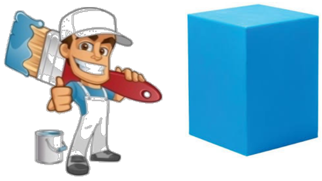
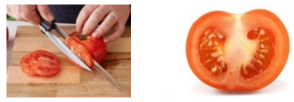
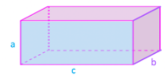
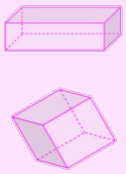
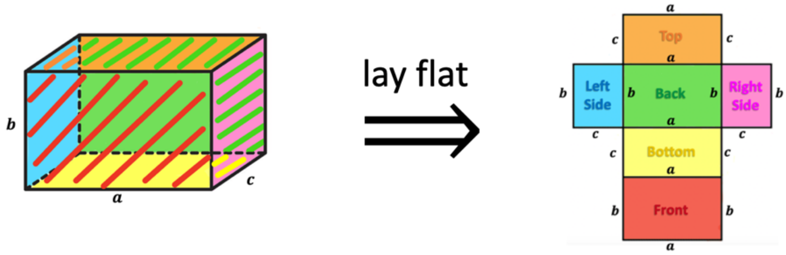
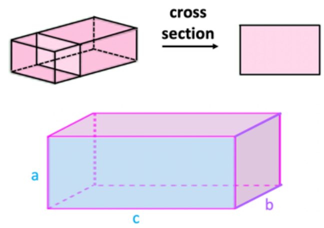
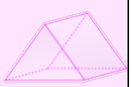
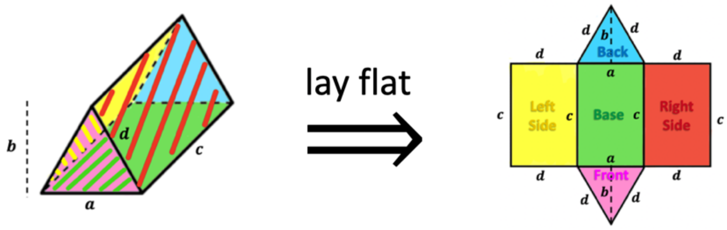
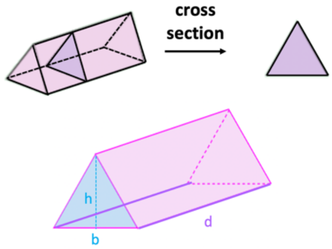
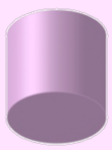
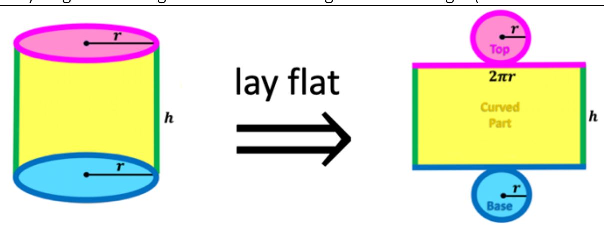
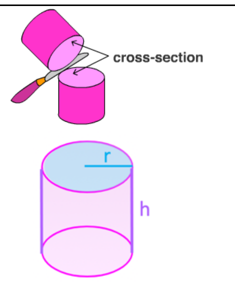
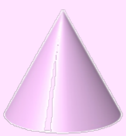
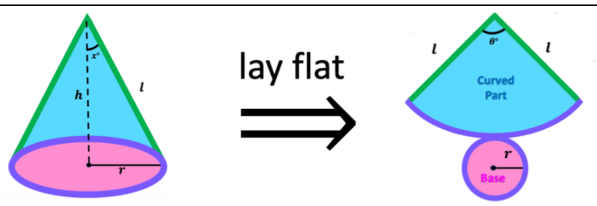
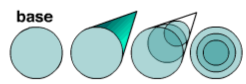
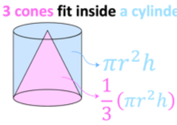
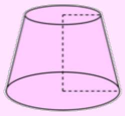
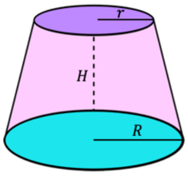
What is surface area?	What is volume?
Surface area is the area that a surface covers (the area of all faces added together). For example, how much paint to use when painting a box. The best way to think about surface area is the amount of space covering the outside of a shape i.e. everything you can touch on the outside of the shape. In order go find the surface area of 3D shapes we must lay them flat (consider the net) and then add up the areas of all the flat shapes formed.  surface area = add up all the individual areas of each shape when laid flat	Volume is the amount of space contained within a shape. For example, how much water a box can store or how much water in a swimming pool. Volume is about what fits inside/how much space an object has which is known as capacity.  It is important to find understand what a cross section is. A cross section is like a view into the inside of something made by cutting through it. For example, this is a cross-section of a tomato.  What is depth? How far back a shape goes. **Volume =area cross section x depth**  Volume = area of cross section × depth = area of rectangle × depth= acb  Volume = area of cross section × depth = area of rectangle × depth= abc

Shape	Surface Area	Volume
Cuboid/ Cube 	 area of top + area of bottom + area of front + area of back + area of left side + area of right side =ac + ac + ab +ab + bc + bc	 area of cross section × depth = area of rectangle × depth = acb
Prism 	 area of base rectangle + area of left rectangle + area of right rectangle+ area of back triangle + area of front triangle =ac + cd + cd + $\frac{1}{2}$bd + $\frac{1}{2}$bd Note: You'll need to use Pythagoras if not given the base or height of the triangle (we need these lengths for the area)	 area of cross section × depth = area of triangle × depth= $\frac{1}{2}bhc$
Cylinder 	 area of top circle + area of bottom circle + area of a rectangle $= \pi r^2 + \pi r^2 + 2\pi rh$ $$2\pi r^2 + 2\pi rh$$	 area of cross section × depth =area of a circle ×depth= $\pi r^2 h$ $$\pi r^2 h$$
Cone 	 area of circle + area of sector $= \pi r^2$ + area of sector The problem here is that we don't readily know the angle θ in order to find the area of the sector part. We have to do quite a bit of work to find it. Instead, we must we just memorise the formula. $$\pi r^2 + \pi rl$$ Note: You'll need to use Pythagoras if not given the height, radius or slant length.	Here the cross sections are not the same throughout the shape  So, we cannot use the formula area of cross section ×depth =area of a circle ×depth. Instead, we rely on a formula. If you know the volume of a cylinder, you can easily remember the formula for the volume of a cone. 3 cones **fit inside** a cylinder  $$\frac{1}{3}\pi r^2 h$$

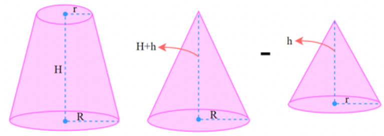
Frustrum





You are never asked to find the surface area of a frustrum, but it has been included for completeness. You only have to know how to find the volume.

