AP® Calculus BC Practice Exam

From the 2014 Administration

Contents

Note: This publication shows the page numbers that appeared in the *2013–14 AP Exam Instructions* book and in the actual exam. This publication was not repaginated to begin with page 1.

Exam Instructions

The following contains instructions taken from the *2013–14 AP Exam Instructions* book.

AP® Calculus AB/BC Exam

Regularly Scheduled Exam Date: Wednesday morning, May 7, 201

Late-Testing Exam Date: Thursday morning, May 22, 2014

Section I Total Time, Calculus AB: 1 hr. 45 min. Section II Total Time, Calculus AB: 1 hr. 30 min.
Section I Total Time, Calculus BC: 1 hr. 45 min. Section II Total Time, Calculus BC: 1 hr. 30 min.

What Proctors Need to Bring to This Exam

- Exam packets
- Answer sheets
- AP Student Packs
- *2013-14 AP Coordinator's Manual*
- This book — *AP Exam Instructions*
- School Code and Home-School/Self-Study Codes
- Extra graphing calculators
- Pencil sharpener
- Extra No. 2 pencils with erasers
- Extra pens with black or dark blue ink
- Extra paper
- Stapler
- Watch
- Signs for the door to the testing room
  - "Exam in Progress"
  - "Cell phones are prohibited in the testing room"

If you are giving the regularly scheduled AP Calculus AB or BC Exam:

- You may seat students **four feet (approximately 1.2 meters) apart** because these exams have scrambled multiple-choice sections. This allows you to test more students in fewer testing rooms.
- See page 10 for a sample seating plan, including form codes and serial numbers, that shows how exams should be distributed to ensure that students seated next to each other are not given the same form of the exam.
- Administrators and proctors must continue to be vigilant about opportunities for cheating.

If you are giving the alternate AP Calculus AB or BC Exam for late testing:

- You must seat students **no less than five feet (approximately 1.5 meters) apart** because these exams do not have scrambled multiple-choice sections.

Graphing calculators are required to answer some of the questions on the AP Calculus Exams. Before starting the exam administration, make sure each student has a graphing calculator from the approved list on page 45 of the *2013-14 AP Coordinator's Manual*. If a student does not have a graphing calculator from the approved list, you may provide one from your supply. If the student does not want to use the calculator you provide or does not want to use a calculator at all, he or she must hand copy, date, and sign the release statement on page 43 of the *2013-14 AP Coordinator's Manual*.

During the administration of Section I, Part B, and Section II, Part A, students may have no more than two graphing calculators on their desks. Calculators may not be shared. **Calculator memories do not need to be cleared before or after the exam.** Students with Hewlett-Packard 48–50 Series and Casio FX-9860 graphing calculators may use cards designed for use with these

calculators. Proctors should make sure infrared ports (Hewlett-Packard) are not facing each other. **Since graphing calculators can be used to store data, including text, proctors should monitor that students are using their calculators appropriately. Attempts by students to use the calculator to remove exam questions and/or answers from the room may result in the cancellation of AP Exam scores.**

The AP Calculus AB Exam and the AP Calculus BC Exam should be administered simultaneously. They may be administered in separate rooms, or in the same room if it is more convenient.

SECTION I: Multiple Choice

Do not begin the exam instructions below until you have completed the appropriate General Instructions for your group.

These exams include survey questions. The time allowed for the survey questions is in addition to the actual test-taking time.

Make sure you begin the exams at the designated time.

If you are giving the regularly scheduled exam, say:

It is Wednesday morning, May 7, and you will be taking either the AP Calculus AB Exam or the AP Calculus BC Exam.

If you are giving the alternate exam for late testing, say:

It is Thursday morning, May 22, and you will be taking either the AP Calculus AB Exam or the AP Calculus BC Exam.

In a moment, you will open the packet that contains your exam materials.

By opening this packet, you agree to all of the AP Program's policies and procedures outlined in the *2013-14 Bulletin for AP Students and Parents*. Please check to make sure you have the correct exam: Calculus AB or Calculus BC. Raise your hand if you do not have the correct exam. . . .

You may now remove the shrinkwrap from your exam packet and take out the Section I booklet, but do not open the booklet or the shrinkwrapped Section II materials. Put the white seals aside. . . .

Carefully remove the AP Exam label found near the top left of your exam booklet cover. Now place it on page 1 of your answer sheet on the light blue box near the top right-hand corner that reads "AP Exam Label."

If students accidentally place the exam label in the space for the number label or vice versa, advise them to leave the labels in place. They should not try to remove the label; their exam will be processed correctly.

Read the statements on the front cover of Section I and look up when you have finished. . . .

Sign your name and write today's date. Look up when you have finished. . . .

Now print your full legal name where indicated. Are there any questions? . . .

Turn to the back cover and read it completely. Look up when you have finished. . . .

Are there any questions? . . .

Section I is the multiple-choice portion of the exam. You may never discuss these specific multiple-choice questions at any time in any form with anyone, including your teacher and other students. If you disclose these questions through any means, your AP Exam score will be canceled. Are there any questions? . . .

You must complete the answer sheet using a No. 2 pencil only. Mark all of your responses beginning on page 2 of your answer sheet, one response per question. Completely fill in the circles. If you need to erase, do so carefully and completely. No credit will be given for anything written in the exam booklet. Scratch paper is not allowed, but you may use the margins or any blank space in the exam booklet for scratch work.

Section I is divided into two parts. Each part is timed separately, and you may work on each part only during the time allotted for it. Calculators are not allowed in Part A. Please put your calculators under your chair. Are there any questions? . . .

You have 55 minutes for Part A. Part A questions are numbered 1 through 28. Mark your responses for these questions on page 2 of your answer sheet. Open your Section I booklet and begin.

Note Start Time here _____. Note Stop Time here _____. Check that students are marking their answers in pencil on page 2 of their answer sheets and that they are not looking beyond Part A. The line of A's at the top of each page will assist you in monitoring students' work. After 55 minutes, say:

Stop working on Part A and turn to page 22 in your Section I booklet. . . .

On that page, you should see an area marked "PLACE SEAL HERE." Making sure all of your other exam materials, including your answer sheet, are out of the way, take one of your seals and press it on that area and then fold the seal over the open edge to the front cover. Be sure you don't seal the Part B section of the booklet or let the seal touch anything except the marked areas. . . .

After all students have sealed Part A, say:

Graphing calculators are required for Part B. You may get your calculators from under your chair and place them on your desk. Part B questions are numbered 76 through 92. Fold your answer sheet so only page 3 is showing and mark your responses for these questions on that page. You have 50 minutes for Part B. You may begin.

Note Start Time here _____. Note Stop Time here _____. Check that students have sealed their booklets properly and are now working on Part B. The large B's in an alternating shaded pattern at the top of each page will assist you in monitoring their work. Proctors should

make sure that students are using their calculators appropriately. Proctors should also make sure Hewlett-Packard calculators' infrared ports are not facing each other. After 50 minutes, say:

> **Stop working and turn to page 38. You have 3 minutes to answer Questions 93–96. These are survey questions and will not affect your score. You may not go back to work on any of the exam questions. . . .**

Give students approximately 3 minutes to answer the survey questions. Then say:

> **Close your booklet and put your answer sheet on your desk, face up. Make sure you have your AP number label and an AP Exam label on page 1 of your answer sheet. I will now collect your answer sheet.**

Collect an answer sheet from each student. Check that each answer sheet has an AP number label and an AP Exam label. Then say:

> **Now you must seal your Section I booklet. Remove the remaining white seals from the backing and press one on each area of your exam booklet cover marked "PLACE SEAL HERE." Fold each seal over the back cover. When you have finished, place the booklet on your desk, face up. I will now collect your Section I booklet. . . .**

Collect a Section I booklet from each student. Check that each student has signed the front cover of the sealed Section I booklet.

There is a 10-minute break between Sections I and II. When all Section I materials have been collected and accounted for and you are ready for the break, say:

> **Please listen carefully to these instructions before we take a 10-minute break. Everything you placed under your chair at the beginning of the exam must stay there. Leave your shrinkwrapped Section II packet on top of your desk during the break. You are not allowed to consult teachers, other students, or textbooks about the exam during the break. You may not make phone calls, send text messages, use your calculators, check email, use a social networking site, or access any electronic or communication device. Remember, you are not allowed to discuss the multiple-choice section of this exam. If you do not follow these rules, your score could be canceled. Are there any questions? . . .**
>
> **You may begin your break. Testing will resume at _____.**

SECTION II: Free Response

After the break, say:

> **May I have everyone's attention? Place your Student Pack on your desk. . . .**
>
> **You may now remove the shrinkwrap from the Section II packet, but do not open the Section II exam booklet until you are told to do so. . . .**
>
> **Read the bulleted statements on the front cover of the exam booklet. Look up when you have finished. . . .**

Now place an AP number label on the shaded box. If you don't have any AP number labels, write your AP number in the box. Look up when you have finished. . . .

Read the last statement. . . .

Using your pen, print the first, middle and last initials of your legal name in the boxes and print today's date where indicated. This constitutes your signature and your agreement to the statements on the front cover. . . .

