Limits

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Questions in past papers often come up combined with other topics.
Topic tags have been given for each question to enable you to know if you can do the question or whether you need to wait to cover the additional topic(s).

Scan the QR code(s) or click the link for instant detailed model solutions!

Question 1

Qualification: AP Calculus AB

Areas: Limits and Continuity, Integration, Applications of Integration

Subtopics: Continuities and Discontinuities, Calculating Limits Algebraically, Average Value of a Function, Properties of Integrals, Integration Technique – Standard Functions, Differentiability

Paper: Part B-Non-Calc / Series: 2003 / Difficulty: Very Hard / Question Number: 6

6. Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \le x \le 3 \\ 5 - x & \text{for } 3 < x \le 5. \end{cases}$$

(a) Is f continuous at $x = 3$? Explain why or why not.

(b) Find the average value of $f(x)$ on the closed interval $0 \le x \le 5$.

(c) Suppose the function g is defined by

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \le x \le 3 \\ mx + 2 & \text{for } 3 < x \le 5, \end{cases}$$

where k and m are constants. If g is differentiable at $x = 3$, what are the values of k and m ?

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Question 2

Qualification: AP Calculus AB

Areas: Differential Equations, Limits and Continuity

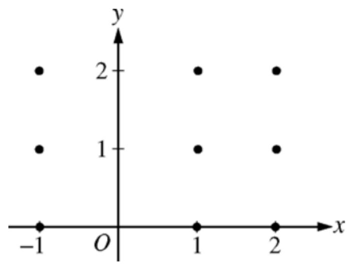
Subtopics: Sketching Slope Field, Particular Solution of Differential Equation, Initial Conditions in Differential Equation, Integration Technique – Standard Functions, Calculating Limits Algebraically

Paper: Part B-Non-Calc / Series: 2008 / Difficulty: Easy / Question Number: 5

5. Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

(c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x \to \infty} f(x)$.

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Question 3

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation, Limits and Continuity

Subtopics: Local or Relative Minima and Maxima, Differentiation Technique - Quotient Rule, Points Of Inflection, Calculating Limits Algebraically, Tangents To Curves

Paper: Part B-Non-Calc / Series: 2008 / Difficulty: Easy / Question Number: 6

6. Let f be the function given by $f(x) = \dfrac{\ln x}{x}$ for all $x > 0$. The derivative of f is given by $f'(x) = \dfrac{1 - \ln x}{x^2}$.

(a) Write an equation for the line tangent to the graph of f at $x = e^2$.

(b) Find the x-coordinate of the critical point of f. Determine whether this point is a relative minimum, a relative maximum, or neither for the function f. Justify your answer.

(c) The graph of the function f has exactly one point of inflection. Find the x-coordinate of this point.

(d) Find $\lim\limits_{x \to 0^+} f(x)$.

Question 4

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Integration

Subtopics: Continuities and Discontinuities, Average Value of a Function, Interpreting Meaning in Applied Contexts, Modelling Situations, Calculating Limits Algebraically, Accumulation of Change

Paper: Part A-Calc / Series: 2011-Form-B / Difficulty: Easy / Question Number: 2

2. A 12,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \dfrac{600t}{t+3} & \text{for } 0 \le t \le 5 \\ 1000e^{-0.2t} & \text{for } t > 5 \end{cases}$$

(a) Is r continuous at $t = 5$? Show the work that leads to your answer.

(b) Find the average rate at which water is draining from the tank between time $t = 0$ and time $t = 8$ hours.

(c) Find $r'(3)$. Using correct units, explain the meaning of that value in the context of this problem.

(d) Write, but do not solve, an equation involving an integral to find the time A when the amount of water in the tank is 9000 liters.

Question 5

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation, Limits and Continuity

Subtopics: Rates of Change (Average), Tangents To Curves, Global or Absolute Minima and Maxima, L'Hôpital's Rule, Calculating Limits Algebraically, Differentiation Technique – Product Rule, Differentiation Technique – Exponentials, Differentiation Technique – Trigonometry

Paper: Part B-Non-Calc / Series: 2018 / Difficulty: Somewhat Challenging / Question Number: 5

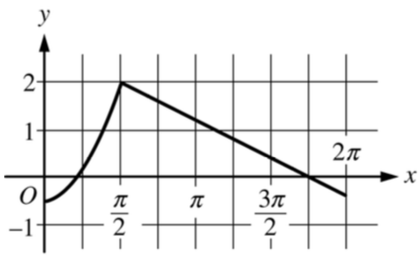
5. Let f be the function defined by $f(x) = e^x \cos x$.

(a) Find the average rate of change of f on the interval $0 \le x \le \pi$.

(b) What is the slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$?

(c) Find the absolute minimum value of f on the interval $0 \le x \le 2\pi$. Justify your answer.

(d) Let g be a differentiable function such that $g\left(\frac{\pi}{2}\right) = 0$. The graph of g', the derivative of g, is shown below. Find the value of $\lim_{x \to \pi/2} \frac{f(x)}{g(x)}$ or state that it does not exist. Justify your answer.