$\pi(r+R)\sqrt{(R-r)^2+h^2}+\pi r^2+\pi R^2$

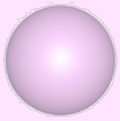


If given H and $\boldsymbol{h}$: $\frac{1}{3}\pi R^2(H+h)-\frac{1}{3}\pi r^2 h$

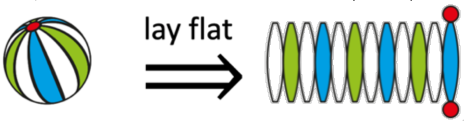
Use similar triangles if not give one of the radii. This should make sense as it is the area of the bigger cone take away the area of the smaller cone.

If not given $\boldsymbol{h}$: $\frac{1}{3}\pi H(R^2+Rr+r^2)$

Sphere



A sphere has no flat surfaces, it is a continuous curve. We can't easily add up the areas.



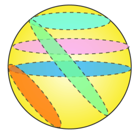
However, we can do the following**.** Take an orange. Cut the orange in half. Using one half of the orange only, draw four circles around it on a single piece of paper. Peel all of the orange and place all the peeled pieces in each of the circles

The surface area of an orange or any sphere covers four drawn out circles.

$4\times$area of the circle $= 4$ oranges $= 4\pi r^2$

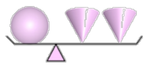
$$4\pi r^2$$

Like with a cone, the cross sections are not the same throughout the shape, so we cannot use the formula area of a circle $\times$depth.

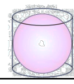


$$\frac{4}{3}\pi r^3$$

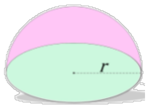
Cool Fact: The volume of a sphere is the same as the volume of 2 cones (with the same diameters)



A sphere is also $\frac{2}{3}$ of the volume of a cylinder



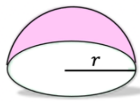
Hemi-Sphere



Watch out for **hemispheres**! When we half a sphere we must also add on the circle that we expose by cutting it. The green circle below is now covering the outside of the shape.

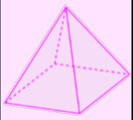
$\frac{4\pi r^2}{2}+\pi r^2 = 3\pi r^2$

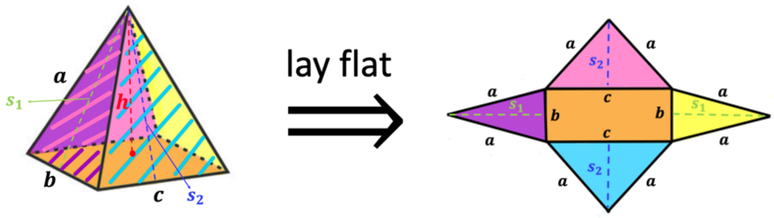
$$3\pi r^2$$



$$\frac{2}{3}\pi r^3$$

Pyramid



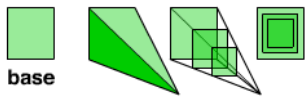


area of base rectangle + area of pink triangle + area of blue triangle+ area of purple triangle + area of yellow triangle

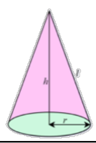
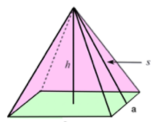
$= bc + \frac{1}{2}as_2 + \frac{1}{2}as_2 + \frac{1}{2}as_1 + \frac{1}{2}as_1$

Note: You'll need to use 3D trig knowledge to find the any of the lengths if not given them

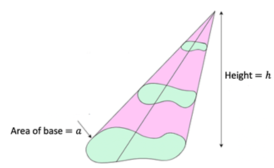
Again, the cross sections are not the same throughout the shape.



It doesn't matter whether the base is a

circle	a square/rectangle
	

or even an irregular base such a wobbly one!



The volume is still given by $\frac{1}{3}$(area of base) $\times$height

Note: You'll need to use 3D trig knowledge to find the height of a pyramid if not given it

$$\frac{1}{3}(\text{base area})(\text{height})$$

Did you know that the volume of a pyramid also applies to the volume of a cone? V=$\frac{1}{3}(\pi r^2)h$

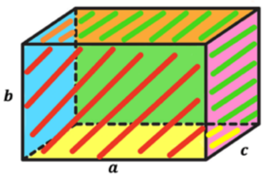
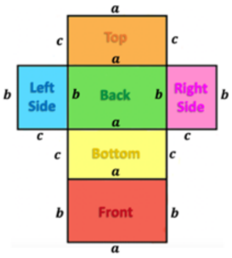
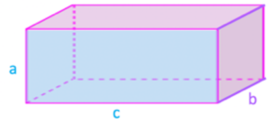
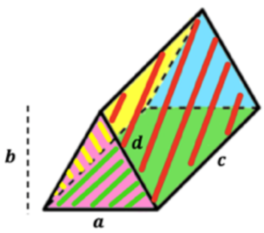
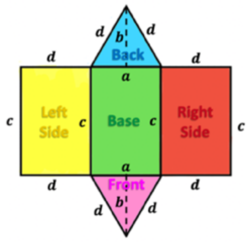
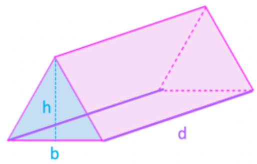
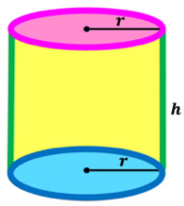
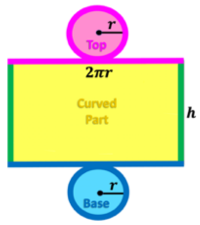
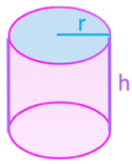
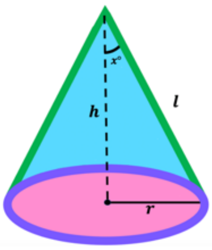
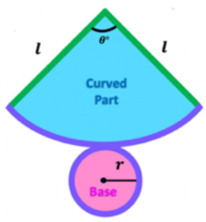
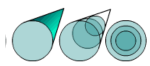
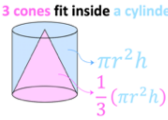
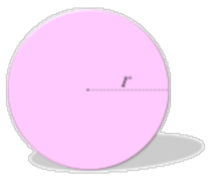
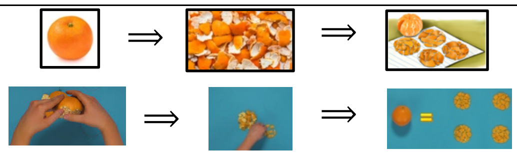
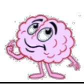
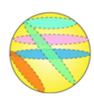
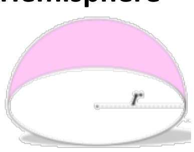
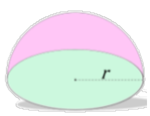
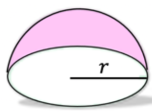
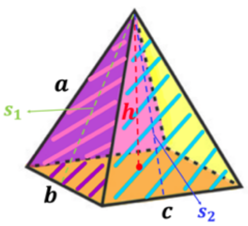
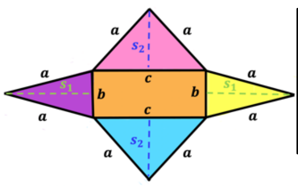
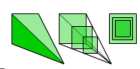
Some cool facts:

There is a nice way that you can remember the difference between the volume and surface area of a sphere if you have covered integration and differentiation. The integral of the surface area of a sphere is equal to the volume and the derivative of the volume gives you the surface area.

