Simulation Free Response

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Topic tags have been given for each question to enable you to know if you can do the question or whether you need to wait to cover the additional topic(s).

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Question 1

Qualification: AP Statistics

Areas: Probability

Subtopics: Simulation Using Random Digit Table, Simulation, Sampling Methods

Paper: Part-A / Series: 2001 / Difficulty: Somewhat Challenging / Question Number: 3

3. Every Monday a local radio station gives coupons away to 50 people who correctly answer a question about a news fact from the previous day's newspaper. The coupons given away are numbered from 1 to 50, with the first person receiving coupon 1, the second person receiving coupon 2, and so on, until all 50 coupons are given away. On the following Saturday, the radio station randomly draws numbers from 1 to 50 and awards cash prizes to the holders of the coupons with these numbers. Numbers continue to be drawn without replacement until the total amount awarded first equals or exceeds $300. If selected, coupons 1 through 5 each have a cash value of $200, coupons 6 through 20 each have a cash value of $100, and coupons 21 through 50 each have a cash value of $50.

   (a) Explain how you would conduct a simulation using the random number table provided below to estimate the distribution of the number of prize winners each week.

   (b) Perform your simulation 3 times. (That is, run 3 trials of your simulation.) Start at the leftmost digit in the first row of the table and move across. Make your procedure clear so that someone can follow what you did. You must do this by marking directly on or above the table. Report the number of winners in each of your 3 trials.

72749	13347	65030	26128	49067	02904	49953	74674	94617	13317
81638	36566	42709	33717	59943	12027	46547	61303	46699	76423
38449	46438	91579	01907	72146	05764	22400	94490	49833	09258

Question 2

Qualification: AP Statistics

Areas: Probability, Hypothesis Testing

Subtopics: Simulation, One Sample T Test for a Mean

Paper: Part-B / Series: 2005-Form-B / Difficulty: Hard / Question Number: 6

6. Regulations require that product labels on containers of food that are available for sale to the public accurately state the amount of food in those containers. Specifically, if milk containers are labeled to have 128 fluid ounces and the mean number of fluid ounces of milk in the containers is at least 128, the milk processor is considered to be in compliance with the regulations. The filling machines can be set to the labeled amount. Variability in the filling process causes the actual contents of milk containers to be normally distributed. A random sample of 12 containers of milk was drawn from the milk processing line in a plant, and the amount of milk in each container was recorded.

   (a) The sample mean and standard deviation of this sample of 12 containers of milk were 127.2 ounces and 2.1 ounces, respectively. Is there sufficient evidence to conclude that the packaging plant is not in compliance with the regulations? Provide statistical justification for your answer.

   Inspectors decide to study a particular filling machine within this plant further. For this machine, the amount of milk in the containers has a mean of 128.0 fluid ounces and a standard deviation of 2.0 fluid ounces.

   (b) What is the probability that a randomly selected container filled by this machine contains at least 125 fluid ounces?

   (c) An inspector will randomly select 12 containers filled by this machine and record the amount of milk in each. What is the probability that the minimum (smallest amount of milk) recorded in the 12 containers will be at least 125 fluid ounces? (Note: In order for the minimum to be at least 125 fluid ounces, each of the 12 containers must contain at least 125 fluid ounces.)

   An analyst wants to use simulation to investigate the sampling distribution of the minimum. This analyst randomly generates 150 samples, each consisting of 12 observations, from a normal distribution with mean 128 and standard deviation 2 and finds the minimum for each sample. The 150 minimums (sorted from smallest to largest) are shown on the next page.

Question 3

Qualification: AP Statistics

Areas: Hypothesis Testing, Chi-Squared Test

Subtopics: Power of the Test, State Hypotheses, Hypothesis Testing - One Tailed, Simulation, Chi-Squared Test For Variance

Paper: Part-B / Series: 2006 / Difficulty: Hard / Question Number: 6

6. A manufacturer of thermostats is concerned that the readings of its thermostats have become less reliable (more variable). In the past, the variance has been 1.52 degrees Fahrenheit (F) squared. A random sample of 10 recently manufactured thermostats was selected and placed in a room that was maintained at 68°F. The readings for those 10 thermostats are given in the table below.

