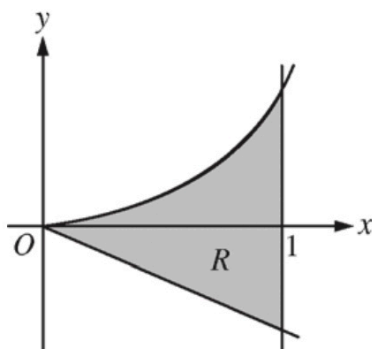


**2014 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS**

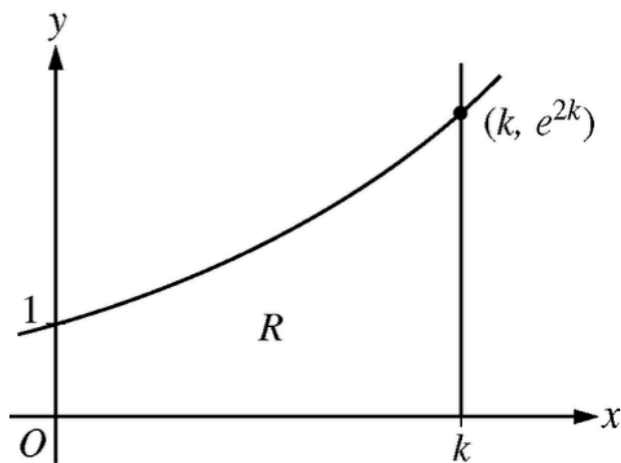


5. Let  $R$  be the shaded region bounded by the graph of  $y = xe^{x^2}$ , the line  $y = -2x$ , and the vertical line  $x = 1$ , as shown in the figure above.
- Find the area of  $R$ .
  - Write, but do not evaluate, an integral expression that gives the volume of the solid generated when  $R$  is rotated about the horizontal line  $y = -2$ .
  - Write, but do not evaluate, an expression involving one or more integrals that gives the perimeter of  $R$ .

# 2011 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS

## CALCULUS BC SECTION II, Part B Time—60 minutes Number of problems—4

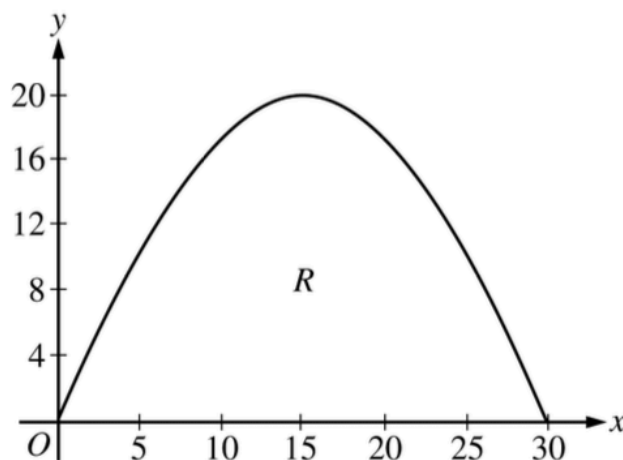
No calculator is allowed for these problems.



3. Let  $f(x) = e^{2x}$ . Let  $R$  be the region in the first quadrant bounded by the graph of  $f$ , the coordinate axes, and the vertical line  $x = k$ , where  $k > 0$ . The region  $R$  is shown in the figure above.
- (a) Write, but do not evaluate, an expression involving an integral that gives the perimeter of  $R$  in terms of  $k$ .
- (b) The region  $R$  is rotated about the  $x$ -axis to form a solid. Find the volume,  $V$ , of the solid in terms of  $k$ .
- (c) The volume  $V$ , found in part (b), changes as  $k$  changes. If  $\frac{dk}{dt} = \frac{1}{3}$ , determine  $\frac{dV}{dt}$  when  $k = \frac{1}{2}$ .

**CALCULUS BC**  
**SECTION II, Part A**  
**Time—45 minutes**  
**Number of problems—3**

**A graphing calculator is required for some problems or parts of problems.**



1. A baker is creating a birthday cake. The base of the cake is the region  $R$  in the first quadrant under the graph of  $y = f(x)$  for  $0 \leq x \leq 30$ , where  $f(x) = 20\sin\left(\frac{\pi x}{30}\right)$ . Both  $x$  and  $y$  are measured in centimeters. The region  $R$  is shown in the figure above. The derivative of  $f$  is  $f'(x) = \frac{2\pi}{3}\cos\left(\frac{\pi x}{30}\right)$ .
- (a) The region  $R$  is cut out of a 30-centimeter-by-20-centimeter rectangular sheet of cardboard, and the remaining cardboard is discarded. Find the area of the discarded cardboard.
- (b) The cake is a solid with base  $R$ . Cross sections of the cake perpendicular to the  $x$ -axis are semicircles. If the baker uses 0.05 gram of unsweetened chocolate for each cubic centimeter of cake, how many grams of unsweetened chocolate will be in the cake?
- (c) Find the perimeter of the base of the cake.