Turn to the back cover and complete Item 1 under "Important Identification Information." Print the first two letters of your last name and the first letter of your first name in the boxes. Look up when you have finished. . . .

In Item 2, print your date of birth in the boxes. . . .

In Item 3, write the school code you printed on the front of your Student Pack in the boxes. . . .

Read Item 4. . . .

Are there any questions? . . .

I need to collect the Student Pack from anyone who will be taking another AP Exam. You may keep it only if you are not taking any other AP Exams this year. If you have no other AP Exams to take, place your Student Pack under your chair now. . . .

While Student Packs are being collected, read the information on the back cover of the exam booklet, paying careful attention to the bulleted statements in the instructions. Do not open the exam booklet or break the seals in the exam booklet until you are told to do so. Look up when you have finished. . . .

Collect the Student Packs. Then say:

Are there any questions? . . .

Section II also has two parts that are timed separately. You are responsible for pacing yourself, and may proceed freely from one question to the next within each part. Graphing calculators are required for Part A, so you may keep your calculators on your desk. You must write your answers in the appropriate space in the exam booklet using a No. 2 pencil or a pen with black or dark blue ink. Do not break the seals for Part B at this time.
Are there any questions? . . .

You have 30 minutes to answer the questions in Part A. If you need more paper during the exam, raise your hand. At the top of each extra piece of paper you use, be sure to write only your AP number and the number of the question you are working on. Do not write your name. Open your exam booklet and begin.

Note Start Time here _____. Note Stop Time here _____. Check that students are working on Part A only and writing their answers in their exam booklets using pencils or pens with black or dark blue ink. The pages for the Part A questions are marked with large 1s or 2s at the top of each page to assist you in monitoring their work. After 20 minutes, say:

There are 10 minutes remaining in Part A.

After 10 minutes, say:

Stop working on Part A. Calculators are not allowed for Part B. Please put all of your calculators under your chair. . . .

Turn to page 13. You have 1 hour for Part B. During this time you may go back to Part A, but you may not use your calculator. Remember to show your work, and write your answer to each part of each problem in the appropriate space in the exam booklet. Are there any questions? . . .

Using your finger, break open the seals on Part B. Do not peel the seals away from the booklet. You may begin Part B.

Note Start Time here _____. Note Stop Time here _____. After 50 minutes, say:

There are 10 minutes remaining in Part B.

After 10 minutes, say:

Stop working and close your exam booklet. Place it on your desk, face up. . . .

If any students used extra paper for the free-response section, have those students staple the extra sheet/s to the first page corresponding to that question in their exam booklets. Then say:

Remain in your seat, without talking, while the exam materials are collected. . . .

Collect a Section II exam booklet from each student. Check for the following:

- Exam booklet front cover: The student placed an AP number label on the shaded box, and printed his or her initials and today's date.
- Exam booklet back cover: The student completed the "Important Identification Information" area.

When all exam materials have been collected and accounted for, return to students any electronic devices you may have collected before the start of the exam.

If you are giving the regularly scheduled exam, say:

You may not discuss or share these specific free-response questions with anyone unless they are released on the College Board website in about two days. Your AP score results will be available online in July.

If you are giving the alternate exam for late testing, say:

None of the questions in this exam may ever be discussed or shared in any way at any time. Your AP score results will be available online in July.

If any students completed the AP number card at the beginning of this exam, say:

Please remember to take your AP number card with you. You will need the information on this card to view your scores and order AP score reporting services online.

Then say:

You are now dismissed.

All exam materials should be put in secure storage until they are returned to the AP Program after your school's last administration. Before storing materials, check the "School Use Only" section on page 1 of the answer sheet and:

- Fill in the appropriate section number circle in order to access a separate AP Instructional Planning Report (for regularly scheduled exams only) or subject score roster at the class section or teacher level. See "Post-Exam Activities" in the *2013-14 AP Coordinator's Manual*.
- Check your list of students who are eligible for fee reductions and fill in the appropriate circle on their registration answer sheets.

Student Answer Sheet for the Multiple-Choice Section

Use this section to capture student responses. (Note that the following answer sheet is a sample, and may differ from one used in an actual exam.)

AP Answer Sheet 2014

AP Number Label
(from Student Pack)

B123456789T

AP Exam Label
(from Section I Booklet)

CollegeBoard

COMPLETE THIS AREA AT EVERY EXAM. USE NO. 2 PENCIL ONLY

To maintain the security of the exam and the validity of my AP score, I will allow no one else to see the multiple-choice questions. I will seal the multiple-choice booklet when asked to do so, and I will not discuss these questions with anyone at any time after completing the section. I am aware of and agree to the AP Program's policies and procedures as outlined in the *2013-14 Bulletin for AP Students and Parents*, including using testing accommodations (e.g., extended time, computer, etc.) only if I have been preapproved by College Board Services for Students with Disabilities.

A. SIGNATURE Sign your legal name as it will appear on your college applications. Date

B. LEGAL NAME Omit apostrophes, Jr., II.

Legal Last Name — First 15 Letters | Legal First Name — First 12 Letters | MI

C. YOUR AP NUMBER

D. EXAM DATE Month Day

E. EXAM START TIME AM PM

F. MULTIPLE-CHOICE BOOKLET SERIAL NUMBER S

G. ONLINE PROVIDER CODE

H. AP EXAM I AM TAKING USING THIS ANSWER SHEET Exam Name: Form: Form Code:

773898

SCHOOL USE ONLY
Section Number 1 2 3 4 5 6 7 8 9
Fee Reduction Granted 1 Option 1 2 Option 2

COMPLETE THIS AREA ONLY ONCE.

I. AREA CODE AND PHONE NUMBER

INTERNATIONAL PHONE

J. SCHOOL YOU ATTEND
SCHOOL CODE
School Name
City
State
Country

K. DATE OF BIRTH
Month: Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec
Day
Year

L. SOCIAL SECURITY NUMBER (Optional)

M. COLLEGE TO RECEIVE YOUR AP SCORE REPORT
COLLEGE CODE
Using the college code listed in the AP Student Pack, indicate the **ONE** college that you want to receive your AP score report.
College Name
City
State
Country

N. CURRENT GRADE LEVEL
- Not yet in 9th grade
- 9th
- 10th
- 11th
- 12th
- No longer in high school

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COMPLETE THIS AREA AT EACH EXAM (IF APPLICABLE)

O. SURVEY QUESTIONS — Answer the survey questions in the AP Student Pack. Do not put responses to exam questions in this sec.

1	A B C D E F G H I	4	A B C D E F G H I	7	A B C D E F G H I
2	A B C D E F G H I	5	A B C D E F G H I	8	A B C D E F G H I
3	A B C D E F G H I	6	A B C D E F G H I	9	A B C D E F G H I

P. LANGUAGE — Do not complete this section unless instructed to do so.

If this answer sheet is for the French Language and Culture, German Language and Culture, Italian Language and Culture, Spanish Language and Culture, or Spanish Literature and Culture Exam, please answer the following questions. Your responses will not affect your score.

1. Have you lived or studied for one month or more in a country where the language of the exam you are now taking is spoken?

○ Yes ○ No

2. Do you regularly speak or hear the language at home?

○ Yes ○ No

QUESTIONS 1–75

Indicate your answers to the exam questions in this section (pages 2 and 3). Mark only one response per question. If a question has only four answer options, do not mark option E. Answers written in the multiple-choice booklet will not be scored.

COMPLETE MARK ● EXAMPLES OF INCOMPLETE MARKS 

You must use a No. 2 pencil and marks must be complete. Do not use a mechanical pencil. It is very important that you fill in the entire circle darkly and completely. If you change your response, erase as completely as possible. Incomplete marks or erasures may affect your score.

1	A B C D E	26	A B C D E	51	A B C D E
2	A B C D E	27	A B C D E	52	A B C D E
3	A B C D E	28	A B C D E	53	A B C D E
4	A B C D E	29	A B C D E	54	A B C D E
5	A B C D E	30	A B C D E	55	A B C D E
6	A B C D E	31	A B C D E	56	A B C D E
7	A B C D E	32	A B C D E	57	A B C D E
8	A B C D E	33	A B C D E	58	A B C D E
9	A B C D E	34	A B C D E	59	A B C D E
10	A B C D E	35	A B C D E	60	A B C D E
11	A B C D E	36	A B C D E	61	A B C D E
12	A B C D E	37	A B C D E	62	A B C D E
13	A B C D E	38	A B C D E	63	A B C D E
14	A B C D E	39	A B C D E	64	A B C D E
15	A B C D E	40	A B C D E	65	A B C D E
16	A B C D E	41	A B C D E	66	A B C D E
17	A B C D E	42	A B C D E	67	A B C D E
18	A B C D E	43	A B C D E	68	A B C D E
19	A B C D E	44	A B C D E	69	A B C D E
20	A B C D E	45	A B C D E	70	A B C D E
21	A B C D E	46	A B C D E	71	A B C D E
22	A B C D E	47	A B C D E	72	A B C D E
23	A B C D E	48	A B C D E	73	A B C D E
24	A B C D E	49	A B C D E	74	A B C D E
25	A B C D E	50	A B C D E	75	A B C D E

DO NOT WRITE IN THIS AREA

QUESTIONS 76–120

Be sure each mark is dark and completely fills the circle. If a question has only four answer options, do not mark option E.