$\int$ surface area of a sphere $=\int 4\pi r^2 dr = \frac{4\pi r^3}{3} =$ volume of a sphere and derivative of volume of a sphere $= \frac{d}{dr}\left(\frac{4\pi r^3}{3}\right) = 4\pi r^2 =$ surface area of a sphere

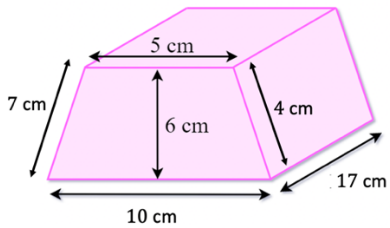
Going off topic from 3D shapes, did you know that you can do the same thing for 2D shapes with the area of a circle to get the circumference? $A=\pi r^2 \Rightarrow \frac{dA}{dr} = 2\pi r$

Another cool fact: The volume of a cone + volume of sphere = volume of a cylinder (if all 3 shapes have the same height and dia

3D Shapes Summary	Surface Area (SA)	Volume (V)
Cuboid  Note: a cube is where all sides are the same length	 area of top + area of bottom + area of front + area of back + area of left side + area of right side $= ac + ac + ab + ab + bc + bc$	 area of cross section × depth = area of rectangle × depth $= acb$
Triangular Prism 	 area of base rectangle + area of left rectangle + area of right rectangle + area of back triangle + area of front triangle $= ac + cd + cd + \frac{1}{2}bd + \frac{1}{2}bd$	 area of cross section × depth = area of triangle × depth $= \frac{1}{2}bhc$
Cylinder 	 area of top circle + area of bottom circle + area of a rectangle $= \pi r^2 + \pi r^2 + 2\pi rh$	 area of cross section × depth =area of a circle ×depth $= \pi r^2 h$
Cone 	The problem here is that we don't readily know the angle θ in order to find the area of the sector part. We have to do quite a bit of work to find it. Instead, we just memorise the formula.  area of sector + area of circle Remember $= \pi rl + \pi r^2$ 	No uniform cross section now (all different sized circles), so we have a formula instead  To help remember the formula: **3 cones fit inside a cylinder**  $= \frac{1}{3}\pi r^2 h$ Remember 
Sphere 	 Remember $= 4\pi r^2$ 	No uniform cross sections (all different sized circles), so again we have a formula instead  $= \frac{4}{3}\pi r^3$ Remember 
Hemisphere 	 Half the S.A. of a sphere + the EXPOSED circle $2\pi r^2 + \pi r^2 = 3\pi r^2$	 We half the volume of a sphere $= \frac{2}{3}\pi r^3$
Pyramid 	 area of base rectangle + area of pink triangle + area of blue triangle + area of purple triangle + area of yellow triangle $= bc + \frac{1}{2}as_2 + \frac{1}{2}as_2 + \frac{1}{2}as_1 + \frac{1}{2}as_1$ Note: You'll need to use 3D trig knowledge to find the any of the lengths if not given them	No uniform cross sections (all different sized squares or rectangles), so again we have a formula instead  $= \frac{1}{3}(\text{base area})(\text{height})$ Remember  Did you know that the volume of a pyramid also applies to the volume of a cone? $V = \frac{1}{3}(\pi r^2)h$

Examples

Find the surface area and volume of the following shape



Answer

Surface area:

The surface area is the area of every side added together.

Imagine every side laid flat and add up all their individual areas

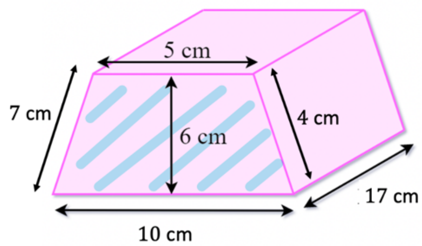
This is the area of 2 trapezia + area of 4 rectangles

The 2 trapezia are located at the front and back and the 4 rectangles are located on the left, right, top and bottom

$$= \frac{1}{2}(5+10)(6) + \frac{1}{2}(5+10)(6) + 4\,(17) + 7\,(17) + 5(17) + 10(17)$$

$$= 532\ cm^2$$

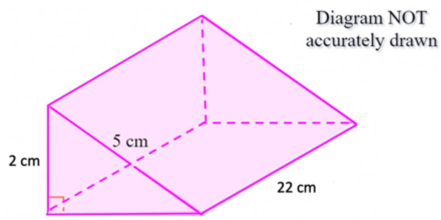
Volume:



Area of cross section = area of blue shaded trapezium $= \frac{1}{2}(5+10)(6) = 45\ cm^2$

area of cross section × depth
$= 45 \times 17 = 765\ cm^3$

The diagram shows a prism with length 22 cm. The cross section of the prism is a right-angled triangle with sides 2 cm and 5 cm.



i. Calculate the total surface area of the prism
ii. Calculate the volume of the prism

Answer

Surface area:

The surface area is the area of every side added together.
Imagine every side laid flat and add up all their individual areas

This is the area of 2 identical triangles + area of 3 different rectangles

We don't have the base of the triangle

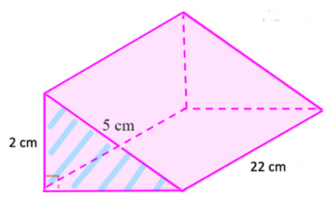
We need to use Pythagoras to find the base

$$base = \sqrt{5^2 - 2^2} = \sqrt{21}$$

Surface area $= \frac{1}{2}(\sqrt{21})(2) + \frac{1}{2}(\sqrt{21})(2) + \sqrt{21}(22) + 2(22) + 5(22)$

$$= 264.0\ cm^2$$

Volume:

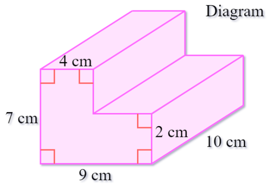


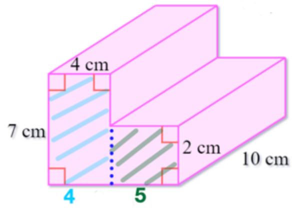
Area of cross section = area of blue shaded triangle $= \frac{1}{2}(\sqrt{21})(2) = \sqrt{21}\ cm^2$

Volume = area of cross section × depth
$= \sqrt{21} \times 22 = 100.8\ cm^3$

What happens when we have two shapes together?