## 2008 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)

### CALCULUS BC

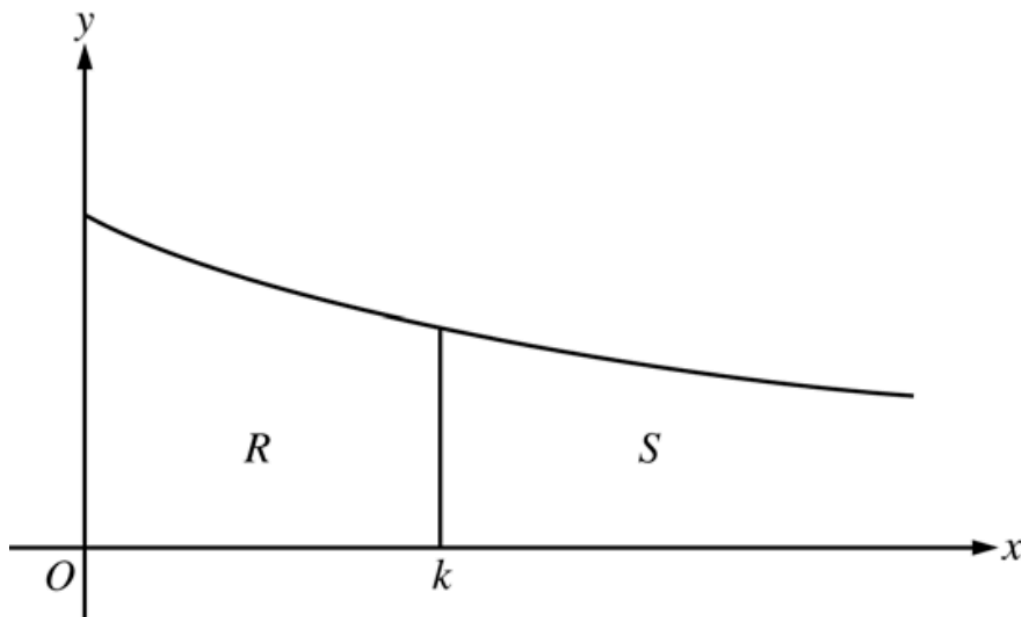
### SECTION II, Part B

Time—45 minutes

Number of problems—3

**No calculator is allowed for these problems.**

- 
4. Let  $f$  be the function given by  $f(x) = kx^2 - x^3$ , where  $k$  is a positive constant. Let  $R$  be the region in the first quadrant bounded by the graph of  $f$  and the  $x$ -axis.
- (a) Find all values of the constant  $k$  for which the area of  $R$  equals 2.
  - (b) For  $k > 0$ , write, but do not evaluate, an integral expression in terms of  $k$  for the volume of the solid generated when  $R$  is rotated about the  $x$ -axis.
  - (c) For  $k > 0$ , write, but do not evaluate, an expression in terms of  $k$ , involving one or more integrals, that gives the perimeter of  $R$ .



6. Consider the graph of the function  $f$  given by  $f(x) = \frac{1}{x+2}$  for  $x \geq 0$ , as shown in the figure above. Let  $R$  be the region bounded by the graph of  $f$ , the  $x$ - and  $y$ -axes, and the vertical line  $x = k$ , where  $k \geq 0$ .
- (a) Find the area of  $R$  in terms of  $k$ .
- (b) Find the volume of the solid generated when  $R$  is revolved about the  $x$ -axis in terms of  $k$ .
- (c) Let  $S$  be the unbounded region in the first quadrant to the right of the vertical line  $x = k$  and below the graph of  $f$ , as shown in the figure above. Find all values of  $k$  such that the volume of the solid generated when  $S$  is revolved about the  $x$ -axis is equal to the volume of the solid found in part (b).

## 2004 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)

5. Let  $g$  be the function given by  $g(x) = \frac{1}{\sqrt{x}}$ .

(a) Find the average value of  $g$  on the closed interval  $[1, 4]$ .

(b) Let  $S$  be the solid generated when the region bounded by the graph of  $y = g(x)$ , the vertical lines  $x = 1$  and  $x = 4$ , and the  $x$ -axis is revolved about the  $x$ -axis. Find the volume of  $S$ .

(c) For the solid  $S$ , given in part (b), find the average value of the areas of the cross sections perpendicular to the  $x$ -axis.

(d) The average value of a function  $f$  on the unbounded interval  $[a, \infty)$  is defined to be  $\lim_{b \rightarrow \infty} \left[ \frac{\int_a^b f(x) dx}{b - a} \right]$ . Show

that the improper integral  $\int_4^{\infty} g(x) dx$  is divergent, but the average value of  $g$  on the interval  $[4, \infty)$  is finite.