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AP® Statistics
2003 Scoring Guidelines
Form B

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Question 1

Solution

Part (a):

The point P does have a large influence on the regression line. When P is removed from the data set, the slope of the line changes from 0.4919 to 0.1500, the intercept changes from 8.107 to 11.123, and the value of R^2 drops from 47.6% to 2.5%. Also, the slope is significantly different from 0 when the point P is included in the data set and is not significantly different from 0 when the point P is excluded from the data set.

Part (b):

The regression line for the corrected data will have a negative slope rather than a positive slope, and the intercept would be much larger for the corrected data.

Scoring

Part (a) is

Essentially correct (E) if the student

1. identifies the point P as influential
   AND
2. explains that there have been changes in at least 2 of the following:
   - Slope
   - Statistical significance of the slope
   - Intercept
   - Regression equation
   - Value of R^2 (or R^2_{adj})

OR

mentions the change in one of the values above and discusses clearly how point P is extreme in the *x* direction (if just "extreme," needs to also explain why that implies the potential for influence).

NOTE: *r* (0.69 to 0.16) can be mentioned as well, but is not counted separately from R^2 unless the student provides clearly distinguishable interpretations of each.

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Question 1 (cont'd)

Partially correct (P) if the student does one of the following
- Identifies Point P as influential but with weak justification (something changes).
- Identifies Point P as influential but only one change is noted.
- Confuses S as the slope on the computer output and thus states that the point P is not influential since the slope doesn't change much.

Incorrect (I) if the student only answers "yes" or refers to all of the numbers changing but provides no indication of understanding of what the numbers represent.

Part (b) is

Essentially correct (E) if the student indicates that the sign of the slope would change from positive to negative. The student should explicitly compare the 2 graphs. The student does not need to comment on the change in the intercept.

Partially correct (P) if the student does one of the following
- Indicates that the slope will change (including "slope is lower"), but fails to explicitly state that the sign changes from positive to negative.
- Comments only that the value of the correlation changes from positive to negative.
- Comments that the line "flattens."

Incorrect (I) if
- Student only comments on the intercept.
- Response is very poorly communicated (e.g., "line is negative," "data are positive," "data are weak").

4 Complete Response (EE)

Both parts essentially correct.

3 Substantial Response (EP or PE)

One part essentially correct and the other part partially correct

2 Developing Response (EI or IE or PP)

One part essentially correct and the other part incorrect
OR
Both parts partially correct

1 Minimal Response (PI or IP)

One part partially correct

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Question 2

Solution

Part (a): $P(age\ 31-45) = \frac{89}{207} = 0.42995$

Part (b): $P(age\ 31-45 \mid income\ over\ 50,000) = \frac{35}{96} = 0.36458$

Part (c):

If annual income and age were independent, the probabilities in (a) and (b) would be equal. Since these probabilities are not equal, annual income and age category are not independent for adults in this sample.

Scoring

Part (a) is scored as either essentially correct (E) (may be minor arithmetic errors) or incorrect (I).

Part (b) is

Essentially correct (E) if the conditional probability is correctly calculated.

Partially correct (P) if the student reverses the conditioning, calculating

$P(income\ over\ 50,000 \mid age\ 31-45) = \frac{35}{89} = 0.3933$

OR

calculates the correct probability for the wrong column, e.g., $\frac{32}{64}$

Incorrect (I) if the student calculates the joint probability: $\frac{35}{207} = 0.169$

Part (c) is

Essentially correct (E) if the student

1. indicates that the two variables are not independent
   AND
2. the explanation is tied to the fact that the probabilities in parts (a) and (b) are not equal (the answer must be based on parts (a) and (b))

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Question 2 (cont'd)

Partially correct (P) if the student indicates that the two variables are not independent, but the explanation is incorrect, or is not based on the answers to parts (a) and (b); i.e., performing new correct calculations instead of referring to those in parts (a) and (b). For example: determining the probability of the intersection and comparing to the two individual probabilities

$\left(\frac{35}{207} = 0.169, \text{ which does not equal } \frac{96}{207} \cdot \frac{89}{207} = (0.43)(0.46)\right)$, reversing conditions

$\left(\text{e.g., } \frac{96}{207} = 0.464, \text{ which does not equal } \frac{35}{89} = 0.393\right)$, or other conditional probability comparisons.