Thermostat	1	2	3	4	5	6	7	8	9	10
Temperature (°F)	66.8	67.8	70.6	69.3	65.9	66.2	68.1	68.6	67.9	67.2

(a) State the null and alternative hypotheses that the manufacturer is interested in testing.

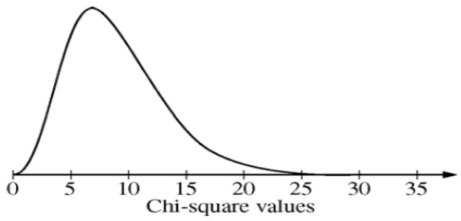
It can be shown that if the population of thermostat temperatures is normally distributed, the sampling distribution of $\frac{(n-1)s^2}{\sigma^2}$ follows a chi-square distribution with $n-1$ degrees of freedom.

(b) Calculate the value of $\frac{(n-1)s^2}{1.52}$ for these data.

(c) Assume that the population of thermostat temperatures follows a normal distribution. Use the test statistic $\frac{(n-1)s^2}{1.52}$ from part (b) and the chi-square distribution to test the hypotheses in part (a).

(d) For the test conducted in part (c), what is the smallest value of the test statistic that would have led to the rejection of the null hypothesis at the 5 percent significance level?

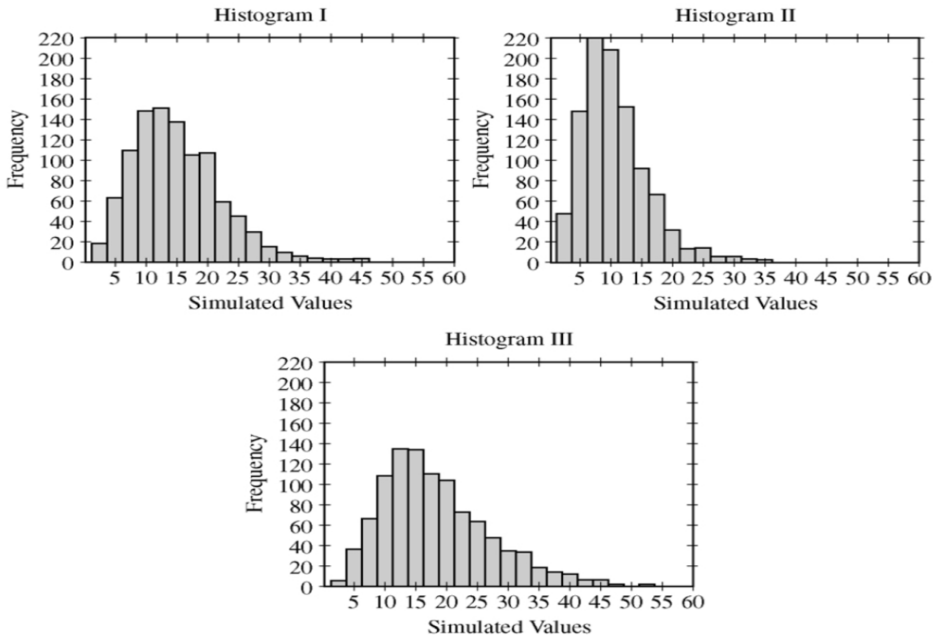
Mark this value of the test statistic on the graph of the chi-square distribution below. Indicate the region that contains all of the values that would have led to the rejection of the null hypothesis.



(e) Using simulation, 1,000 samples, each of size 10, were randomly generated from 3 populations with different variances. Each population was normally distributed with mean 68 and variance greater than 1.52.

The histograms below show the simulated sampling distribution of $\frac{(n-1)s^2}{1.52}$ for each population.

Mark the region identified in part (d) on each of the histograms below.



(f) Based on the regions that you marked in part (e), identify the simulated sampling distribution that corresponds to the population with the largest variance. Then identify the simulated sampling distribution that corresponds to the population with the smallest variance. Justify your choices.

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Question 4

Qualification: AP Statistics

Areas: Probability

Subtopics: Simulation

Paper: Part-A / Series: 2014 / Difficulty: Somewhat Challenging / Question Number: 2

2. Nine sales representatives, 6 men and 3 women, at a small company wanted to attend a national convention. There were only enough travel funds to send 3 people. The manager selected 3 people to attend and stated that the people were selected at random. The 3 people selected were women. There were concerns that no men were selected to attend the convention.