76	A B C D E	91	A B C D E	106	A B C D E
77	A B C D E	92	A B C D E	107	A B C D E
78	A B C D E	93	A B C D E	108	A B C D E
79	A B C D E	94	A B C D E	109	A B C D E
80	A B C D E	95	A B C D E	110	A B C D E
81	A B C D E	96	A B C D E	111	A B C D E
82	A B C D E	97	A B C D E	112	A B C D E
83	A B C D E	98	A B C D E	113	A B C D E
84	A B C D E	99	A B C D E	114	A B C D E
85	A B C D E	100	A B C D E	115	A B C D E
86	A B C D E	101	A B C D E	116	A B C D E
87	A B C D E	102	A B C D E	117	A B C D E
88	A B C D E	103	A B C D E	118	A B C D E
89	A B C D E	104	A B C D E	119	A B C D E
90	A B C D E	105	A B C D E	120	A B C D E

For Students Taking AP Biology

Write your answer in the boxes at the top of the griddable area and fill in the corresponding circles. Mark only one circle in any column. You will receive credit only if the circles are filled in correctly.

121 122 123 124 125 126

ETS USE ONLY

SELECTED MEDIA EXAMS	R	W	O	OTHER EXAMS	R	W	O
PT02				TOTAL			
PT03				Subscore (if applicable)			
PT04				Subscore (if applicable)			

Exam		0 1 2 3 4 5 6 7 8 9
		0 1 2 3 4 5 6 7 8 9
Exam		0 1 2 3 4 5 6 7 8 9
		0 1 2 3 4 5 6 7 8 9

DO NOT WRITE IN THIS AREA

COMPLETE THIS AREA ONLY ONCE.

Q. YOUR MAILING ADDRESS Use the address abbreviations from your AP Student Pack. Fill in only one circle per column. Indicate a space in your address by leaving a blank box; do not grid that column.

STREET ADDRESS (include street number, street name, apartment number, etc.)

CITY

ZIP OR POSTAL CODE

COUNTRY CODE

STATE

AK	HI	MI	NY	VT	
AL	IA	MN	OH	WA	
AR	ID	MS	OR	WV	
AZ	IL	MT	PA	WY	
CA	IN	NC	RI	Puerto Rico	
CO	KS	ND	SC		
CT	KY	NE	SD	AA	
DC	LA	NH	TN	AE	
DE	MA	NJ	TX	AP	
FL	MD	NM	UT	Other	
GA	ME	NV	VA		

R. FOR STUDENTS OUTSIDE THE UNITED STATES ONLY If your address does not fit in the spaces provided in Item Q, fill in as many circles as you can, then fill in the circle in Item R and print the remainder of your address in the spaces provided.

Address | City | State or Province | Country | ZIP or Postal Code

T. EMAIL ADDRESS By providing your email address, you are granting the College Board permission to use your email in accordance with the policies in the *2013-14 Bulletin for AP Students and Parents*.

S. STUDENT IDENTIFIER (Student ID Number)

U. STUDENT SEARCH SERVICE®

Colleges and scholarship programs may request your information to inform you of educational opportunities and financial aid.

Would you like us to supply your information?

- Yes
- No

If you don't answer and previously chose to participate in this service, we will continue providing your information.

V. SEX

- Female
- Male

W. WHICH LANGUAGE DO YOU KNOW BEST?

- English
- English and another language about the same
- Another language

X. ETHNICITY/RACE

- American Indian or Alaska Native
- Asian, Asian American or Pacific Islander
- Black or African American
- Mexican or Mexican American
- Puerto Rican
- Other Hispanic, Latino or Latin American
- White
- Other

Y. PARENTAL EDUCATION LEVEL

Father/Male Guardian	Mother/Female Guardian	
○	○	Grade school
○	○	Some high school
○	○	High school diploma or equivalent
○	○	Business or trade school
○	○	Some college
○	○	Associate or two-year degree
○	○	Bachelor's or four-year degree
○	○	Some graduate or professional school
○	○	Graduate or professional degree

Section I: Multiple-Choice Questions

This is the multiple-choice section of the 2014 AP exam.
It includes cover material and other administrative instructions
to help familiarize students with the mechanics of the exam.
(Note that future exams may differ in look from the following content.)

AP® Calculus BC Exam

SECTION I: Multiple Choice

2014

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour, 45 minutes
Number of Questions
45
Percent of Total Score
50%
Writing Instrument
Pencil required

Part A

Number of Questions
28
Time
55 minutes
Electronic Device
None allowed

Part B

Number of Questions
17
Time
50 minutes
Electronic Device
Graphing calculator required

PLACE SEAL HERE

Instructions

Section I of this exam contains 45 multiple-choice questions and 4 survey questions. For Part A, fill in only the circles for numbers 1 through 28 on page 2 of the answer sheet. For Part B, fill in only the circles for numbers 76 through 92 on page 3 of the answer sheet. The survey questions are numbers 93 through 96.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding circle on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question	Sample Answer
Chicago is a (A) state (B) city (C) country (D) continent (E) village	Ⓐ ● Ⓒ Ⓓ Ⓔ

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

Your total score on the multiple-choice section is based only on the number of questions answered correctly. Points are not deducted for incorrect answers or unanswered questions.

Form I
Form Code 4JBP6-S

CALCULUS BC
SECTION I, Part A
Time—55 minutes
Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

1. $\displaystyle\int \frac{x^3 + 5}{x^2}\,dx =$

(A) $1 - \dfrac{10}{x^3} + C$

(B) $\dfrac{3x}{4} + \dfrac{15}{x^2} + C$

(C) $\dfrac{x^2}{2} - \dfrac{5}{x} + C$

(D) $\dfrac{x^2}{2} - \dfrac{5}{3x^3} + C$

(E) $-\dfrac{x^3}{4} - 5 + C$

---

2. What is the slope of the line tangent to the graph of $y = \ln(2x)$ at the point where $x = 4$?

(A) $\dfrac{1}{8}$ (B) $\dfrac{1}{4}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{3}{4}$ (E) 4

3. $\lim_{x \to 0} \dfrac{x^2}{1 - \cos x}$ is

(A) -2 (B) 0 (C) 1 (D) 2 (E) nonexistent

---

4. $\displaystyle\int \frac{1}{x^2 - 7x + 10}\,dx =$

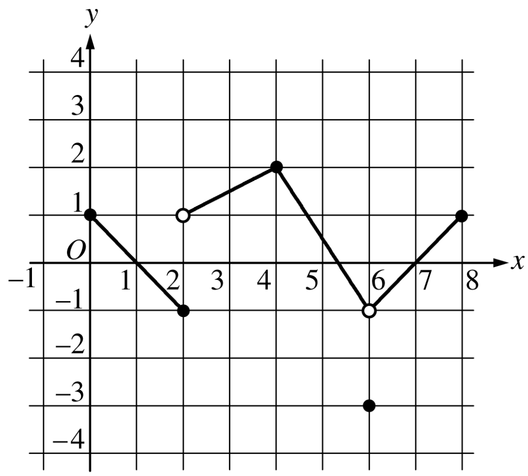
(A) $\ln\left|(x-2)(x-5)\right| + C$

(B) $\frac{1}{3}\ln\left|(x-2)(x-5)\right| + C$

(C) $\frac{1}{3}\ln\left|\dfrac{2x-7}{(x-2)(x-5)}\right| + C$

(D) $\frac{1}{3}\ln\left|\dfrac{x-2}{x-5}\right| + C$

(E) $\frac{1}{3}\ln\left|\dfrac{x-5}{x-2}\right| + C$



5. The figure above shows the graph of the function f. Which of the following statements are true?

I. $\lim_{x \to 2^-} f(x) = f(2)$

II. $\lim_{x \to 6^-} f(x) = \lim_{x \to 6^+} f(x)$

III. $\lim_{x \to 6} f(x) = f(6)$

(A) II only

(B) III only

(C) I and II only

(D) II and III only

(E) I, II, and III

6. The infinite series $\sum_{k=1}^{\infty} a_k$ has nth partial sum $S_n = (-1)^{n+1}$ for $n \geq 1$. What is the sum of the series $\sum_{k=1}^{\infty} a_k$?

(A) -1

(B) 0

(C) $\frac{1}{2}$

(D) 1

(E) The series diverges.

7. Let f be the function defined by $f(x) = \begin{cases} x^2 + 2 & \text{for } x \le 3, \\ 6x + k & \text{for } x > 3. \end{cases}$

If f is continuous at $x = 3$, what is the value of k ?

(A) -7 (B) 2 (C) 3 (D) 7 (E) There is no such value of k.

---

8. $\int_0^1 x\sqrt{1 + 8x^2}\, dx =$

(A) $\frac{1}{24}$ (B) $\frac{13}{12}$ (C) $\frac{9}{8}$ (D) $\frac{52}{3}$ (E) 18

9. The function f has a first derivative given by $f'(x) = x(x-3)^2(x+1)$. At what values of x does f have a relative maximum?