Incorrect (I) if the student fails to give a numerical justification to support the argument.
OR
Incorrect if the student does one of the following

- performs an incorrect additional calculation
- says the variables are independent based entirely on the context.
- performs a chi-square test ($\chi^2 = 5.38$, p-value = 0.496) since this addresses independence in the population instead of the sample
- only states "yes, independent" with no justification

NOTE: If either of the probabilities calculated in (a) or (b) are incorrect, part (c) should be scored as if those probabilities were correct. For example, if the student incorrectly calculated the same answer for parts (a) and (b), part (c) would be scored as correct if the student states that you can't tell if the two variables are independent because you would need to check all age-gender combinations.

4 Complete Response (EEE)

All three parts essentially correct

3 Substantial Response (EEP, EPE, EPP, IEE)

Part (a) essentially correct and parts (b) and (c) at least partially correct
OR
Part (a) incorrect and parts (b) and (c) essentially correct

2 Developing Response (EEI, EIE, EPI, EIP, IEP, IPE, IPP)

Part (a) essentially correct and one (but not both) of parts (b) and (c) correct
OR
Part (a) incorrect and both parts (b) and (c) at least partially correct

1 Minimal Response (EII, IPI, IIP, IEI, IIE)

Part (a) essentially correct and parts (b) and (c) incorrect
OR
Part (a) incorrect and one of parts (b) and (c) partially correct

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Question 3

Solution

Part (a):

This study is an experiment. The researchers imposed treatments and subjects were randomly assigned to the two treatment groups.

Part (b):

The two-proportion z test could be used to compare the proportion of volunteers who get the flu for the two conditions. The hypotheses would be

$$H_0 : p_T - p_C = 0 \quad versus \quad H_a : p_T - p_C < 0$$

OR

$$H_0 : p_C - p_T = 0 \quad versus \quad H_a : p_C - p_T > 0$$

OR

$$H_0 : p_T = p_C \quad versus \quad H_a : p_T < p_C$$

OR

$$H_0 : p_T = p_C \quad versus \quad H_a : p_C > p_T$$

where

p_T is the proportion of those receiving vitamin C (from the population of students who would volunteer for such a study) who contract the flu (or the probability that such a student receiving vitamin C contracts the flu).

and

p_C is the proportion of those receiving placebo (from the population of all students who would volunteer for such a study) who contract the flu.

Note: Could also define p as the proportion who do *not* contract the flu and then reverse the direction of the alternative hypothesis statement.

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Question 3 (cont'd)

Scoring

This problem has 3 elements; part (a) is one element and part (b) is divided into two elements (naming the test and stating the hypotheses). Each element is scored as either essentially correct (E), partially correct (P), or incorrect (I).

Element 1 (part (a)) is

Essentially correct (E) if the student

1. concludes that the study is an experiment
AND
2. the explanation is tied to the fact that the researchers *imposed* a treatment (controlled which medicine)
OR
states that there was random assignment of subjects to treatments (random division into two groups)

Partially correct (P) if the student indicates that the study is an experiment, but the explanation is missing or does not include either that the researcher imposes treatments or that there is random assignment of subjects to treatments (e.g., "experimenters controlled factors").

Incorrect (I) if the student says one of the following:

- this is not an experiment because the study used volunteers or because the subjects were not randomly selected.
- this is not an experiment because the experimenters did not control everything

Element 2 (naming the test) is

Essentially correct (E) if the student identifies the two-sample z test for proportions (all 3 parts of the name are needed)

Partially correct (P) if the student does either of the following

- identifies a chi-square test (homogeneity)
- only gives two components of the name (e.g., "2 sample z test")
NOTE: If student says "two sample p test," grade holistically.

Incorrect (I) if the student identifies any test involving means

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Question 3 (cont'd)

Element 3 (stating the hypotheses) is

Essentially correct (E) if the student does any of the following

- gives a correct pair of one-sided hypotheses. If the student uses p_T and p_C (for treatment and control) or p_C and p_P (for vitamin C and placebo), they need not define the parameters to get credit for this element. If they use p_1 and p_2, they must identify which is treatment and which is control.
- states correct hypotheses for the test described in element 2 (e.g., a two-sided alternative hypothesis or verbal description for the chi-square test, or involving two means for a two-sample *t* test).
- provides correct statement of null and alternative hypotheses in words.