   (a) Calculate the probability that randomly selecting 3 people from a group of 6 men and 3 women will result in selecting 3 women.

   (b) Based on your answer to part (a), is there reason to doubt the manager's claim that the 3 people were selected at random? Explain.

   (c) An alternative to calculating the exact probability is to conduct a simulation to estimate the probability. A proposed simulation process is described below.

   > Each trial in the simulation consists of rolling three fair, six-sided dice, one die for each of the convention attendees. For each die, rolling a 1, 2, 3, or 4 represents selecting a man; rolling a 5 or 6 represents selecting a woman. After 1,000 trials, the number of times the dice indicate selecting 3 women is recorded.

   Does the proposed process correctly simulate the random selection of 3 women from a group of 9 people consisting of 6 men and 3 women? Explain why or why not.

Question 5

Qualification: AP Statistics

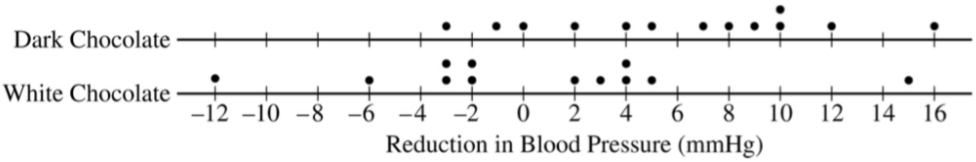
Areas: Hypothesis Testing

Subtopics: Determining the Median, Difference in Sample Means, Comparing Medians , Dotplots, Comparing Distributions, Simulation

Paper: Part-A / Series: 2022 / Difficulty: Somewhat Challenging / Question Number: 5

5. Studies have shown that foods rich in compounds known as flavonoids help lower blood pressure. Researchers conducted a study to investigate whether there was a greater reduction in blood pressure for people who consumed dark chocolate, which contains flavonoids, than people who consumed white chocolate, which does not contain flavonoids. Twenty-five healthy adults agreed to participate in the study and add 3.5 ounces of chocolate to their daily diets. Of the 25 participants, 13 were randomly assigned to the dark chocolate group and the rest were assigned to the white chocolate group. All participants had their blood pressure recorded, in millimeters of mercury (mmHg), before adding chocolate to their daily diets and again 30 days after adding chocolate to their daily diets.

The reduction in blood pressure (before minus after) for each of the participants in the two groups is shown in the dotplots below.

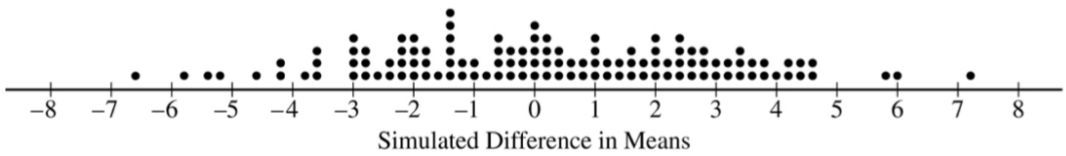


(a) Determine and compare the medians of the reduction in blood pressure for the two groups.

The researchers found the mean reduction in blood pressure for those who consumed dark chocolate is $\bar{x}_{dark} = 6.08$ mmHg and the mean reduction in blood pressure for those who consumed white chocolate is $\bar{x}_{white} = 0.42$ mmHg.

(b) One researcher indicated that because the difference in sample means of 5.66 mmHg is greater than 0 there is convincing statistical evidence to conclude that the population mean blood pressure reduction for those who consume dark chocolate is greater than for those who consume white chocolate. Why might the researcher's conclusion, based only on the difference in sample means of 5.66 mmHg, not necessarily be true?

A simulation was conducted to investigate whether there is a greater reduction of blood pressure for those who consume dark chocolate than for those who consume white chocolate. The simulation was conducted under the assumption that no difference exists. The results of 120 trials of the simulation are shown in the following dotplot.



(c) Use the results of the simulation to determine whether the results from the 25 participants in the study provide convincing statistical evidence, at a 5 percent level of significance, that adding dark chocolate to a daily diet will result in a greater reduction in blood pressure, on average, than adding white chocolate to a daily diet. Justify your answer.

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