(A) -1 only (B) 0 only (C) -1 and 0 only (D) -1 and 3 only (E) -1, 0, and 3

10. What is the sum of the series $\sum_{n=1}^{\infty} \frac{(-2)^n}{e^{n+1}}$?

(A) $\frac{-2}{e^2 - 2e}$ (B) $\frac{-2}{e^2 + 2e}$ (C) $\frac{-2}{e + 2}$ (D) $\frac{e}{e + 2}$ (E) The series diverges.

GO ON TO THE NEXT PAGE.

$$f(x) = \begin{cases} 2x + 5 & \text{for } x < -1 \\ -x^2 + 6 & \text{for } x \geq -1 \end{cases}$$

11. If f is the function defined above, then $f'(-1)$ is

(A) -2 (B) 2 (C) 3 (D) 5 (E) nonexistent

---

12. Let f be the function given by $f(x) = 9^x$. If four subintervals of equal length are used, what is the value of the right Riemann sum approximation for $\int_0^2 f(x)\,dx$?

(A) 20 (B) 40 (C) 60 (D) 80 (E) 120

13. A rectangular area is to be enclosed by a wall on one side and fencing on the other three sides. If 18 meters of fencing are used, what is the maximum area that can be enclosed?

(A) $\frac{9}{2}$ m^2 (B) $\frac{81}{4}$ m^2 (C) 27 m^2 (D) 40 m^2 (E) $\frac{81}{2}$ m^2

---

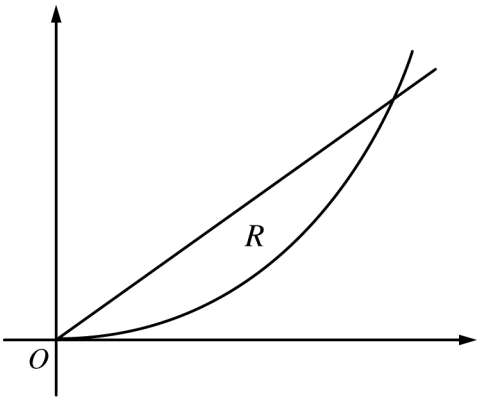
14. Let $P(x) = 3 - 3x^2 + 6x^4$ be the fourth-degree Taylor polynomial for the function f about $x = 0$. What is the value of $f^{(4)}(0)$?

(A) 0 (B) $\frac{1}{4}$ (C) 6 (D) 24 (E) 144

15. Suppose $\ln x - \ln y = y - 4$, where y is a differentiable function of x and $y = 4$ when $x = 4$. What is the value of $\frac{dy}{dx}$ when $x = 4$?

(A) 0 (B) $\frac{1}{5}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) $\frac{17}{5}$

GO ON TO THE NEXT PAGE.



16. Let R be the region in the first quadrant that is bounded by the polar curves $r = \theta$ and $\theta = k$, where k is a constant, $0 < k < \frac{\pi}{2}$, as shown in the figure above. What is the area of R in terms of k?

(A) $\frac{k^3}{6}$ (B) $\frac{k^3}{3}$ (C) $\frac{k^3}{2}$ (D) $\frac{k^2}{4}$ (E) $\frac{k^2}{2}$

17. Which of the following is the Maclaurin series for e^{3x} ?

(A) $1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$

(B) $3 + 9x + \frac{27x^2}{2} + \frac{81x^3}{3!} + \frac{243x^4}{4!} + \cdots$

(C) $1 - 3x + \frac{9x^2}{2} - \frac{27x^3}{3!} + \frac{81x^4}{4!} - \cdots$

(D) $1 + 3x + \frac{3x^2}{2} + \frac{3x^3}{3!} + \frac{3x^4}{4!} + \cdots$

(E) $1 + 3x + \frac{9x^2}{2} + \frac{27x^3}{3!} + \frac{81x^4}{4!} + \cdots$

GO ON TO THE NEXT PAGE.

18. $\displaystyle\int_1^{\infty} \frac{x^2}{\left(x^3+2\right)^2}\,dx$ is

(A) $-\frac{1}{9}$ (B) $\frac{1}{9}$ (C) $\frac{1}{3}$ (D) 1 (E) divergent

---

19. For what values of x does the graph of $y = 3x^5 + 10x^4$ have a point of inflection?

(A) $x = -\frac{8}{3}$ only

(B) $x = -2$ only

(C) $x = 0$ only

(D) $x = 0$ and $x = -\frac{8}{3}$

(E) $x = 0$ and $x = -2$

20. If $f'(x) = \dfrac{(x-2)^3\left(x^2-4\right)}{16}$ and $g(x) = f\left(x^2-1\right)$, what is $g'(2)$?

(A) 2 (B) $\frac{5}{4}$ (C) $\frac{5}{8}$ (D) $\frac{5}{16}$ (E) 0

---

x	1	3	5	7
$f(x)$	4	6	7	5
$f'(x)$	2	1	0	–1

21. The table above gives selected values for a differentiable function f and its first derivative. Using a left Riemann sum with 3 subintervals of equal length, which of the following is an approximation of the length of the graph of f on the interval $[1, 7]$?

(A) 6 (B) 34 (C) $2\sqrt{3} + 2\sqrt{2} + 2$ (D) $2\sqrt{5} + 2\sqrt{2} + 2$ (E) $2\sqrt{5} + 4\sqrt{2} + 2$

GO ON TO THE NEXT PAGE.

22. What is the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n \cdot 2^n}$?

(A) $1 < x < 5$

(B) $1 \le x < 5$

(C) $1 \le x \le 5$

(D) $2 < x < 4$

(E) $2 \le x \le 4$

---

23. What is the particular solution to the differential equation $\frac{dy}{dx} = xy^2$ with the initial condition $y(2) = 1$?

(A) $y = e^{\frac{x^2}{2} - 2}$

(B) $y = e^{\frac{x^2}{2}}$

(C) $y = -\frac{2}{x^2}$

(D) $y = \frac{2}{6 - x^2}$

(E) $y = \frac{6 - x^2}{2}$

GO ON TO THE NEXT PAGE.

24. Which of the following series converge?

I. $1 + (-1) + 1 + \cdots + (-1)^{n-1} + \cdots$

II. $1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1} + \cdots$

III. $1 + \frac{1}{3} + \frac{1}{3^2} + \cdots + \frac{1}{3^{n-1}} + \cdots$

(A) I only

(B) II only

(C) III only

(D) II and III only

(E) I, II, and III

---

25. What is the slope of the line tangent to the polar curve $r = \cos\theta$ at the point where $\theta = \frac{\pi}{6}$?

(A) $-\sqrt{3}$ (B) $-\frac{1}{\sqrt{3}}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{\sqrt{3}}{2}$ (E) $\sqrt{3}$

26. For $x > 0$, $\dfrac{d}{dx}\displaystyle\int_1^{\sqrt{x}} \frac{1}{1+t^2}\,dt =$

(A) $\dfrac{1}{2\sqrt{x}(1+x)}$ (B) $\dfrac{1}{2\sqrt{x}\left(1+\sqrt{x}\right)}$ (C) $\dfrac{1}{1+x}$ (D) $\dfrac{\sqrt{x}}{1+x}$ (E) $\dfrac{1}{1+\sqrt{x}}$

GO ON TO THE NEXT PAGE.

27. What is the coefficient of x^2 in the Taylor series for $\sin^2 x$ about $x = 0$?

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

---

28. The function h is given by $h(x) = x^5 + 3x - 2$ and $h(1) = 2$. If h^{-1} is the inverse of h, what is the value of $\left(h^{-1}\right)'(2)$?

(A) $\frac{1}{83}$ (B) $\frac{1}{8}$ (C) $\frac{1}{2}$ (D) 1 (E) 8

CALCULUS BC
SECTION I, Part B
Time—50 minutes
Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76–92.

YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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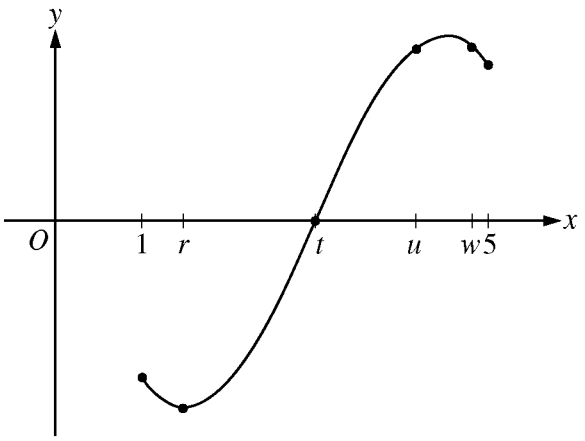

Graph of f

76. The figure above shows the graph of the differentiable function f for $1 \le x \le 5$. Which of the following could be the x-coordinate of a point at which the line tangent to the graph of f is parallel to the secant line through the points $(1, f(1))$ and $(5, f(5))$?

(A) r (B) t (C) u (D) w (E) There is no such point.

77. The number of antibodies y in a patient's bloodstream at time t is increasing according to a logistic differential equation. Which of the following could be the differential equation?

(A) $\frac{dy}{dt} = 0.025t$

(B) $\frac{dy}{dt} = 0.025t(5000 - t)$

(C) $\frac{dy}{dt} = 0.025y$

(D) $\frac{dy}{dt} = 0.025(5000 - y)$

(E) $\frac{dy}{dt} = 0.025y(5000 - y)$

GO ON TO THE NEXT PAGE.