Diagram NOT accurately drawn

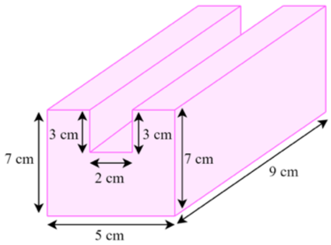


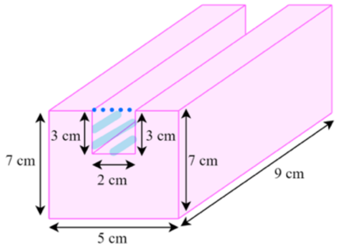


Let's colour code the cross section for ease of explaining the area of it

Area of cross section = area of blue shaded rectangle plus area of green shaded rectangle $= 7(4) + 5(2) = 38\ cm^2$

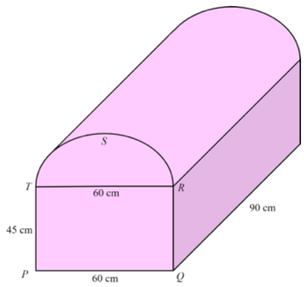
Volume =area of cross section ×depth $= 38 \times 10 = 380\ cm^3$

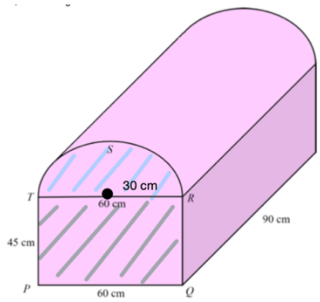




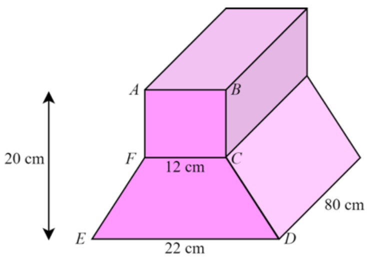
Area of cross section= area of entire pink rectangle − area of mini pink rectangle $= 7(5) - 3(2) = 29\ cm^2$

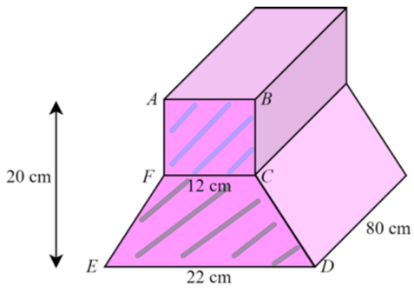
Volume =area of cross section ×depth $= 29 \times 9 = 261\ cm^3$





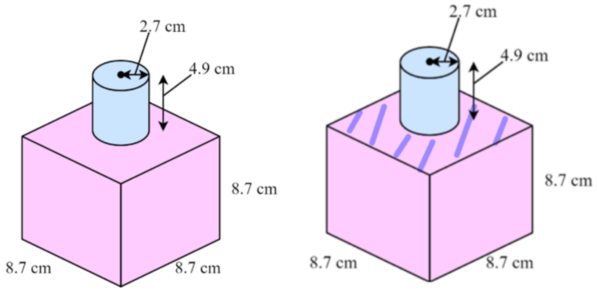
Area of cross section = area of blue shaded semi-circle + area of green shaded rectangle $= \frac{1}{2}\pi(30)^2 + 60(45) = 4113.7\ cm^2$
Volume =area of cross section ×depth $= 4113.7176 \times 90 = 370{,}234.5\ cm^3$





Area of cross section = area of blue shaded square + area of green shaded trapezium $= 12(12) + \frac{1}{2}(12 + 22)(8) = 280cm^2$

Volume =area of cross section ×depth $= 280 \times 80 = 22{,}4000\ cm^3$



Here we don't care about the light blue base for the cylinder and need to take off the base of the cylinder when we find the area of the top of the cube

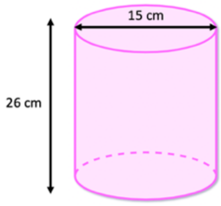
Surface area = area of dark blue top of cube + light blue cylinder + 5 sides of pink box

$$= (8.7^2 - \pi(2.7)^2) + (\pi(2.7)^2 + 2\pi(2.7)(4.9)) + 5(8.7^2) = 537\ cm^2$$

Note: You could have done this quicker by realising that all we need to do is add the full cube surface area and the curved surface area of the cylinder. The missing base of the cylinder cancels out with the exposed top of the cylinder.

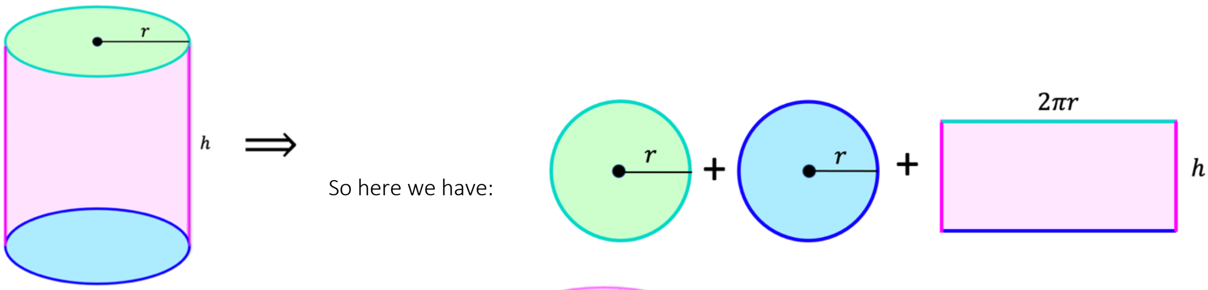
$$6(8.7^2) + 2\pi(2.7)(4.9) = 537\ cm^2$$

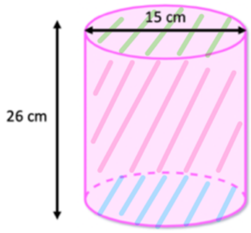
Find the surface area and volume of the following shape.



i.

The surface area is the area of every side added together. Imagine every side laid flat and add up all their individual areas. The green shaded region becomes a rectangle when laid flat. It general when we unfold a cylinder it looks like:





Surface Area = area of green shaded circle + area of blue shaded circle + area of pink shaded rectangle

$$= \pi(7.5)^2 + \pi(7.5)^2 + 2\pi(7.5)(26) = 1578.7\ cm^2$$

Note: You also learn the formula $2\pi r^2 + 2\pi rh$ for the surface area of a cylinder which you could memorise and have used straight away instead

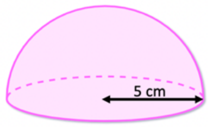
ii.

Area of cross section = area of green shaded circle $= \pi(7.5)^2 = 176.715\ cm^2$

Volume = area of cross section × depth $= 176.715 \times 26 = 4594.6\ cm^3$

Note: You also learn the formula $\pi r^2 h$ for the surface area of a cylinder which you could memorise and have used straight away instead

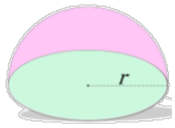
The diagram shows a solid hemisphere of radius 5.



Find the total surface area and volume of the solid hemisphere.

Volume

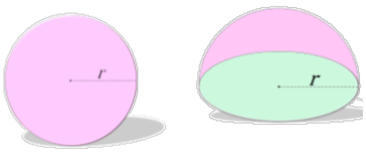
It would now be quite long to lay the sides flat and work out the area.