Partially correct (P) if either

- the direction of the inequality in the alternative hypothesis is incorrect, or if a two sided alternative is specified, or
- p_1 and p_2 are used in the hypotheses without specifying which refers to the treatment group and which refers to the control group

Incorrect (I) if the student does any of the following

- specifies sample proportions in the hypotheses (unless defined as parameters).
- uses proportions in the hypotheses and means in the test procedure or vice versa.
- reverses the hypotheses.

NOTE: Elements 2 and 3 could be correct if a one-sample test for a proportion is specified and the hypotheses are given as

$H_0 : p_T = p_0$ *versus* $H_a : p_T < p_0$

and p_0 is defined to be the true proportion in the population of untreated volunteers who get the flu. But to receive credit for this solution p_0 must be correctly defined.

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Question 3 (cont'd)

4 **Complete Response (3E)**

All three parts essentially correct

3 **Substantial Response (2E 1P)**

Two parts essentially correct and 1 part partially correct

2 **Developing Response (2E 0P or 1E 2P or 3P)**

2 parts essentially correct and no parts partially correct
OR
One part essentially correct and 2 parts partially correct
OR
3 parts partially correct

1 **Minimal Response (1E 1P or 1E 0P or 0E 2P)**

One part essentially correct and either 0 or 1 parts partially correct
OR
No parts essentially correct and 2 parts partially correct

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Question 4

Solution

Part (a):

Assign each subject a number from 001 to 300 and then use a random number table or a random number generator to select 150 of the 300 for the new filter group. The other 150 would be assigned to the standard filter group.
OR
For each subject, flip a coin. If the coin lands H, assign the subject to the new filter group; otherwise assign the subject to the standard filter group. Continue in this way until one of the groups has 150 subjects. Assign all remaining subjects to the other group.

Part (b):

Without a comparison group, the cholesterol level could change overall, but we would not be able to determine whether the observed change was due to some other extraneous variable that changed during the 10-week period. For example, diet might change with time of the year, and the diet might result in changes in cholesterol changes. So a change in cholesterol would not be attributable to the new coffee filter. The addition of a control group enables the researchers to assess the mean change in cholesterol level due to the coffee filter, as opposed to just determining if the cholesterol level changed. The control group eliminates the confounding variable of another change that might have occurred over the 10-week period.

Part (c):

The two-sample t test for means or mean differences would be used (or the two-sample z test for means).

Part (d):

If it is known that smoking is related to changes in cholesterol level, it would be best to control for smoking by using only nonsmokers. This eliminates smoking as a source of variability, creating more homogenous groups, enabling more direct comparisons between the treatment and control groups and more precise estimates of the treatment effects (though we will only be able to generalize the results to nonsmokers).

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Question 4 (cont'd)

Scoring

Part (a) is

Essentially correct (E) if the student describes a method that is
1. based on random assignment and
2. will result in equal numbers of subjects in each group

Partially correct (P) if the student does any of the following
- describes a method based on random assignment, but that does not ensure an equal number of subjects in each group.
- attempts to describe a method of random assignment that ensures 150 subjects in each group, but the explanation of random assignment is not clear or incomplete. This may include assigning numbers to the subjects and selecting the even numbered subjects for the treatment group, if it is clear the student believes this will randomize the groups.

Incorrect (I) if the student describes any method not based on random assignment. For example, allows the subjects to self-select themselves into the 2 groups, or just referring to "random assignment" but not describing the method of assignment.

Part (b) is

Essentially correct (E) if the student
1. describes the need for a comparison group and
2. explains how the control group allows the researchers to attribute the change in cholesterol to the new filter as opposed to natural variability or another confounding variable (that a simple before-after measurement would not be sufficient to conclude that the change is due to the new filter).

OR

discusses "confounding" or "lurking" variables and describes the variable in such a way that it could lead to the change in the cholesterol levels during the 10 week time period for these subjects.

Partially correct (P) if the student does one of the following

- indicates that the inclusion of a control group allows the new and standard filters to be compared, but the explanation does not adequately explain the *need* for a comparison group to control for other changes during the 10 week period.

- refers to controlling for "confounding or lurking variables" or "factors," and describes a reasonable variable that could change cholesterol but does not discuss the need for comparison with the control group to eliminate these factors as potential explanations for the change in cholesterol with the new filter.