78. What is the area of the region enclosed by the graphs of $y = \dfrac{1}{1+x^2}$ and $y = x^2 - \dfrac{1}{3}$?

(A) 0.786 (B) 0.791 (C) 1.582 (D) 1.837 (E) 1.862

---

79. A vase has the shape obtained by revolving the curve $y = 2 + \sin x$ from $x = 0$ to $x = 5$ about the x-axis, where x and y are measured in inches. What is the volume, in cubic inches, of the vase?

(A) 10.716 (B) 25.501 (C) 33.666 (D) 71.113 (E) 80.115

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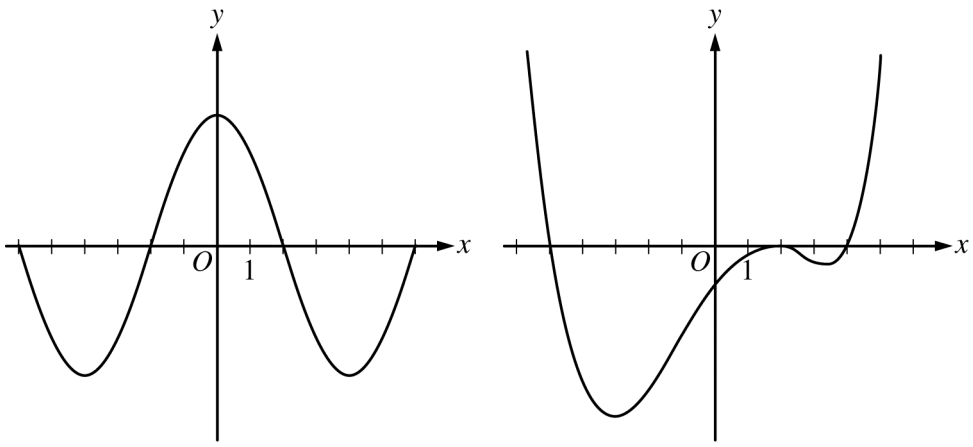


Graph of f Graph of g

80. The graphs of two differentiable functions f and g are shown above. Given $p(x) = f(x)g(x)$, which of the following statements about $p'(-2)$ is true?

(A) $p'(-2) < 0$

(B) $p'(-2) = 0$

(C) $p'(-2) > 0$

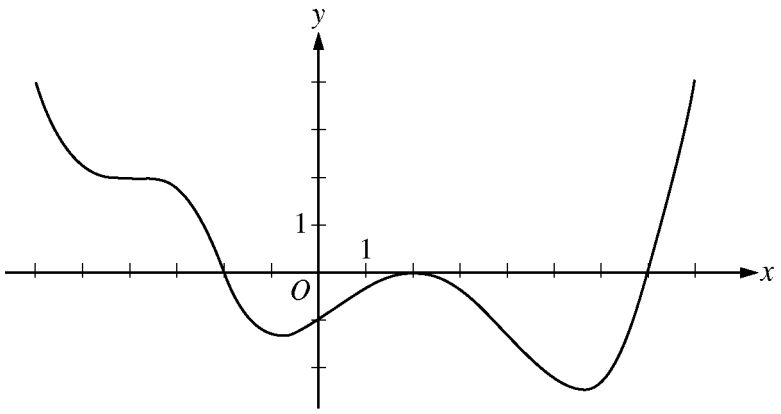
(D) $p'(-2)$ is undefined.

(E) There is not enough information given to conclude anything about $p'(-2)$.

---

81. At time $t = 0$ years, a forest preserve has a population of 1500 deer. If the rate of growth of the population is modeled by $R(t) = 2000e^{0.23t}$ deer per year, what is the population at time $t = 3$?

(A) 3987 (B) 5487 (C) 8641 (D) 10,141 (E) 12,628



Graph of f'

82. The figure above shows the graph of f', the derivative of function f, for $-6 < x < 8$. Of the following, which best describes the graph of f on the same interval?

(A) 1 relative minimum, 1 relative maximum, and 3 points of inflection

(B) 1 relative minimum, 1 relative maximum, and 4 points of inflection

(C) 2 relative minima, 1 relative maximum, and 2 points of inflection

(D) 2 relative minima, 1 relative maximum, and 4 points of inflection

(E) 2 relative minima, 2 relative maxima, and 3 points of inflection

x	$f'(x)$
1	0.2
1.5	0.5
2	0.9

83. The table above gives values of f', the derivative of a function f. If $f(1) = 4$, what is the approximation to $f(2)$ obtained by using Euler's method with a step size of 0.5 ?

(A) 2.35

(B) 3.65

(C) 4.35

(D) 4.70

(E) 4.80

84. A sphere is expanding in such a way that the area of any circular cross section through the sphere's center is increasing at a constant rate of $2\,\text{cm}^2/\text{sec}$. At the instant when the radius of the sphere is 4 centimeters, what is the rate of change of the sphere's volume? (The volume V of a sphere with radius r is given by $V = \frac{4}{3}\pi r^3$.)

(A) $8\,\text{cm}^3/\text{sec}$

(B) $16\,\text{cm}^3/\text{sec}$

(C) $8\pi\,\text{cm}^3/\text{sec}$

(D) $64\pi\,\text{cm}^3/\text{sec}$

(E) $128\pi\,\text{cm}^3/\text{sec}$

85. For $t \geq 0$, the components of the velocity of a particle moving in the xy-plane are given by the parametric equations $x'(t) = \dfrac{1}{t+1}$ and $y'(t) = ke^{kt}$, where k is a positive constant. The line $y = 4x + 3$ is parallel to the line tangent to the path of the particle at the point where $t = 2$. What is the value of k?

(A) 0.072 (B) 0.433 (C) 0.495 (D) 0.803 (E) 0.828

---

t (hours)	0	1	2	3	4	5	6
$s(t)$ (miles)	0	25	55	92	150	210	275

86. The table above gives the distance $s(t)$, in miles, that a car has traveled at various times t, in hours, during a 6-hour trip. The graph of the function s is increasing and concave up. Based on the information, which of the following could be the velocity of the car, in miles per hour, at time $t = 3$?

(A) 37 (B) 49 (C) 58 (D) 65 (E) 92

GO ON TO THE NEXT PAGE.

87. If $0 < b_n < a_n$ for $n \geq 1$, which of the following must be true?

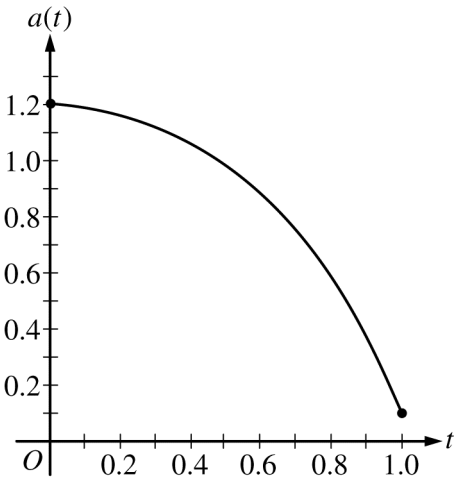
(A) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} b_n$ converges.

(B) If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} b_n = 0$.

(C) If $\sum_{n=1}^{\infty} b_n$ diverges, then $\lim_{n\to\infty} a_n = 0$.

(D) If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

(E) If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.



88. A particle moves along the x-axis so that its acceleration $a(t)$ is given by the graph above for all values of t where $0 \le t \le 1$. At time $t = 0$, the velocity of the particle is $-\frac{1}{2}$. Which of the following statements must be true?

(A) The particle passes through $x = 0$ for some t between $t = 0$ and $t = 1$.

(B) The velocity of the particle is 0 for some t between $t = 0$ and $t = 1$.

(C) The velocity of the particle is negative for all values of t between $t = 0$ and $t = 1$.

(D) The velocity of the particle is positive for all values of t between $t = 0$ and $t = 1$.

(E) The velocity of the particle is less than $-\frac{1}{2}$ for all values of t between $t = 0$ and $t = 1$.

89. The function f is given by $f(x) = \int_1^x \sqrt{t^3 + 2}\, dt$. What is the average rate of change of f over the interval $[0, 3]$?

(A) 1.324 (B) 1.497 (C) 1.696 (D) 2.266 (E) 2.694

---

90. A particle moves along a line so that its velocity is given by $v(t) = -t^3 + 2t^2 + 2^{-t}$ for $t \geq 0$. For what values of t is the speed of the particle increasing?

(A) $(0, 0.177)$ and $(1.256, \infty)$

(B) $(0, 1.256)$ only

(C) $(0, 2.057)$ only

(D) $(0.177, 1.256)$ only

(E) $(0.177, 1.256)$ and $(2.057, \infty)$

GO ON TO THE NEXT PAGE.

91. Line ℓ is tangent to the graph of $y = \cos x$ at the point $(k, \cos k)$, where $0 < k < \pi$. For what value of k does line ℓ pass through the origin?

(A) 0.860

(B) 1.571

(C) 2.356

(D) 2.798

(E) There is no such value of k.