The circle at the base is easy and just πr^2, but the pink curved part takes a bit longer to find. Instead, you're meant to memorise the formula for a sphere: $4\pi r^2$

Here we want a hemisphere though to we half the area: $\frac{4\pi r^2}{2} = 2\pi r^2$

The problem is that when we have the hemisphere we expose the green circle



So, we need to add it back on. Surface area of a hemisphere is $2\pi r^2 + \pi r^2 = 3\pi r^2$

Here we have $r = 5$

Surface area $= 3\pi(5)^2 = 235.6\ cm^2$

Surface Area

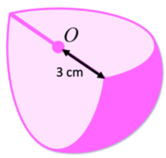
We can no longer use the formula that volume is cross section time depth since we no longer have a uniform cross section here. It changes dependent on where we are inside the cone. So, we memorise the formula for a sphere $= \frac{4}{3}\pi r^3$

Here we want a hemisphere though to we half the volume: $\frac{\frac{4}{3}\pi r^3}{2} = \frac{4}{6}\pi r^3 = \frac{2}{3}\pi r^3$

Here we have $r = 5$

volume$= \frac{2}{3}\pi(5)^3 = 261.8\ cm^3$

The shape below is one quarter of a solid sphere, centre O. Find the surface area and volume of the following.



Volume

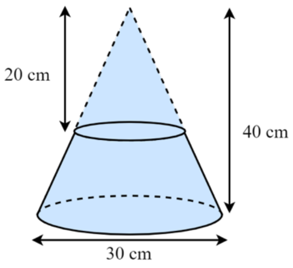
volume$= \frac{\frac{4}{3}\pi(3)^3}{4} = \frac{1}{3}\pi(3)^3 = 9\pi$

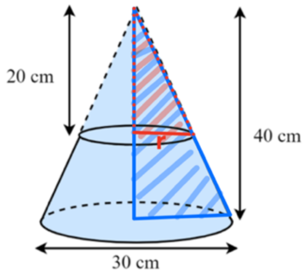
Surface Area

Notice how we expose 2 semi circles that weren't there before

surface area$= \frac{4\pi r^2}{4} + \frac{\pi r^2}{2} + \frac{\pi r^2}{2} = 2\pi r^2$

Find the volume of the frustrum





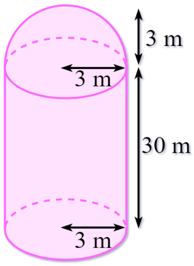
Since we are given the dimensions of both cones we can use the fact that it is volume of the bigger cone minus the volume of the smaller cone, so all you need to know is the formula for the volume of a cone $\frac{1}{3}\pi r^2 h$

Here it looks like we don't have enough info as the radius of the cone is missing. We use similar shapes to get r. These shapes are similar so

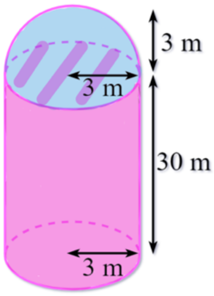
$$\frac{40}{15} = \frac{20}{r}$$
$$40r = 300$$
$$r = 7.5$$
$$\text{Volume} = \frac{1}{3}\pi(15)^2(40) - \frac{1}{3}\pi(7.5)^2(20) = 5890.2\ cm^3$$

What happens when we have two shapes together such as cones, spheres or cylinders?

Find the surface area and volume of the following shape:



Volume

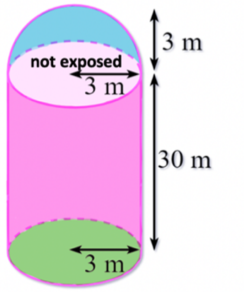


Volume of cylinder = $\pi(3)^2(30) = \pi(9)(30) = 270\pi$

Volume of hemisphere= $\frac{\frac{4}{3}\pi(3)^3}{2} = \frac{2}{3}\pi(3)^3 = \frac{2}{3}(27)\pi = 18\pi$

$= 288\pi$

Surface Area



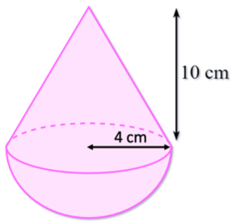
Surface area of hemisphere = $2\pi(3)^2 = 18\pi$

Surface area of cylinder= $2\pi(3)(30) + \pi(3)^2 = 189\pi$

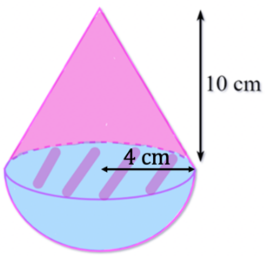
Note: it is not $+2\pi r^2$ since only the dark green circle is exposed, not the light pink

$= 207\pi$

Find the surface area and volume of the following shape:



Volume

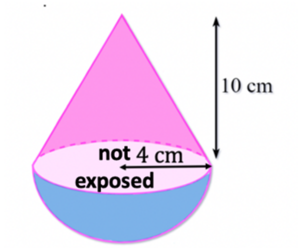


Volume of cone= $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(4)^2(10) = \frac{160}{3}\pi$

Volume of a hemisphere= $\frac{2}{3}\pi r^3 = \frac{2}{3}\pi(4)^3 = \frac{128}{3}\pi$

Total volume $= 96\pi$

Surface Area



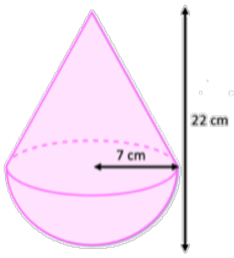
$4^2 + 10^2 = l^2$

$l = \sqrt{116}$

Surface area of cone= $\pi r l = \pi(4)\sqrt{116} = \pi(4)(\sqrt{116}) = 4\sqrt{116}\pi = 185.6$

Surface area of hemisphere= $\frac{4\pi r^2}{2} = \frac{4\pi(4)^2}{2} = 32\pi$

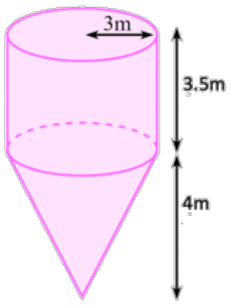
Total surface area= 235.875



When finding surface area remember that the circle of the cone is not exposed and neither is the circle of the hemisphere

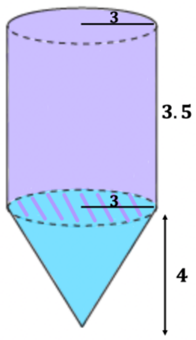
Total surface area= $\pi r l + 2\pi r^2 = \pi(7)\sqrt{274} + 2\pi(7)^2 = 671.9\ cm^2$

Total volume= $\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{1}{3}\pi(7)^2(15) + \frac{2}{3}\pi(7)^3 = 1488.1\ cm^3$



Volume

The volume is everything inside the shape.



