0 = 1$
>
> $x^0 = 1$
>
> $(2x)^0 = 1$
>
> $\left(\frac{2}{3}\right)^0 = 1$

Rule 2: Raising a fraction to a power:

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

Apply the power to both the numerator and denominator

> Simplify $\left(\frac{2}{3}\right)^3$
>
> $\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$

Note: If more than 1 "element" inside the bracket we then use multiplication rule 3

> Simplify $\left(\frac{2x}{3y^2}\right)^3$
>
> $\left(\frac{2x}{3y^2}\right)^3 = \frac{(2x)^3}{(3y^2)^3} = \frac{(2)^3(x)^3}{(3)^3(y^2)^3}$
>
> $= \frac{2^3x^3}{3^3y^6} = \frac{8x^3}{27y^6}$

Rule 3: Raising negative numbers to a power:

$(\text{positive number})^{\text{even power}} = +$

$(\text{posiitve number})^{\text{odd power}} = +$

but

$(\text{negative number})^{\text{even power}} = +$

$(\text{negative number})^{\text{odd power}} = -$

> Example 1:
>
> Simplify $(-2)^4$ versus $-(2)^4$
>
> $(-2)^4 = -2 \times -2 \times -2 \times -2 = 16$
>
> $-2^4 = -(2 \times 2 \times 2 \times 2) = -16$
>
> They are not the same thing!
>
> $(-2)^4 = 16$ and $-(2)^4 = -16$
>
> Example 2:
>
> Simplify $(-2)^3$ versus $-(2)^3$
>
> $(-2)^3 = -2 \times -2 \times -2 = -8$
>
> $-2^3 = -(2 \times 2 \times 2) = -8$
>
> Here they are the same thing!
>
> $(-2)^3 = -8$ and $-(2)^3 = -8$

> Simplify $2(3)^2$
>
> $2(3)^2$ does not equal 6^2
>
> We must do the power 3^2 first BIDMAS/BODMAS)
>
> $2(3)^2 = 2(9) = 18$

Negative Powers

Rule 1: $x^{-n} = \frac{1}{x^n}$ and $(ab)^{-n} = \frac{1}{(ab)^n}$

The easiest way to think of this rule is that if we move terms between the numerator and denominator, the POWER of what is being moved changes(swaps/reverses) its sign (a positive becomes a negative and vice versa).

> Simplify 2^{-3}
>
> $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
>
> Moving the power -3 from the numerator to the denominator reverses the sign (in other words $-$ became $+$).
>
> Note: 2^{-3} means $\frac{2^{-3}}{1}$ hence the power was in the numerator originally and negative. We then moved it to the denominator and it became positive. There is a 1 in the numerator since the 2^{-3} moved, leaving only a 1.

> Get rid of the negative powers in $\frac{2x^2y^{-3}}{3z^{-4}}$
>
> $\frac{2x^2y^{-3}}{3z^{-4}}$
>
> The constants 2 and 3 stay where they are since they are and so can the x^2 term since it doesn't have a negative power. Remember for terms with negative powers that anything that moves between numerator and denominator changes the sign of its power
>
> $\frac{2x^2z^4}{3y^3}$

Rule 2: Raising fractions to negative powers

$$\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n = \frac{y^n}{x^n}$$

We flip the fraction and change the sign

Why is this rule true?

Way 1: Flip the fraction and the power becomes positive $\left(\frac{y}{x}\right)^n$

Now raise both to the power n giving $\frac{y^n}{x^n}$

Way 2: Apply the power to both the numerator and denominator first to get $\frac{x^{-n}}{y^{-n}}$. Now deal with the negative powers $\Longrightarrow \frac{y^n}{x^n}$

Way 3: Get rid of the negative power first by writing over 1

$$\frac{1}{\left(\frac{x}{y}\right)^n} = \left(\frac{y}{x}\right)^n \quad \text{or} \quad \left(\frac{1}{\frac{x}{y}}\right)^n = \left(\frac{y}{x}\right)^n$$

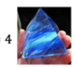
Now raise both to the power n giving $\frac{y^n}{x^n}$

Notice how writing a fraction over 1 just flips it (i.e. way 1)

> Simplify $\left(\frac{64}{125}\right)^{-\frac{2}{3}}$
>
> $\left(\frac{64}{125}\right)^{-\frac{2}{3}}$. Flip the fraction $\left(\frac{125}{64}\right)^{\frac{2}{3}}$
>
> $\left(\frac{125}{64}\right)^{\frac{2}{3}} = \frac{125^{\frac{2}{3}}}{64^{\frac{2}{3}}} = \frac{25}{16}$

The miners go underground (to the denominator)....

Example 3: $\left(\frac{1}{4}\right)^{-1} = \frac{4}{1} = 4$



Example 4: $\left(\frac{3}{2}\right)^{-1} = \frac{2}{3}$

Example 1: $3^{-1} = \frac{3^{-1}}{1} = \frac{1}{3}$

Example 2: $2^{-3} = \frac{2^{-3}}{1} = \frac{1}{2^3} = \frac{1}{8}$

... and send anything from down there up to the surface (numerator)

Example 5: $\left(\frac{2}{5}\right)^{-3} = \left(\frac{5}{2}\right)^3 = \frac{125}{8}$

Example 6: Get rid of the negative powers

$\frac{2x^2z^{-4}}{y^{-3}} = \frac{2x^2y^3}{z^4}$

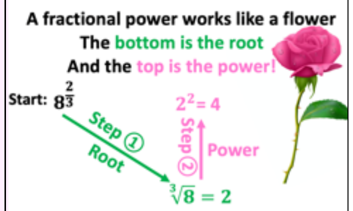
Example 7: $\left(\frac{4a^3}{6b^{-2}}\right)^{-2} = \left(\frac{6b^{-2}}{4a^3}\right)^2 = \frac{36b^{-4}}{16a^6} = \frac{9}{4a^6b^4}$

Rational Powers (Fractional Powers)

$$a^{\frac{n}{m}} = \left(\sqrt[m]{a}\right)^n$$

"ROOT AND THEN POWER

Note: $x^{\frac{1}{m}} = \sqrt[m]{x}$



> Simplify $27^{\frac{2}{3}}$
>
> $27^{\frac{2}{3}}$
>
> Root
>
> $\left(\sqrt[3]{27}\right)^2$
>
> $(3)^2$
>
> Power
>
> $= 3^2$
>
> $= 9$

> Simplify $\left(\frac{64x^6z^{12}}{27y^3}\right)^{\frac{1}{3}}$
>
> $\left(\frac{64x^6z^{12}}{27y^3}\right)^{\frac{1}{3}} = \frac{(64x^6z^{12})^{\frac{1}{3}}}{(27y^3)^{\frac{1}{3}}}$
>
> $= \frac{64^{\frac{1}{3}}(x^6)^{\frac{1}{3}}(z^{12})^{\frac{1}{3}}}{27^{\frac{1}{3}}(y^3)^{\frac{1}{3}}}$
>
> $= \frac{64^{\frac{1}{3}}x^2z^4}{27^{\frac{1}{3}}y} = \frac{4x^2z^4}{3y}$

Common Mistakes

$$\sqrt{x} = x^{\frac{1}{2}}$$

We drop the 2 for square root. When nothing is written to the left of the root it means square root