Incorrect (I) if the student fails to indicate the need for a comparison group and does not describe a good confounding variable in context.

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Question 4 (cont'd)

Parts (c) and (d) are scored together. These are essentially correct (E) if both parts (c) and (d) are answered correctly. Part (cd) is partially correct (P) if only one of parts (c) or (d) is answered correctly.

Part (c) is

Correct if the student indicates a procedure to compare means for two independent samples. The student can refer to a two sample *t* test.

Incorrect if the student states only "two sample z test" or any other test procedure.

Part (d) is

Correct if the student explains how smoking could be related to initial cholesterol levels at the start of the study (there is something different about smokers) and indicates a desire for more homogenous groups or recognizes that focusing only on nonsmokers will reduce variability (akin to blocking). Student may use the word "confounding" if (through explanation) they indicate they intended an extraneous variable.

Incorrect if the student does either of the following

- Describes smoking as a confounding variable.
- Only states that smoking is related to cholesterol.

4 Complete Response (3E)

All three parts essentially correct

3 Substantial Response (2E 1P)

2 Developing Response (2E 0P or 1E 2P or 3P)

1 Minimal Response (1E 1P or 1E 0P or 0E 2P)

NOTE: A response with 1E and 1P can be graded holistically based on the strength of the responses in parts (b) and (d).

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Question 5

Solution

Part (a):

P(a number on all 3 spins) = [P(number)]^3 since the outcomes are independent from spin to spin

$$= \left(\frac{3}{4}\right)^3 = 0.4219$$

Part (b):

Winnings	0	900	1000	1300
Probability	0.25	0.25	0.25	0.25

E(winnings) = $\sum x_i p_i$ = 0(0.25)+900(0.25)+1000(0.25)+1300(0.25) = 800

Or

E(winnings on 4th spin) = -800(0.25) + 100(0.25) + 200(0.25) + 500(0.25) = 0

So E(winnings) = initial amount + E(winnings on 4th spin) = 800 + 0 = 800

Part (c):

Element 1: States a correct pair of hypotheses

H_0: The four outcomes are equally likely $\left(\text{or } p_1 = p_2 = p_3 = p_4 = \frac{1}{4}\right)$

H_a: The four outcomes are not equally likely (or at least one p_i differs from $\frac{1}{4}$)

Element 2: Identifies a correct test (by name or by formula) and checks appropriate conditions.

Chi-square test (for goodness of fit) $\chi^2 = \sum \frac{(Obs - Exp)^2}{Exp}$

Conditions: Outcomes of spins of the wheel are independent and large sample size.

The problem states that successive spins of the wheel are independent.

The expected counts are all equal to 25, which is greater than 5 (or 10), so the sample size is large enough to proceed.

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Question 5 (cont'd)

Element 3: Correct mechanics, including the value of the test statistic, df, and p-value (or rejection region)

Expected counts: 25 for each of the 4 cells

$$\chi^2 = \sum \frac{(Obs - Exp)^2}{Exp} = \frac{(33-25)^2}{25} + \cdots + \frac{(26-25)^2}{25} = 4.24$$

df = 4-1 = 3 p-value = .2367
(from tables p-value > 0.10, from Graphing Calculator: p-value = 0.23669,
from table rejection region for $\alpha = 0.05$ is 7.81, $\alpha = 0.01$ is 11.34)

Element 4: Using the results of the statistical test, states a correct conclusion in the context of the problem.

Because the p-value is greater than the stated α (or because the p-value is large, or because the test statistic does not fall in the rejection region), fail to reject H_0. There is not convincing evidence that the four outcomes on the wheel are not equally likely. That is, we don't have convincing evidence against the conjecture that the four outcomes on the wheel are equally likely.

