

## 2019 AP<sup>®</sup> STATISTICS FREE-RESPONSE QUESTIONS

3. A medical researcher surveyed a large group of men and women about whether they take medicine as prescribed. The responses were categorized as never, sometimes, or always. The relative frequency of each category is shown in the table.

	Never	Sometimes	Always	Total
Men	0.0564	0.2016	0.2120	0.4700
Women	0.0636	0.1384	0.3280	0.5300
Total	0.1200	0.3400	0.5400	1.0000

- (a) One person from those surveyed will be selected at random.
- What is the probability that the person selected will be someone whose response is never and who is a woman?
  - What is the probability that the person selected will be someone whose response is never or who is a woman?
  - What is the probability that the person selected will be someone whose response is never given that the person is a woman?
- (b) For the people surveyed, are the events of being a person whose response is never and being a woman independent? Justify your answer.
- (c) Assume that, in a large population, the probability that a person will always take medicine as prescribed is 0.54. If 5 people are selected at random from the population, what is the probability that at least 4 of the people selected will always take medicine as prescribed? Support your answer.

## 2018 AP<sup>®</sup> STATISTICS FREE-RESPONSE QUESTIONS

3. Approximately 3.5 percent of all children born in a certain region are from multiple births (that is, twins, triplets, etc.). Of the children born in the region who are from multiple births, 22 percent are left-handed. Of the children born in the region who are from single births, 11 percent are left-handed.
- (a) What is the probability that a randomly selected child born in the region is left-handed?
  - (b) What is the probability that a randomly selected child born in the region is a child from a multiple birth, given that the child selected is left-handed?
  - (c) A random sample of 20 children born in the region will be selected. What is the probability that the sample will have at least 3 children who are left-handed?

## 2014 AP<sup>®</sup> STATISTICS FREE-RESPONSE QUESTIONS

2. Nine sales representatives, 6 men and 3 women, at a small company wanted to attend a national convention. There were only enough travel funds to send 3 people. The manager selected 3 people to attend and stated that the people were selected at random. The 3 people selected were women. There were concerns that no men were selected to attend the convention.
- (a) Calculate the probability that randomly selecting 3 people from a group of 6 men and 3 women will result in selecting 3 women.
- (b) Based on your answer to part (a), is there reason to doubt the manager's claim that the 3 people were selected at random? Explain.
- (c) An alternative to calculating the exact probability is to conduct a simulation to estimate the probability. A proposed simulation process is described below.

Each trial in the simulation consists of rolling three fair, six-sided dice, one die for each of the convention attendees. For each die, rolling a 1, 2, 3, or 4 represents selecting a man; rolling a 5 or 6 represents selecting a woman. After 1,000 trials, the number of times the dice indicate selecting 3 women is recorded.

Does the proposed process correctly simulate the random selection of 3 women from a group of 9 people consisting of 6 men and 3 women? Explain why or why not.

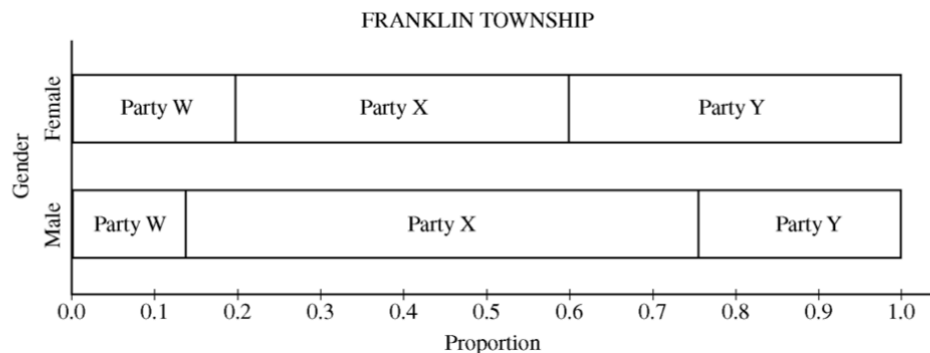
**2011 AP® STATISTICS FREE-RESPONSE QUESTIONS**