Volume of cylinder = $\pi(3)^2(3.5) = 31.5\pi$

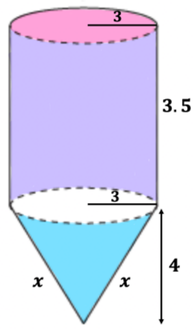
Volume of cone= $\frac{1}{3}\pi(3)^2(4) = 12\pi$

Total volume $= 136.7\ cm^3$

Surface Area

Let's first find the height of the cone at the bottom using Pythagoras

$$3^2 + 4^2 = x^2$$
$$x^2 = \sqrt{25} = 5$$



When finding surface area remember that we care about the areas of everything on the OUTSIDE. The white bottom circle of the cylinder is not exposed and neither is white circle of the cone.

Surface area of cylinder= $2\pi(3)(3.5) + \pi(3)^2 = 30\pi$

Surface area of cone= $\pi r l = \pi(3)(5) = 15\pi$

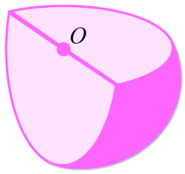
(note: it is not $+\pi r^2$ since the circle is not exposed)

Total surface area $= 45\pi = 141.4\ m^2$

Working Backwards:

Sometimes we are given the volume/surface area of the of the shape. We can use this to work backwards and solve for r (or another unknown). Once we have r we can find the volume of the hemisphere and then add the volume together to find the total volume.

Shape S is one quarter of a solid sphere, centre O. The volume of S is $576\pi\ cm^3$

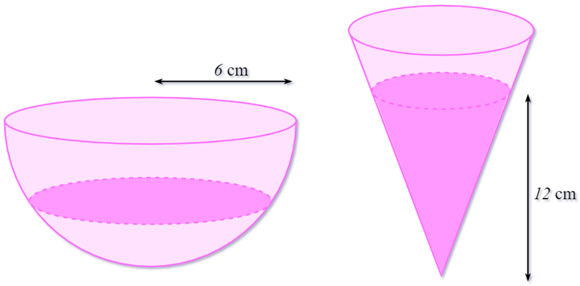


Shape of S

Notice how we expose 2 semi circles that weren't there before

surface area$=\frac{4\pi r^2}{4}+\frac{\pi r^2}{2}+\frac{\pi r^2}{2}=2\pi r^2$

volume$=\frac{\frac{4}{3}\pi r^3}{4}=\frac{1}{3}\pi r^3$



The water is poured into a hollow cone.
The depth of the water is the cone is 12 cm.
Work out the radius of the surface of the water in the cone.

volume of sphere$=\frac{4}{3}\pi(6^3)=288\pi$

Volume of hemisphere$=\frac{288\pi}{2}=144\pi$

Volume of Water in hemisphere$=\frac{2}{5}(144\pi)=\frac{288}{5}\pi$

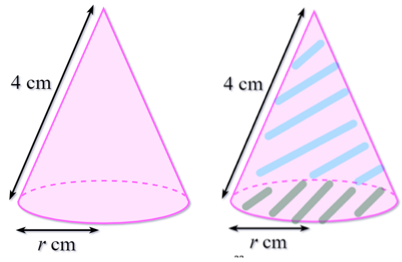
Volume of water in cone is also $\frac{288}{5}\pi$

$\frac{1}{3}\pi r^2(12)=\frac{288}{5}\pi$

$4r^2=\frac{288}{5}$

$r^2=14.4$

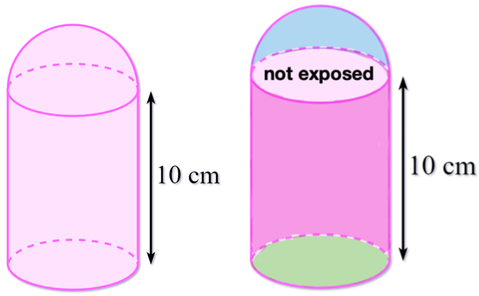
$r=3.80$



Surface area of cone= $\pi r l + \pi r^2 = \frac{33}{4}\pi$

We know l so lets fill this in and then use algebra to solve for r

$$\pi r(4) + \pi r^2 = \frac{33}{4}\pi$$
$$4\pi r + \pi r^2 = \frac{33}{4}\pi$$
$$4r + r^2 = \frac{33}{4}$$
$$4r + r^2 = \frac{33}{4}$$
$$4r^2 + 16r - 33 = 0$$
$$(2r + 11)(2r - 3) = 0$$
$$r \neq -\frac{11}{2}, r = \frac{3}{2}$$



Surface area of hemisphere = $2\pi r^2$

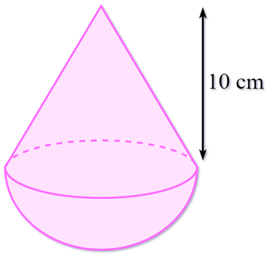
Surface area of cylinder= $2\pi r h + \pi r^2$ (note: it is not πr^2 since only the dark green circle is exposed, not the light pink)

We are given the surface area of the hemisphere and can work backwards to find r:

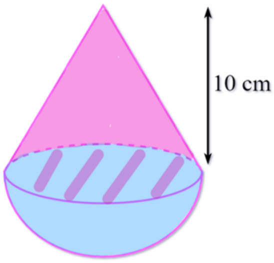
$$2\pi r^2 = 32\pi$$
$$2r^2 = 32$$
$$r^2 = 16$$
$$r = 4$$

Total surface area= $2\pi r^2 + 2\pi r h + \pi r^2 = 32\pi + 2\pi(4)(10) + \pi(4)^2 = 32\pi + 80\pi + 16\pi = 128\pi$

The solid shape is made from a hemisphere and a cone.



The radius of the hemisphere is equal to the radius of the cone
The cone has a height of 10 cm
The volume of the cone is $270\pi\ cm^3$
Work out the total volume of the solid shape
Give your answer in terms of π



Volume of cone= $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2(10) = \frac{10}{3}\pi r^2$. We now need to solve for r.

$$\frac{10}{3}\pi r^2 = 270\pi$$
$$\frac{10}{3}r^2 = 270$$
$$r^2 = 81$$
$$r = 9$$

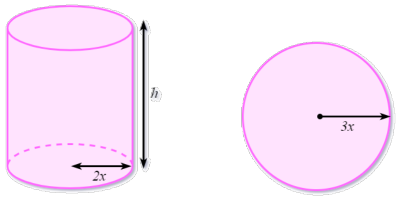
Volume of a cone = $\frac{10}{3}\pi(9)^2 = 270\pi$

Volume of a hemisphere= $\frac{2}{3}\pi r^3 = \frac{2}{3}\pi(9)^3 = 486\pi$

Total volume = $270\pi + 486\pi = 756\pi$

Algebraic Side Lengths:

The diagram below shows a cylinder and a sphere. The radius of the base of the cylinder is $2x$ cm and the height of the cylinder is h cm. The radius of the sphere $3x$ cm. The volume of the cylinder is equal to the volume of the sphere.