Scoring

Part (a): 1 for correct answer (including Binomial calculation) = 0.4219

$\frac{1}{2}$ if answer is $\frac{3}{4}$ or $\left(\frac{1}{4}\right)^3 = 0.0156$ or $\left(\frac{1}{4}\right)^3 (3) = 0.047$ or 0.4219 with no work

Part (b): 1 if the expected value, 800, is correct (except for minor computational errors)

$\frac{1}{2}$ if expected value is computed as

- 800 + expected winnings on one spin, or 800 + 200 = 1000
- E(outcome on one spin)=200 but then solution breaks down
- E(winnings on 4th spin)=0 but then solution breaks down
- With fairly major computational errors $\left(\text{e.g., } \frac{3200}{3}\right)$

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Question 5 (cont'd)

0 if

- answer of 800 is given but no work is shown or bad logic, e.g., 4(200)
- expected value formula is given but no calculations are done
- outcomes are set up correctly but no expected value is calculated

Part (c): $\frac{1}{2}$ for each element of the test that is correct

1. statement of hypotheses

2. identification of test and check of sample size condition

3. correct mechanics $\chi^2 = 4.24$, df=3, p-value=0.2367, and/or $\chi^2_{3,0.05} = 7.81$

4. statement of conclusion (fail to reject)

If both an α and a p-value are given, the linkage is implied. If no α is given, the solution must be explicit about the linkage by giving a correct interpretation of the p-value or explaining how the conclusion follows from the p-value.

NOTE: If the p-value in element 3 is incorrect but the conclusion is consistent with the computed p-value, element 4 can be considered as correct.

4 **Complete Response**
Score of 4 from parts (a) through (c)

3 **Substantial Response**
Score of 3 from parts (a) through (c)

2 **Developing Response**
Score of 2 from parts (a) through (c)

1 **Minimal Response**
Score of 1 from parts (a) through (c)

IF A PAPER IS BETWEEN TWO SCORES (FOR EXAMPLE, 2 ½ PARTS) USE A HOLISTIC APPROACH TO DETERMINE WHETHER TO SCORE UP OR DOWN DEPENDING ON THE STRENGTH OF THE RESPONSE AND COMMUNICATION.

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Question 6

Solution

Part (a): Part (a) is scored based on four elements

Element 1: Identifies appropriate confidence interval by name or by formula and checks the appropriate conditions.

One sample confidence interval for a proportion $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Conditions: random sample from a large population and large sample size. Since the sample of size 2000 was a random sample from the population of all patients at the HMO, it is reasonable to consider the sample of 40 to 44 year old females (n=370) as a random sample of the 40 to 44 year old female patients of the HMO.

$\hat{p} = 0.10 \qquad n\hat{p} = 370(0.10) = 37 \qquad n(1-\hat{p}) = 370(0.90) = 333$

Since both $n\hat{p}$ and $n(1-\hat{p})$ are both greater than 5 (or 10), the sample size is large enough to proceed.

Since these conditions are met, if is reasonable to proceed with the following calculations and interpretations.

Element 2: Correct mechanics

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.10 \pm 1.96\sqrt{\frac{(0.10)(0.90)}{370}} = 0.10 \pm 0.03 = (0.07, 0.13)$$

Graphing calculator: (0.06943, 0.13057)

Element 3: Interpretation of the interval

We can be 95% confident that the true proportion of this HMO's 40 to 44 year old female patients who contracted the disease is between 0.07 and 0.13.

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Question 6 (cont'd)

Element 4: Interpretation of the confidence level

Ninety five percent of all possible random samples of size 370 from this population will result in a confidence interval that includes the true population proportion of this HMO's 40 to 44 year old female patients who contracted the disease. Can also say, approximately 95% of a large number of intervals will contain the population proportion.
OR
The method used to produce this interval will fail to capture the population proportion about 5% of the time.

Part (b):

The width of the interval is determined by the magnitude of $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, which depends on $\hat{p}$ and n. If the sample proportions are equal, then the confidence interval widths will be the same if the sample sizes are the same for all 8 age-gender groups. Thus, we need to take random samples of size 250 from each of the 8 groups.

Part (c):

The width of the interval is proportional to $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$. The interval widths for all of the groups will be the same if $\frac{\hat{p}(1-\hat{p})}{n}$ is the same for each group. This will happen when the sample size is proportional to $\hat{p}(1-\hat{p})$.