---

x	2	4
$f(x)$	7	13
$g(x)$	2	9
$g'(x)$	1	7
$g''(x)$	5	8

92. The table above gives selected values of twice-differentiable functions f and g, as well as the first two derivatives of g. If $f'(x) = 3$ for all values of x, what is the value of $\int_2^4 f(x)g''(x)\,dx$?

(A) 63 (B) 69 (C) 78 (D) 84 (E) 103

GO ON TO THE NEXT PAGE.

END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART B ONLY.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

MAKE SURE YOU HAVE DONE THE FOLLOWING.

- **PLACED YOUR AP NUMBER LABEL ON YOUR ANSWER SHEET**
- **WRITTEN AND GRIDDED YOUR AP NUMBER CORRECTLY ON YOUR ANSWER SHEET**
- **TAKEN THE AP EXAM LABEL FROM THE FRONT OF THIS BOOKLET AND PLACED IT ON YOUR ANSWER SHEET**

AFTER TIME HAS BEEN CALLED, TURN TO PAGE 38 AND ANSWER QUESTIONS 93–96.

Section II: Free-Response Questions

This is the free-response section of the 2014 AP exam.
It includes cover material and other administrative instructions
to help familiarize students with the mechanics of the exam.
(Note that future exams may differ in look from the following content.)

AP® Calculus AB Exam

SECTION II: Free Response

2014

DO NOT OPEN THIS BOOKLET OR BREAK THE SEALS ON PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour, 30 minutes
Number of Questions
6
Percent of Total Score
50%
Writing Instrument
Either pencil or pen with black or dark blue ink
Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions
2
Time
30 minutes
Electronic Device
Graphing calculator required
Percent of Section II Score
33.3%

Part B

Number of Questions
4
Time
60 minutes
Electronic Device
None allowed
Percent of Section II Score
66.6%

IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name ☐☐
   First letter of your first name ☐
2. Date of birth
   ☐☐ Month ☐☐ Day ☐☐☐☐ Year
3. Six-digit school code
   ☐☐☐☐☐☐
4. Unless I check the box below, I grant the College Board the unlimited right to use, reproduce, and publish my free-response materials, both written and oral, for educational research and instructional purposes. My name and the name of my school will not be used in any way in connection with my free-response materials. I understand that I am free to mark "No" with no effect on my score or its reporting.
   No, I do not grant the College Board these rights. ☐

Instructions

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit. Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Form I
Form Code 4JBP6-S

66

CALCULUS BC
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

GO ON TO THE NEXT PAGE.

t (hours)	0	0.4	0.8	1.2	1.6	2.0	2.4
$v(t)$ (miles per hour)	0	11.8	9.5	17.2	16.3	16.8	20.1

1. Ruth rode her bicycle on a straight trail. She recorded her velocity $v(t)$, in miles per hour, for selected values of t over the interval $0 \le t \le 2.4$ hours, as shown in the table above. For $0 < t \le 2.4$, $v(t) > 0$.

   (a) Use the data in the table to approximate Ruth's acceleration at time $t = 1.4$ hours. Show the computations that lead to your answer. Indicate units of measure.

   (b) Using correct units, interpret the meaning of $\int_0^{2.4} v(t)\,dt$ in the context of the problem. Approximate $\int_0^{2.4} v(t)\,dt$ using a midpoint Riemann sum with three subintervals of equal length and values from the table.

Do not write beyond this border.

Continue problem 1 on page 5.

(c) For $0 \le t \le 2.4$ hours, Ruth's velocity can be modeled by the function g given by $g(t) = \dfrac{24t + 5\sin(6t)}{t + 0.7}$. According to the model, what was Ruth's average velocity during the time interval $0 \le t \le 2.4$?

(d) According to the model given in part (c), is Ruth's speed increasing or decreasing at time $t = 1.3$? Give a reason for your answer.

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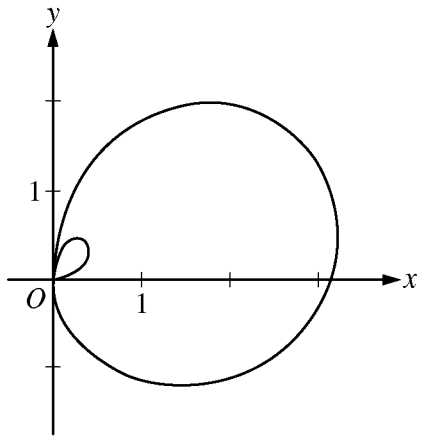


2. Consider the polar curve defined by the function $r(\theta) = \theta\cos\theta$, where $0 \le \theta \le \frac{3\pi}{2}$. The derivative of r is given by $\frac{dr}{d\theta} = \cos\theta - \theta\sin\theta$. The figure above shows the graph of r for $0 \le \theta \le \frac{3\pi}{2}$.

(a) Find the area of the region enclosed by the inner loop of the curve.

Do not write beyond this border.

Continue problem 2 on page 7.

(b) For $0 \le \theta \le \frac{3\pi}{2}$, find the greatest distance from any point on the graph of r to the origin. Justify your answer.

(c) There is a point on the curve at which the slope of the line tangent to the curve is $\frac{2}{2-\pi}$. At this point, $\frac{dy}{d\theta} = \frac{1}{2}$. Find $\frac{dx}{d\theta}$ at this point.

Do not write beyond this border.

END OF PART A OF SECTION II

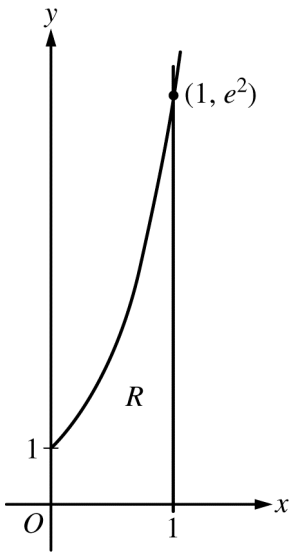
IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS BC
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.

DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.

NO CALCULATOR ALLOWED



3. Let $f(x) = e^{2x}$. Let R be the region in the first quadrant bounded by the graph of $y = f(x)$ and the vertical line $x = 1$, as shown in the figure above.

   (a) Write an equation for the line tangent to the graph of f at $x = 1$.

Continue problem 3 on page 15.

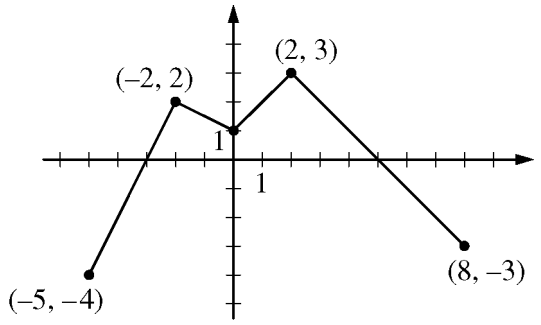
NO CALCULATOR ALLOWED

(b) Find the area of R.

(c) Region R forms the base of a solid whose cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

Do not write beyond this border.

NO CALCULATOR ALLOWED



Graph of f

4. The continuous function f is defined on the interval $-5 \le x \le 8$. The graph of f, which consists of four line segments, is shown in the figure above. Let g be the function given by $g(x) = 2x + \int_{-2}^{x} f(t)\,dt$.

(a) Find $g(0)$ and $g(-5)$.

(b) Find $g'(x)$ in terms of $f(x)$. For each of $g''(4)$ and $g''(-2)$, find the value or state that it does not exist.

Do not write beyond this border.

Continue problem 4 on page 17.

NO CALCULATOR ALLOWED

(c) On what intervals, if any, is the graph of g concave down? Give a reason for your answer.

(d) The function h is given by $h(x) = g\left(x^3 + 1\right)$. Find $h'(1)$. Show the work that leads to your answer.

Do not write beyond this border.

NO CALCULATOR ALLOWED

5. A toy train moves along a straight track set up on a table. The position $x(t)$ of the train at time t seconds is measured in centimeters from the center of the track. At time $t = 1$, the train is 6 centimeters to the left of the center, so $x(1) = -6$. For $0 \le t \le 4$, the velocity of the train at time t is given by $v(t) = 3t^2 - 12$, where $v(t)$ is measured in centimeters per second.

(a) For $0 \le t \le 4$, find $x(t)$.

(b) Find the total distance traveled by the train during the time interval $0 \le t \le 4$.

Do not write beyond this border.

Continue problem 5 on page 19.

NO CALCULATOR ALLOWED

(c) A toy bus moving on the same table has position given by $(x(t), y(t))$. Here, $x(t)$ is the function found in part (a), and $y(t) = 2 + 12\sin\left(\frac{t}{4}\right)$ is the distance from the bus to the train track, in centimeters. Write, but do not evaluate, an integral expression that gives the total distance traveled by the bus during the time interval $0 \le t \le 4$.

Do not write beyond this border.

NO CALCULATOR ALLOWED

6. Let $a_n = \dfrac{1}{n \ln n}$ for $n \geq 3$.

(a) Let f be the function given by $f(x) = \dfrac{1}{x \ln x}$. For $x \geq 3$, f is continuous, decreasing, and positive. Use the integral test to show that $\sum_{n=3}^{\infty} a_n$ diverges.