2. The table below shows the political party registration by gender of all 500 registered voters in Franklin Township.

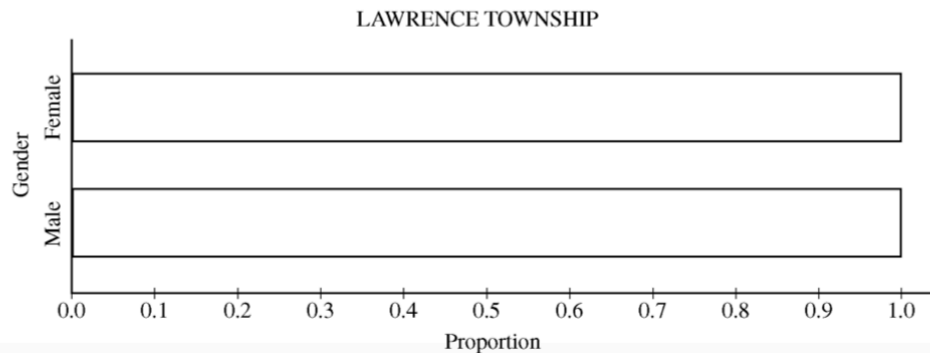
PARTY REGISTRATION–FRANKLIN TOWNSHIP

	Party W	Party X	Party Y	Total
Female	60	120	120	300
Male	28	124	48	200
Total	88	244	168	500

- (a) Given that a randomly selected registered voter is a male, what is the probability that he is registered for Party Y?
- (b) Among the registered voters of Franklin Township, are the events “is a male” and “is registered for Party Y” independent? Justify your answer based on probabilities calculated from the table above.
- (c) One way to display the data in the table is to use a segmented bar graph. The following segmented bar graph, constructed from the data in the party registration–Franklin Township table, shows party-registration distributions for males and females in Franklin Township.



In Lawrence Township, the proportions of all registered voters for Parties W, X, and Y are the same as for Franklin Township, and party registration is independent of gender. Complete the graph below to show the distributions of party registration by gender in Lawrence Township.



5. An advertising agency in a large city is conducting a survey of adults to investigate whether there is an association between highest level of educational achievement and primary source for news. The company takes a random sample of 2,500 adults in the city. The results are shown in the table below.

Primary Source for News	HIGHEST LEVEL OF EDUCATIONAL ACHIEVEMENT			Total
	Not High School Graduate	High School Graduate But Not College Graduate	College Graduate	
Newspapers	49	205	188	442
Local television	90	170	75	335
Cable television	113	496	147	756
Internet	41	401	245	687
None	77	165	38	280
Total	370	1,437	693	2,500

- (a) If an adult is to be selected at random from this sample, what is the probability that the selected adult is a college graduate or obtains news primarily from the internet?
- (b) If an adult who is a college graduate is to be selected at random from this sample, what is the probability that the selected adult obtains news primarily from the internet?
- (c) When selecting an adult at random from the sample of 2,500 adults, are the events “is a college graduate” and “obtains news primarily from the internet” independent? Justify your answer.
- (d) The company wants to conduct a statistical test to investigate whether there is an association between educational achievement and primary source for news for adults in the city. What is the name of the statistical test that should be used?

What are the appropriate degrees of freedom for this test?