Express h in terms of x

Volume of cylinder $= \pi r^2 h = \pi(2x)^2(h) = \pi(4x)(h) = 4\pi h x^2$

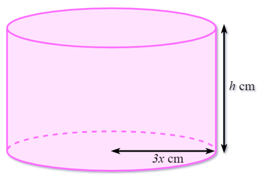
Volume of sphere$= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3x)^3 = \frac{4}{3}\pi(27x^3)$

Volumes are equal $\Longrightarrow 4\pi h x^2 = \frac{4}{3}\pi(27x^3)$

Cancel the π on both sides

$$4hx^2 = \frac{4}{3}(27x^3)$$
$$4hx^2 = 36x^3$$
$$4h = 36x$$
$$h = 9x$$

The diagram shows a solid metal cylinder



The cylinder has a base radius $3x\ cm$ and height h cm

The metal cylinder is melted

All the meta is then used to make 270 spheres

Each sphere has a radius of $\frac{1}{2}x$ cm

Find, an expression, in its simplest form, for h in terms of x

The diagram shows a solid metal cylinder.

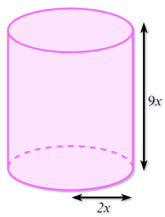


Diagram NOT accurately drawn

The cylinder has a base radius $2x$ and a height $9x$

The cylinder is melted down and made into a sphere of radius

Find an expression for r in terms of x

Volume of cylinder$= \pi r^2 h$

Volume of sphere$= \frac{4}{3}\pi r^3$

Volume of cylinder =volume of sphere

$\pi r^2 h = \frac{4}{3}\pi r^3$

Plug in dimensions given

$\pi(2x)^2(9x) = \frac{4}{3}\pi r^3$

$\pi(4x^2)(9x) = \frac{4}{3}\pi r^3$

$(4x^2)(9x) = \frac{4}{3}r^3$

$36x^3 = \frac{4}{3}r^3$

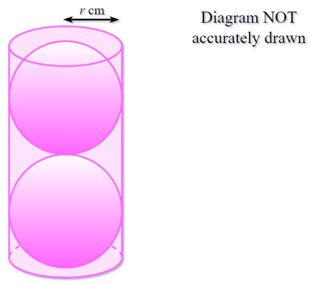
$108x^3 = 4r^3$

$r^3 = 27x^3$

$r = \sqrt[3]{27x^3}$

$r = 3x$

Two solid spheres, each of radius r cm, fit exactly inside a hollow cylinder



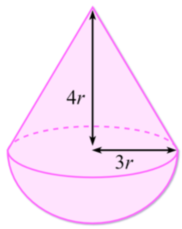
The radius of the cylinder is r cm
The height of the cylinder is equal to $4r\ cm$
The volume of the space inside the cylinder, not occupied by the spheres is $\frac{125}{6}\pi\ cm^3$
Calculate the value of r

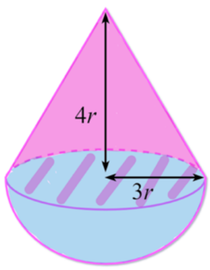
Volume of cylinder$= \pi r^2 h = \pi r^2(4r) = 4\pi r^3$

Volume of 2 spheres$= 2\left(\frac{4}{3}\pi r^3\right) = \frac{8}{3}\pi r^3$

Unoccupied space$= 4\pi r^3 - \frac{8}{3}\pi r^3$
$4\pi r^3 - \frac{8}{3}\pi r^3 = \frac{125}{6}\pi$
$4r^3 - \frac{8}{3}r^3 = \frac{125}{6}$
$12r^3 - 8r^3 = \frac{125}{2}$
$4r^3 = \frac{125}{2}$
$r^3 = \frac{125}{8}$
$r = \sqrt[3]{\frac{125}{8}}$
$r = \frac{\sqrt[3]{125}}{\sqrt[3]{8}} = \frac{5}{2}$

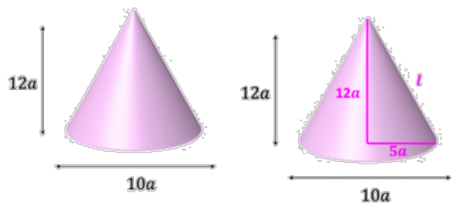
The radius of the hemisphere is r cm
The radius of the base of the cone is $3r$ cm
The height of the cone is $4r$ cm
The volume of the solid shape is $330\pi\ cm^3$
Find the value of r in the form $\sqrt[3]{n}$, where n is an integer





volume of cone $= \frac{1}{3}\pi r^2 h$
volume of hemisphere $= \frac{2}{3}\pi r^3$
volume of cone $+$ volume of hemisphere= Total volume
$\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = 330\pi$
Plug in the dimensions given in the question
$\frac{1}{3}\pi(3r)^2(4r) + \frac{2}{3}\pi(3r)^3 = 330\pi$
$\frac{1}{3}(3r)^2(4r) + \frac{2}{3}(3r)^3 = 330$
$\frac{1}{3}(9r^2)(4r) + \frac{2}{3}(27r^3) = 330$
$(9r^2)(4r) + 2(27r^3) = 990$
$36r^3 + 54r^3 = 990$
$90r^3 = 990$
$r^3 = 11$
$r = \sqrt[3]{11}$

The diameter of the base of the cone is $10a$ cm. The height of the cone is $12a$. cm.The total surface area of the cone is $810\pi\ cm^2$. The volume of the cone is $k\pi\ cm^3$, where k is an integer. Find k



Surface area of a cone: $\pi r^2 + \pi rl$
We need to use Pythagoras to find l first:

$$(12a)^2 + (5a)^2 = l^2$$
$$144a^2 + 25a^2 = l^2$$
$$169a^2 = l^2$$
$$l^2 = 169a^2$$
$$l = 13a$$

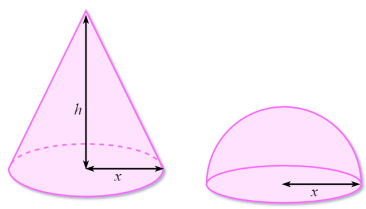
Surface area: $\pi(5a)^2 + \pi(5a)(13a) = 810\pi$

$$25a^2\pi + 65a^2\pi = 810\pi$$
$$25a^2 + 65a^2 = 810$$
$$90a^2 = 810$$
$$a^2 = 9$$
$$a = 3$$

Now that we know a we can easily find the volume. The radius is 5(3)=15 and the height is 12(3)=36

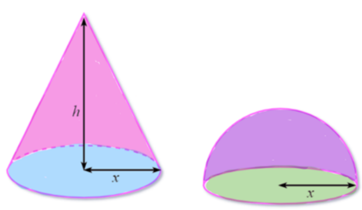
$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(15)^2(36) = 2700\pi$$
$$k = 2700$$