(b) Consider the infinite series $\sum_{n=3}^{\infty} (-1)^{n+1} a_n = \dfrac{1}{3 \ln 3} - \dfrac{1}{4 \ln 4} + \dfrac{1}{5 \ln 5} - \cdots$. Identify properties of this series that guarantee the series converges. Explain why the sum of this series is less than $\dfrac{1}{3}$.

Do not write beyond this border.

Continue problem 6 on page 21.

NO CALCULATOR ALLOWED

(c) Find the interval of convergence of the power series $\sum_{n=3}^{\infty} \frac{(x-2)^{n+1}}{n \ln n}$. Show the analysis that leads to your answer.

Do not write beyond this border.

STOP

END OF EXAM

THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE FRONT AND BACK COVERS OF THE SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER LABEL APPEARS IN THE BOX ON THE COVER.**
- **MAKE SURE YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.**

Multiple-Choice Answer Key

The following contains the answers to
the multiple-choice questions in this exam.

Answer Key for AP Calculus BC Practice Exam, Section I

Question 1: C

Question 2: B

Question 3: D

Question 4: E

Question 5: C

Question 6: E

Question 7: A

Question 8: B

Question 9: A

Question 10: B

Question 11: E

Question 12: C

Question 13: E

Question 14: E

Question 15: B

Question 16: A

Question 17: E

Question 18: B

Question 19: B

Question 20: B

Question 21: D

Question 22: B

Question 23: D

Question 24: C

Question 25: B

Question 26: A

Question 27: D

Question 28: B

Question 76: C

Question 77: E

Question 78: C

Question 79: E

Question 80: A

Question 81: D

Question 82: A

Question 83: C

Question 84: B

Question 85: C

Question 86: B

Question 87: B

Question 88: B

Question 89: E

Question 90: E

Question 91: D

Question 92: A

Free-Response Scoring Guidelines

The following contains the scoring guidelines
for the free-response questions in this exam.

AP® CALCULUS BC
2014 SCORING GUIDELINES

Question 1

t (hours)	0	0.4	0.8	1.2	1.6	2.0	2.4
$v(t)$ (miles per hour)	0	11.8	9.5	17.2	16.3	16.8	20.1

Ruth rode her bicycle on a straight trail. She recorded her velocity $v(t)$, in miles per hour, for selected values of t over the interval $0 \le t \le 2.4$ hours, as shown in the table above. For $0 < t \le 2.4$, $v(t) > 0$.

(a) Use the data in the table to approximate Ruth's acceleration at time $t = 1.4$ hours. Show the computations that lead to your answer. Indicate units of measure.

(b) Using correct units, interpret the meaning of $\int_0^{2.4} v(t)\,dt$ in the context of the problem. Approximate $\int_0^{2.4} v(t)\,dt$ using a midpoint Riemann sum with three subintervals of equal length and values from the table.

(c) For $0 \le t \le 2.4$ hours, Ruth's velocity can be modeled by the function g given by $g(t) = \dfrac{24t + 5\sin(6t)}{t + 0.7}$. According to the model, what was Ruth's average velocity during the time interval $0 \le t \le 2.4$?

(d) According to the model given in part (c), is Ruth's speed increasing or decreasing at time $t = 1.3$? Give a reason for your answer.

(a) $a(1.4) \approx \dfrac{v(1.6) - v(1.2)}{1.6 - 1.2} = \dfrac{16.3 - 17.2}{1.6 - 1.2} = -2.25$ miles/hr^2

2 : { 1 : approximation; 1 : units }

(b) $\int_0^{2.4} v(t)\,dt$ is the total distance Ruth traveled, in miles, from time $t = 0$ to time $t = 2.4$ hours.

$\int_0^{2.4} v(t)\,dt \approx (0.8)(11.8) + (0.8)(17.2) + (0.8)(16.8)$
$= 36.64$ miles

3 : { 1 : interpretation; 1 : midpoint Riemann sum; 1 : approximation }

(c) Average velocity $= \dfrac{1}{2.4}\int_0^{2.4} g(t)\,dt = 14.064$ miles/hr

2 : { 1 : integral; 1 : answer }

(d) Velocity $= g(1.3) = 18.096358 > 0$
Acceleration $= g'(1.3) = 3.761152 > 0$

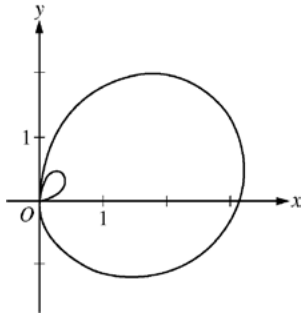
Ruth's speed is increasing at time $t = 1.3$ since velocity and acceleration have the same sign at this time.

2 : conclusion with reason

AP® CALCULUS BC
2014 SCORING GUIDELINES

Question 2

Consider the polar curve defined by the function $r(\theta) = \theta\cos\theta$, where $0 \le \theta \le \frac{3\pi}{2}$. The derivative of r is given by $\frac{dr}{d\theta} = \cos\theta - \theta\sin\theta$. The figure above shows the graph of r for $0 \le \theta \le \frac{3\pi}{2}$.



(a) Find the area of the region enclosed by the inner loop of the curve.

(b) For $0 \le \theta \le \frac{3\pi}{2}$, find the greatest distance from any point on the graph of r to the origin. Justify your answer.

(c) There is a point on the curve at which the slope of the line tangent to the curve is $\frac{2}{2-\pi}$. At this point, $\frac{dy}{d\theta} = \frac{1}{2}$. Find $\frac{dx}{d\theta}$ at this point.

(a) Area $= \frac{1}{2}\int_0^{\pi/2} (r(\theta))^2\, d\theta = 0.127$ (or 0.126)

3 : { 1 : integrand; 1 : limits and constant; 1 : answer }

(b) $r'(\theta) = \cos\theta - \theta\sin\theta = 0 \Rightarrow \theta = 0.860334, 3.425618$

θ	$r(\theta)$
0	0
0.860334	0.561096
3.425618	–3.288371
$\frac{3\pi}{2}$	0

Therefore, the greatest distance from any point on the graph of r to the origin is 3.288.

3 : { 1 : identifies $\theta = 3.425618$ as a candidate; 1 : answer; 1 : justification }

(c) $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$

At the point where the tangent line has slope $\frac{2}{2-\pi}$, $\frac{2}{2-\pi} = \frac{1/2}{dx/d\theta}$.

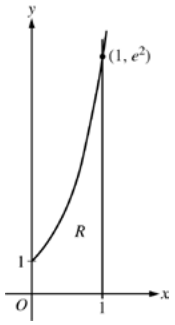
Therefore, $\frac{dx}{d\theta} = \frac{1}{2}\cdot\frac{2-\pi}{2} = \frac{2-\pi}{4}$ at this point.

3 : { 1 : $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$; 1 : equation; 1 : answer }

AP® CALCULUS BC
2014 SCORING GUIDELINES

Question 3

Let $f(x) = e^{2x}$. Let R be the region in the first quadrant bounded by the graph of $y = f(x)$ and the vertical line $x = 1$, as shown in the figure above.



(a) Write an equation for the line tangent to the graph of f at $x = 1$.

(b) Find the area of R.

(c) Region R forms the base of a solid whose cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

(a) $f(1) = e^2$

$f'(x) = 2e^{2x} \Rightarrow f'(1) = 2e^2$

An equation for the tangent line is $y = e^2 + 2e^2(x - 1)$.

$2 : \begin{cases} 1 : f'(1) \\ 1 : \text{answer} \end{cases}$

(b) $\text{Area} = \int_0^1 e^{2x}\,dx = \left[\frac{1}{2}e^{2x}\right]_{x=0}^{x=1} = \frac{1}{2}\left(e^2 - 1\right)$

$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(c) $\text{Volume} = 1 + \int_1^{e^2} \left(1 - \frac{1}{2}\ln y\right)^2 dy$

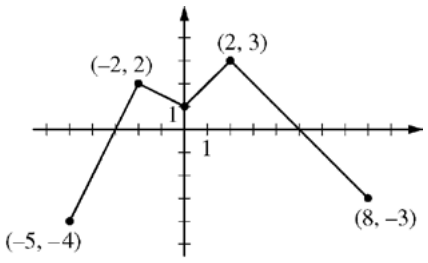
$4 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

AP® CALCULUS BC
2014 SCORING GUIDELINES

Question 4

The continuous function f is defined on the interval $-5 \le x \le 8$. The graph of f, which consists of four line segments, is shown in the figure above.

Let g be the function given by $g(x) = 2x + \int_{-2}^{x} f(t)\,dt$.



Graph of f

(a) Find $g(0)$ and $g(-5)$.

(b) Find $g'(x)$ in terms of $f(x)$. For each of $g''(4)$ and $g''(-2)$, find the value or state that it does not exist.

(c) On what intervals, if any, is the graph of g concave down? Give a reason for your answer.

(d) The function h is given by $h(x) = g\left(x^3 + 1\right)$. Find $h'(1)$. Show the work that leads to your answer.