## 2009 AP<sup>®</sup> STATISTICS FREE-RESPONSE QUESTIONS (Form B)

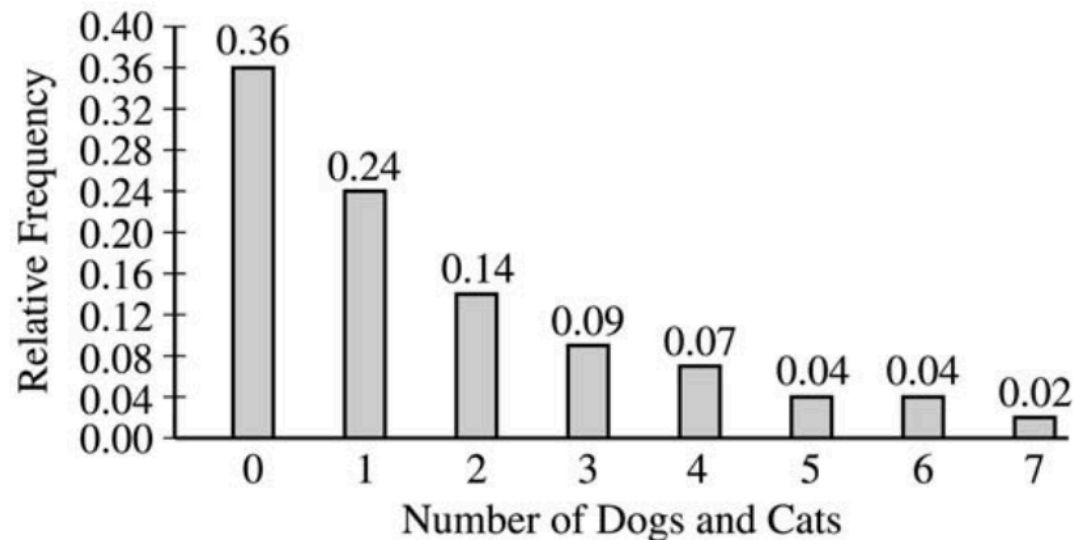
2. The ELISA tests whether a patient has contracted HIV. The ELISA is said to be positive if it indicates that HIV is present in a blood sample, and the ELISA is said to be negative if it does not indicate that HIV is present in a blood sample. Instead of directly measuring the presence of HIV, the ELISA measures levels of antibodies in the blood that should be elevated if HIV is present. Because of variability in antibody levels among human patients, the ELISA does not always indicate the correct result.

As part of a training program, staff at a testing lab applied the ELISA to 500 blood samples known to contain HIV. The ELISA was positive for 489 of those blood samples and negative for the other 11 samples. As part of the same training program, the staff also applied the ELISA to 500 other blood samples known to not contain HIV. The ELISA was positive for 37 of those blood samples and negative for the other 463 samples.

- (a) When a new blood sample arrives at the lab, it will be tested to determine whether HIV is present. Using the data from the training program, estimate the probability that the ELISA would be positive when it is applied to a blood sample that does not contain HIV.
- (b) Among the blood samples examined in the training program that provided positive ELISA results for HIV, what proportion actually contained HIV?
- (c) When a blood sample yields a positive ELISA result, two more ELISAs are performed on the same blood sample. If at least one of the two additional ELISAs is positive, the blood sample is subjected to a more expensive and more accurate test to make a definitive determination of whether HIV is present in the sample. Repeated ELISAs on the same sample are generally assumed to be independent. Under the assumption of independence, what is the probability that a new blood sample that comes into the lab will be subjected to the more expensive test if that sample does not contain HIV?

## 2007 AP<sup>®</sup> STATISTICS FREE-RESPONSE QUESTIONS (Form B)

2. The graph below displays the relative frequency distribution for  $X$ , the total number of dogs and cats owned per household, for the households in a large suburban area. For instance, 14 percent of the households own 2 of these pets.



- (a) According to a local law, each household in this area is prohibited from owning more than 3 of these pets. If a household in this area is selected at random, what is the probability that the selected household will be in violation of this law? Show your work.
- (b) If 10 households in this area are selected at random, what is the probability that exactly 2 of them will be in violation of this law? Show your work.
- (c) The mean and standard deviation of  $X$  are 1.65 and 1.851, respectively. Suppose 150 households in this area are to be selected at random and  $\bar{X}$ , the mean number of dogs and cats per household, is to be computed. Describe the sampling distribution of  $\bar{X}$ , including its shape, center, and spread.

4. Two antibiotics are available as treatment for a common ear infection in children.
- Antibiotic A is known to effectively cure the infection 60 percent of the time. Treatment with antibiotic A costs \$50.
  - Antibiotic B is known to effectively cure the infection 90 percent of the time. Treatment with antibiotic B costs \$80.

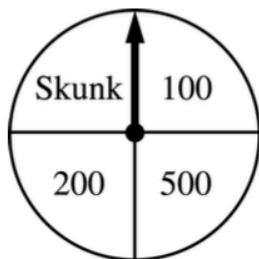
The antibiotics work independently of one another. Both antibiotics can be safely administered to children. A health insurance company intends to recommend one of the following two plans of treatment for children with this ear infection.