The diagram shows a solid cone and solid hemisphere



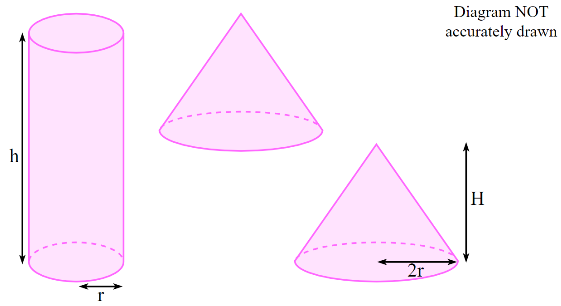
The cone has a base of radius x cm and a height of h cm
The hemisphere has a base of radius x cm
The surface area of the cone is equal to the surface area of the hemisphere
Find an expression for h in terms of x

Surface area of cone $= \pi rl + \pi r^2$
Surface area of hemisphere $= 2\pi r^2 + \pi r^2$



Surface area of cone= Surface area of hemisphere
$\pi rl + \pi r^2 = 2\pi r^2 + \pi r^2$
Plug in the dimensions given in the question
We need to find l first
$l^2 = x^2 + h^2$
$l = \sqrt{x^2 + h^2}$
We can plug in now
$\pi(x)\sqrt{x^2 + h^2} + \pi(x)^2 = 2\pi(x)^2 + \pi(x)^2$
$x\sqrt{x^2 + h^2} + x^2 = 2x^2 + x^2$
$x\sqrt{x^2 + h^2} + x^2 = 3x^2$
$x\sqrt{x^2 + h^2} = 2x^2$
$\sqrt{x^2 + h^2} = 2x$
$x^2 + h^2 = 4x^2$
$h^2 = 3x^2$
$h = \sqrt{3x^2}$
$h = \sqrt{3}x$

Some plasticine is used to make a solid cylinder of base radius r cm and height h cm



The plasticine is then split in half and used to make two identical cones. Each cone has base radius $2r$ cm and height H cm

Express H in terms of h.. Give your answer in its simplest form

Volume of cylinder $= \pi r^2 h$

Volume of cone$= \frac{1}{3}\pi r^2 h$

Volume of cylinder is 2 times the volume of the cone

$\pi r^2 h = 2\left(\frac{1}{3}\pi r^2 h\right)$

Plug in the dimensions given in the question

$\pi r^2 h = \frac{2}{3}\pi(2r)^2(H)$

$\pi r^2 h = \frac{2}{3}\pi(4r^2)(H)$

$r^2 h = \frac{2}{3}(4r^2)(H)$

$r^2 h = \frac{8}{3}(r^2)(H)$

$3r^2 h = 8r^2 H$

$8r^2 H = 3r^2 h$

$H = \frac{3r^2 h}{8r^2}$

$H = \frac{3}{8}h$

The diagram shows a solid cone

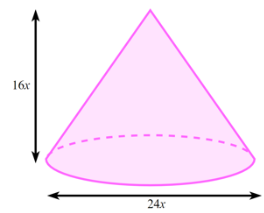
The diameter of the base of the cone is $24x\ cm$

The height of the cone is $16x\ cm$

The curved surface area of the cone is $2106\ \pi\ cm^3$

The volume of the cone is V$\pi\ cm^3$, where V is an integer

Find the value of V



We need x in order to find the volume. We need to find l before we can find x.

We can find l using Pythagroas

$(12x)^2 + (16x)^2 = l^2$

$144x^2 + 256x^2 = l^2$

$400x^2 = l^2$

$l = 20x$

We can use the fact that we know the curved surface area to find x

curved surface area$= \pi r l = 2160\pi$

$\pi(12x)(20x) = 2160\pi$

$240x^2 = 2160$

$x^2 = 9$

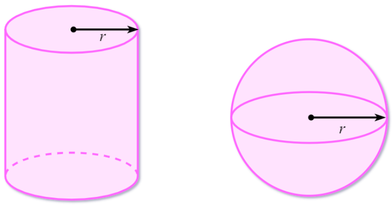
$x = 3$

Now we know x we can find the volume of the cone

Volume of cone $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(12x)^2(16x) = \frac{1}{3}\pi(12 \times 3)^2(16 \times 3) = 20736\pi$

The diagram shows a solid cylinder and a solid sphere

Diagram NOT accurately drawn



The cylinder has radius r
The sphere has radius r
Given that $\frac{\textbf{total surface area of cylinder}}{\text{surface area of sohere}} = 2$

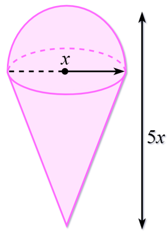
Find the value of $\frac{volme\ of\ cylinder}{volume\ of\ sphere}$

$\frac{\textbf{total surface area of cylinder}}{\text{surface area of sphere}} = \frac{2\pi rh+2\pi r^2}{4\pi r^2} = 2$

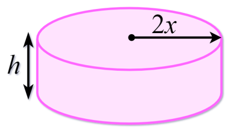
$$\frac{2rh + 2r^2}{4r^2} = 2$$
$$\frac{2h + 2r}{4r} = 2$$
$$2h + 2r = 8r$$
$$2h = 6r$$
$$h = 3r$$

$$\frac{volme\ of\ cylinder}{volume\ of\ sphere} = \frac{\pi r^2 h}{\frac{4}{3}\pi r^3} = \frac{\pi r^2(3r)}{\frac{4}{3}\pi r^3} = \frac{3r^3}{\frac{4}{3}r^3} = \frac{3}{\frac{4}{3}} = 3 \div \frac{4}{3} = 3 \times \frac{3}{4} = \frac{9}{4}$$

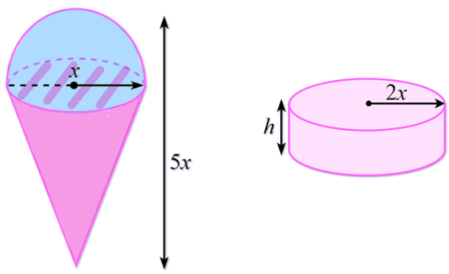
A solid is made by putting a hemisphere on top of a cone



The total height of the solid is $5x$
The radius of the base of the cone is x
The radius of the hemisphere is x



A cylinder has the same volume as the solid
The cylinder has radius $2x$ and height h
All measurements are in centimetres
Find a formula for h in terms of x



Volume of cone $= \frac{1}{3}\pi r^2 h$

Volume of hemisphere$= \frac{2}{3}\pi r^3$

Volume of cylinder $= \pi r^2 h$

volume of cone$+$ volume of hemisphere $=$ volume of a cylinder

$\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \pi r^2 h$

Plug in the dimensions given in the question

$\frac{1}{3}\pi x^2(4x) + \frac{2}{3}\pi x^3 = \pi(2x)^2 h$

$$\frac{1}{3}x^2(4x) + \frac{2}{3}x^3 = (2x)^2h$$
$$\frac{4}{3}x^2(x) + \frac{2}{3}x^3 = 4x^2h$$
$$4x^2(x) + 2x^3 = 3(4x^2h)$$
$$6x^3 = 12x^2h$$
$$12x^2h = 6x^3$$
$$h = \frac{6x^3}{12x^2}$$
$$h = \frac{x}{2}$$