For this to happen, we need (approximately)

$$\sqrt{\frac{(0.05)(0.95)}{n_1}} = \sqrt{\frac{(0.08)(0.92)}{n_2}} = \sqrt{\frac{(0.20)(0.80)}{n_3}} = \sqrt{\frac{(0.35)(0.65)}{n_4}}$$

where $n_1 + n_2 + n_3 + n_4 = 2000$

Solving these equations we get

Age Group	Sample Size	$\sqrt{\frac{p(1-p)}{n}}$
35 to 39	187	0.0159
40 to 44	289	0.0159
45 to 49	629	0.0159
50 to 54	895	0.0159

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Question 6 (cont'd)

OR

it is sufficient to say that since $\sum \hat{p}(1-\hat{p})$ is anticipated to be

$$\sum \hat{p}(1-\hat{p}) = (0.05)(0.95) + (0.08)(0.92) + (0.20)(0.80) + (0.35)(0.65) = 0.0475 + 0.0736 + 0.1600 + 0.2275$$

$$= 0.5086$$

we want

$$n_1 = \left(\frac{0.0475}{0.5086}\right)2000 = 186.79 \quad n_2 = \left(\frac{0.0736}{0.5086}\right)2000 = 289.42$$

$$n_3 = \left(\frac{0.1600}{0.5086}\right)2000 = 629.18 \quad n_4 = \left(\frac{0.2275}{0.5086}\right)2000 = 894.61$$

Scoring

Part (a) is scored based on the number of the four elements that are correct, ½ point for each element.

1. Naming procedure and checking conditions
   (It is OK not to repeat SRS since stated given in the problem statement.)
2. Carrying out the mechanics for a 95% confidence interval (0.0694, 0.1306)
3. Interpreting the confidence interval in context
4. Interpreting the confidence level

Note: Parts 3 and 4 need to be read together (correct interpretations in reverse order should be graded as essentially correct).

Common errors in parts 3 and 4 include: No context in part 3, claiming 95 out of 100 intervals exactly, or describing the interval for a population mean (this last error will only count once against element 3 and element 4).

Parts (b) and (c) are scored as either essentially correct (E), partially correct (P), or incorrect (I). Score essentially correct as 1 point and partially correct as $\frac{1}{2}$ point.

Part (b) is

Essentially correct (E) if the student
1. specifies equal sample sizes of 250. (If an explanation clearly indicates that all samples will be 250, without specifying the actual number, credit will be awarded.)
AND
2. provides justification that appeals to the width of the interval being dependent on $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

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Question 6 (cont'd)

Partially correct (P) if the student does one of the following

- specifies that equal samples sizes of 250 are needed, but does not justify the answer based on $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$.
- justifies that the width of the interval depends on n (through standard error), but fails to state that the sample size is 250 for all 8 groups.
- justifies that the sample sizes need to be equal through the standard error but does not focus on all eight groups. For example: considers either just males and females (1000/1000) or equal gender in given age groups (398/300/177/105), or equal samples sizes in the four age groups (500/500/500/500).

Incorrect (I) if the answer

- Specifies that equal sample sizes are needed but gives with no explanation.

Part (c) is

Essentially correct (E) if

1. the explanation recognizes that the sample sizes should be proportional to $p(1\text{-}p)$
   AND
2. the resulting sample sizes are computed
   OR
   conditions that would allow for the computation of the sample sizes are given (setting up the equations and noting that the sample sizes must sum to 2000). It is not necessary to actually compute the sample sizes.

Partially correct (P) if the student

- correctly computes the sample sizes but the justification is missing or incorrect
  OR
- chooses the samples sizes to be proportional to p (rather than $p(1\text{-}p)$), resulting in sample sizes of 147, 235, 588, 1029. Note, $\frac{0.05}{0.68}(2000) = 147$ etc.
  OR
- states that the sample sizes depend on n and on $\hat{p}$ and $\hat{p}(1-\hat{p})$, but fails to indicate how to carry out the calculation. For example, trying to set the confidence intervals equal to each other and noting that the sum of the n's is 2000.

NOTE: If a student incorrectly solves their equations but obtains plausible answers (between 0 and 2000) then score as essentially correct.

Incorrect (I) if equal sample sizes are recommended.

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2003 SCORING GUIDELINES (Form B)

Question 6 (cont'd)

4 **Complete Response**
Score of 4 on parts (a) through (c)

3 **Substantial Response**
Score of 3 on parts (a) through (c)

2 **Developing Response**
Score of 2 on parts (a) through (c)

1 **Minimal Response**
Score of 1 on parts (a) through (c)

IF A PAPER IS BETWEEN TWO SCORES (FOR EXAMPLE, 2 ½) USE A HOLISTIC APPROACH TO DETERMINE WHETHER TO SCORE UP OR DOWN DEPENDING ON THE STRENGTH OF THE RESPONSE AND COMMUNICATION.

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