Graph of g'

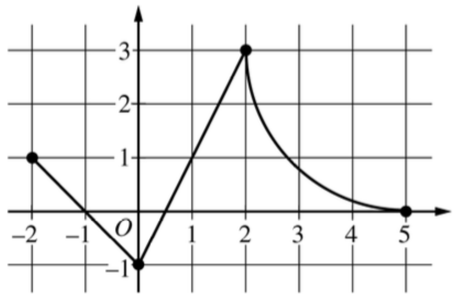
Question 6

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Differentiation, Integration

Subtopics: Integration Technique – Geometric Areas, Fundamental Theorem of Calculus (First), Global or Absolute Minima and Maxima, Calculating Limits Algebraically, Integration Graphs

Paper: Part B-Non-Calc / Series: 2019 / Difficulty: Medium / Question Number: 3



Graph of f

3. The continuous function f is defined on the closed interval $-6 \le x \le 5$. The figure above shows a portion of the graph of f, consisting of two line segments and a quarter of a circle centered at the point $(5, 3)$. It is known that the point $(3, 3 - \sqrt{5})$ is on the graph of f.

(a) If $\int_{-6}^{5} f(x)\,dx = 7$, find the value of $\int_{-6}^{-2} f(x)\,dx$. Show the work that leads to your answer.

(b) Evaluate $\int_{3}^{5} (2f'(x) + 4)\,dx$.

(c) The function g is given by $g(x) = \int_{-2}^{x} f(t)\,dt$. Find the absolute maximum value of g on the interval $-2 \le x \le 5$. Justify your answer.

(d) Find $\lim\limits_{x \to 1} \dfrac{10^x - 3f'(x)}{f(x) - \arctan x}$.

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Question 7

Qualification: AP Calculus AB

Areas: Limits and Continuity, Differentiation

Subtopics: Differentiation Technique – Product Rule, L'Hôpital's Rule, Calculating Limits Algebraically, Differentiation Technique – Chain Rule, Continuities and Discontinuities

Paper: Part B-Non-Calc / Series: 2019 / Difficulty: Very Hard / Question Number: 6

6. Functions f, g, and h are twice-differentiable functions with $g(2) = h(2) = 4$. The line $y = 4 + \frac{2}{3}(x - 2)$ is tangent to both the graph of g at $x = 2$ and the graph of h at $x = 2$.

(a) Find $h'(2)$.

(b) Let a be the function given by $a(x) = 3x^3h(x)$. Write an expression for $a'(x)$. Find $a'(2)$.

(c) The function h satisfies $h(x) = \dfrac{x^2 - 4}{1 - (f(x))^3}$ for $x \neq 2$. It is known that $\lim\limits_{x\to 2} h(x)$ can be evaluated using L'Hospital's Rule. Use $\lim\limits_{x\to 2} h(x)$ to find $f(2)$ and $f'(2)$. Show the work that leads to your answers.

(d) It is known that $g(x) \leq h(x)$ for $1 < x < 3$. Let k be a function satisfying $g(x) \leq k(x) \leq h(x)$ for $1 < x < 3$. Is k continuous at $x = 2$? Justify your answer.

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Question 8

Qualification: AP Calculus AB

Areas: Applications of Differentiation

Subtopics: Kinematics (Displacement, Velocity, and Acceleration), Increasing/Decreasing , Calculating Limits Algebraically

Paper: Part B-Non-Calc / Series: 2022 / Difficulty: Medium / Question Number: 6

6. Particle P moves along the x-axis such that, for time $t > 0$, its position is given by $x_P(t) = 6 - 4e^{-t}$.

Particle Q moves along the y-axis such that, for time $t > 0$, its velocity is given by $v_Q(t) = \frac{1}{t^2}$. At time $t = 1$, the position of particle Q is $y_Q(1) = 2$.

(a) Find $v_P(t)$, the velocity of particle P at time t.

(b) Find $a_Q(t)$, the acceleration of particle Q at time t. Find all times t, for $t > 0$, when the speed of particle Q is decreasing. Justify your answer.

(c) Find $y_Q(t)$, the position of particle Q at time t.

(d) As $t \to \infty$, which particle will eventually be farther from the origin? Give a reason for your answer.

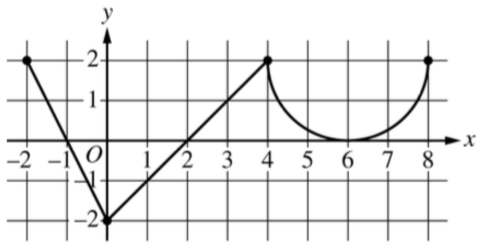
Question 9

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Differentiation

Subtopics: Local or Relative Minima and Maxima, Concavity, Derivative Graphs, Global or Absolute Minima and Maxima, Integration Technique – Geometric Areas, L'Hôpital's Rule, Calculating Limits Algebraically

Paper: Part B-Non-Calc / Series: 2023 / Difficulty: Medium / Question Number: 4



Graph of f'

4. The function f is defined on the closed interval $[-2, 8]$ and satisfies $f(2) = 1$. The graph of f', the derivative of f, consists of two line segments and a semicircle, as shown in the figure.

   (a) Does f have a relative minimum, a relative maximum, or neither at $x = 6$? Give a reason for your answer.

   (b) On what open intervals, if any, is the graph of f concave down? Give a reason for your answer.

   (c) Find the value of $\lim_{x \to 2} \frac{6f(x) - 3x}{x^2 - 5x + 6}$, or show that it does not exist. Justify your answer.

   (d) Find the absolute minimum value of f on the closed interval $[-2, 8]$. Justify your answer.

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