(a) $g(0) = 2 \cdot 0 + \int_{-2}^{0} f(t)\,dt = 3$

$g(-5) = 2 \cdot (-5) + \int_{-2}^{-5} f(t)\,dt = -10 + 3 = -7$

$2 : \begin{cases} 1 : g(0) \\ 1 : g(-5) \end{cases}$

(b) $g'(x) = 2 + f(x)$
$g''(x) = f'(x)$

$g''(4) = f'(4) = -1$

$g''(-2) = f'(-2)$ does not exist.

$3 : \begin{cases} 1 : g'(x) \\ 1 : g''(4) \\ 1 : g''(-2) \text{ does not exist} \end{cases}$

(c) The graph of g is concave down on the intervals $(-2, 0)$ and $(2, 8)$ since $g'(x) = 2 + f(x)$ decreases on those intervals.

1 : intervals and reason

(d) $h'(x) = g'\left(x^3 + 1\right) \cdot 3x^2$

$h'(1) = g'(2) \cdot 3 = (2 + f(2)) \cdot 3$
$= (2 + 3) \cdot 3 = 15$

$3 : \begin{cases} 2 : \text{chain rule} \\ 1 : \text{answer} \end{cases}$

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2014 SCORING GUIDELINES

Question 5

A toy train moves along a straight track set up on a table. The position $x(t)$ of the train at time t seconds is measured in centimeters from the center of the track. At time $t = 1$, the train is 6 centimeters to the left of the center, so $x(1) = -6$. For $0 \le t \le 4$, the velocity of the train at time t is given by $v(t) = 3t^2 - 12$, where $v(t)$ is measured in centimeters per second.

(a) For $0 \le t \le 4$, find $x(t)$.

(b) Find the total distance traveled by the train during the time interval $0 \le t \le 4$.

(c) A toy bus moving on the same table has position given by $(x(t), y(t))$. Here, $x(t)$ is the function found in part (a), and $y(t) = 2 + 12\sin\left(\frac{t}{4}\right)$ is the distance from the bus to the train track, in centimeters. Write, but do not evaluate, an integral expression that gives the total distance traveled by the bus during the time interval $0 \le t \le 4$.

(a) $x(t) = -6 + \int_1^t \left(3u^2 - 12\right) du$

$= -6 + \left[u^3 - 12u\right]_{u=1}^{u=t}$

$= t^3 - 12t + 5$

$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) $v(t) = 3\left(t^2 - 4\right) = 0 \Rightarrow t = -2,\ t = 2$

Distance $= |x(2) - x(0)| + |x(4) - x(2)|$

$= |-11 - 5| + |21 - (-11)|$

$= 16 + 32 = 48$

$3 : \begin{cases} 1 : \text{identifies } t = 2 \\ 1 : \text{considers } x(0),\ x(2), \text{ and } x(4) \\ 1 : \text{answer} \end{cases}$

(c) $y'(t) = 3\cos\left(\frac{t}{4}\right)$

Distance $= \int_0^4 \sqrt{\left(3t^2 - 12\right)^2 + \left(3\cos\left(\frac{t}{4}\right)\right)^2}\, dt$

$3 : \begin{cases} 1 : y'(t) \\ 2 : \text{integral} \end{cases}$

AP® CALCULUS BC
2014 SCORING GUIDELINES

Question 6

Let $a_n = \dfrac{1}{n \ln n}$ for $n \geq 3$.

(a) Let f be the function given by $f(x) = \dfrac{1}{x \ln x}$. For $x \geq 3$, f is continuous, decreasing, and positive.

Use the integral test to show that $\displaystyle\sum_{n=3}^{\infty} a_n$ diverges.

(b) Consider the infinite series $\displaystyle\sum_{n=3}^{\infty} (-1)^{n+1} a_n = \frac{1}{3\ln 3} - \frac{1}{4\ln 4} + \frac{1}{5\ln 5} - \cdots$. Identify properties of this series that guarantee the series converges. Explain why the sum of this series is less than $\dfrac{1}{3}$.

(c) Find the interval of convergence of the power series $\displaystyle\sum_{n=3}^{\infty} \frac{(x-2)^{n+1}}{n \ln n}$. Show the analysis that leads to your answer.

(a) $u = \ln x \Rightarrow du = \dfrac{1}{x}\,dx$

$$\int_3^{\infty} \frac{1}{x \ln x}\,dx = \lim_{b\to\infty} \int_3^b \frac{1}{x \ln x}\,dx = \lim_{b\to\infty} \int_{\ln 3}^{\ln b} \frac{1}{u}\,du = \lim_{b\to\infty} \Big[\ln|u|\Big]_{u=\ln 3}^{u=\ln b}$$
$$= \lim_{b\to\infty} (\ln|\ln b| - \ln|\ln 3|) = \infty$$

Therefore, the series diverges.

3 : { 1 : integral; 1 : antiderivative; 1 : evaluation }

(b) The terms in this alternating series decrease in absolute value and $\displaystyle\lim_{n\to\infty} \frac{1}{n \ln n} = 0$. Therefore, the Alternating Series Test guarantees that this series converges. Furthermore,

$$\frac{1}{3\ln 3} - \frac{1}{4\ln 4} < \text{Sum} < \frac{1}{3\ln 3} < \frac{1}{3}$$

Therefore, the sum of the series is less than $\dfrac{1}{3}$.

2 : { 1 : properties; 1 : explanation }

(c) $$\left|\frac{(x-2)^{n+2}}{(n+1)\ln(n+1)} \cdot \frac{n \ln n}{(x-2)^{n+1}}\right| = \left(\frac{n}{n+1}\right)\left(\frac{\ln n}{\ln(n+1)}\right)|x-2|$$

$$\lim_{n\to\infty} \left(\frac{n}{n+1}\right)\left(\frac{\ln n}{\ln(n+1)}\right)|x-2| = |x-2|$$

$|x-2| < 1 \Rightarrow 1 < x < 3$

When $x = 1$, the series is $\dfrac{1}{3\ln 3} - \dfrac{1}{4\ln 4} + \dfrac{1}{5\ln 5} - \dfrac{1}{6\ln 6} + \cdots$

This series converges by the Alternating Series Test.

When $x = 3$, the series is $\dfrac{1}{3\ln 3} + \dfrac{1}{4\ln 4} + \dfrac{1}{5\ln 5} + \dfrac{1}{6\ln 6} + \cdots$

This series diverges by the integral test, as shown in part (a).
Therefore, the interval of convergence is $1 \leq x < 3$.

4 : { 1: sets up ratio; 1: computes limit of ratio; 1: identifies radius of convergence; 1: analysis and interval of convergence }

Scoring Worksheets

The following provides a scoring worksheets and conversion tables used for calculating a composite score of the exam.

2014 AP Calculus BC Scoring Worksheet

Section I: Multiple Choice

_____	×	1.2000	=	_____
Number Correct (out of 45)				Weighted Section I Score (Do not round)

Section II: Free Response

Question 1	_____ (out of 9)	×	1.0000 =	_____ (Do not round)
Question 2	_____ (out of 9)	×	1.0000 =	_____ (Do not round)
Question 3	_____ (out of 9)	×	1.0000 =	_____ (Do not round)
Question 4	_____ (out of 9)	×	1.0000 =	_____ (Do not round)
Question 5	_____ (out of 9)	×	1.0000 =	_____ (Do not round)
Question 6	_____ (out of 9)	×	1.0000 =	_____ (Do not round)

Sum = _____
Weighted Section II Score (Do not round)

Composite Score

_____	+	_____	=	_____
Weighted Section I Score		Weighted Section II Score		Composite Score (Round to nearest whole number)

AP Score Conversion Chart
Calculus BC

Composite Score Range	AP Score
62-108	5
52-61	4
40-51	3
33-39	2
0-32	1

2014 AP Calculus BC — AB Subscore Scoring Worksheet

Section I: Multiple Choice

Questions (1-2, 5, 7-9, 11-13, 15, 19-20, 23, 26, 28, 76, 78-82, 84, 86, 88-91)

_____ × 1.0000 = _____

Number Correct (out of 27) | Weighted Section I Score (Do not round)

Section II: Free Response

Question 1 _____ × 1.0000 = _____

(out of 9) | (Do not round)

Question 3 _____ × 1.0000 = _____

(out of 9) | (Do not round)

Question 5 _____ × 1.0000 = _____

(out of 9) | (Do not round)

Sum = _____

Weighted Section II Score (Do not round)

Composite Score

_____ + _____ = _____

Weighted Section I Score | Weighted Section II Score | Composite Score (Round to nearest whole number)

AP Score Conversion Chart
Calculus AB Subscore

Composite Score Range	AP Score
33-54	5
26-32	4
19-25	3
14-18	2
0-13	1

AP Calculus BC

The College Board

The College Board is a mission-driven not-for-profit organization that connects students to college success and opportunity. Founded in 1900, the College Board was created to expand access to higher education. Today, the membership association is made up of over 6,000 of the world's leading educational institutions and is dedicated to promoting excellence and equity in education. Each year, the College Board helps more than seven million students prepare for a successful transition to college through programs and services in college readiness and college success — including the SAT® and the Advanced Placement Program®. The organization also serves the education community through research and advocacy on behalf of students, educators, and schools. The College Board is committed to the principles of excellence and equity, and that commitment is embodied in all of its programs, services, activities, and concerns.