- Plan I: Treat with antibiotic A first. If it is not effective, then treat with antibiotic B.
  - Plan II: Treat with antibiotic B first. If it is not effective, then treat with antibiotic A.
- (a) If a doctor treats a child with an ear infection using plan I, what is the probability that the child will be cured?
- If a doctor treats a child with an ear infection using plan II, what is the probability that the child will be cured?
- (b) Compute the expected cost per child when plan I is used for treatment.  
Compute the expected cost per child when plan II is used for treatment.
- (c) Based on the results in parts (a) and (b), which plan would you recommend?  
Explain your recommendation.





## 2003 AP<sup>®</sup> STATISTICS FREE-RESPONSE QUESTIONS (Form B)



5. Contestants on a game show spin a wheel like the one shown in the figure above. Each of the four outcomes on this wheel is equally likely and outcomes are independent from one spin to the next.
- The contestant spins the wheel.
  - If the result is a skunk, no money is won and the contestant's turn is finished.
  - If the result is a number, the corresponding amount in dollars is won. The contestant can then stop with those winnings or can choose to spin again, and his or her turn continues.
  - If the contestant spins again and the result is a skunk, all of the money earned on that turn is lost and the turn ends.
  - The contestant may continue adding to his or her winnings until he or she chooses to stop or until a spin results in a skunk.
- (a) What is the probability that the result will be a number on all of the first three spins of the wheel?
- (b) Suppose a contestant has earned \$800 on his or her first three spins and chooses to spin the wheel again. What is the expected value of his or her total winnings for the four spins?
- (c) A contestant who lost at this game alleges that the wheel is not fair. In order to check on the fairness of the wheel, the data in the table below were collected for 100 spins of this wheel.

Result	Skunk	\$100	\$200	\$500
Frequency	33	21	20	26

Based on these data, can you conclude that the four outcomes on this wheel are not equally likely? Give appropriate statistical evidence to support your answer.

## 2003 AP<sup>®</sup> STATISTICS FREE-RESPONSE QUESTIONS (Form B)

2. A simple random sample of adults living in a suburb of a large city was selected. The age and annual income of each adult in the sample were recorded. The resulting data are summarized in the table below.

Age Category	Annual Income			Total
	\$25,000-\$35,000	\$35,001-\$50,000	Over \$50,000	
21-30	8	15	27	50
31-45	22	32	35	89
46-60	12	14	27	53
Over 60	5	3	7	15
Total	47	64	96	207

- (a) What is the probability that a person chosen at random from those in this sample will be in the 31-45 age category?
- (b) What is the probability that a person chosen at random from those in this sample whose incomes are over \$50,000 will be in the 31-45 age category? Show your work.
- (c) Based on your answers to parts (a) and (b), is annual income independent of age category for those in this sample? Explain.

5. Die A has four 9's and two 0's on its faces. Die B has four 3's and two 11's on its faces. When either of these dice is rolled, each face has an equal chance of landing on top. Two players are going to play a game. The first player selects a die and rolls it. The second player rolls the remaining die. The winner is the player whose die has the higher number on top.
- Suppose you are the first player and you want to win the game. Which die would you select? Justify your answer.
  - Suppose the player using die A receives 45 tokens each time he or she wins the game. How many tokens must the player using die B receive each time he or she wins in order for this to be a fair game? Explain how you found your answer.
- (A fair game is one in which the player using die A and the player using die B both end up with the same number of tokens in the long run.)

3. A laboratory test for the detection of a certain disease gives a positive result 5 percent of the time for people who do not have the disease. The test gives a negative result 0.3 percent of the time for people who have the disease. Large-scale studies have shown that the disease occurs in about 2 percent of the population.
- (a) What is the probability that a person selected at random would test positive for this disease? Show your work.
- (b) What is the probability that a person selected at random who tests positive for the disease does not have the disease? Show your work.

At a certain college 10% of female students in business administration take one mathematics course, 20% take two, 30% take three, and the rest take four. Of those taking just one mathematics course, 15% successfully complete all their business courses on the first attempt, while the success rate rises to 25%, 55%, and 85%, respectively, for those taking two, three, and four mathematics courses.

- (a) What is the probability that a randomly chosen female business administration major will successfully complete all her business courses on the first attempt? Show your work.
- (b) If a randomly chosen female business administration student successfully completes all her business courses on the first attempt, what is the probability that she has taken at least three mathematics courses? Show your work.