Name	Formula	Comments, common mistakes and when to use

Quotient

- $\tan x = \frac{\sin x}{\cos x}$
- $\cot x = \frac{\cos x}{\sin x}$

The formulae hold for any angle as long as they are the same: $\tan 2x = \frac{\sin 2x}{\cos 2x}$ etc

Reciprocal

- $\sec x = \frac{1}{\cos x} \Leftrightarrow \cos x = \frac{1}{\sec x}$
- $\operatorname{cosec} x = \frac{1}{\sin x} \Leftrightarrow \sec x = \frac{1}{\operatorname{cosec} x}$
- $\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x} \Leftrightarrow \tan x = \frac{1}{\cot x}$

The formulae hold for any angles as long as they are the same: $\sec 2x = \frac{1}{\cos 2x}$ etc

Quadratic (Pythagorean)

- $\sin^2 x + \cos^2 x = 1$ ↳ $\cos^2 x = 1 - \sin^2 x$ ↳ $\sin^2 x = 1 - \cos^2 x$
- $1 + \tan^2 x = \sec^2 x.$ ↳ $\tan^2 x = \sec^2 x - 1$
- $1 + \cot^2 x = \operatorname{cosec}^2 x$ ↳ $\cot^2 x = \operatorname{cosec}^2 x - 1$

- Re-arrangements are just as important as the original formula!!!
- Proof: Divide all of $\sin^2 x + \cos^2 x = 1$ by $\sin^2 x$ or $\cos^2 x$ to get 2nd and 3rd formulae
- Don't forget you can to replace $\sin^2 x + \cos^2 x$ with 1 and vice versa
- The formulae hold for any angles as long as they are the same: $\sin^2 2x + \cos^2 2x = 1$etc

Double Angle

- $\sin 2x = 2\sin x \cos x$
- $\cos 2x = \cos^2 x - \sin^2 x$
  $= 2\cos^2 x - 1$ ↳ $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
  $= 1 - 2\sin^2 x$ ↳ $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
- $\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$ ↳ $\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$

- Re-arrangements are just as important as the original formula!!
- Proof: write as $\sin(a + a)$ and use addition formula etc
- The formulae hold for any doubling relationship between the angles. Notice how the coefficients don't change

$\sin 4x = 2\sin 2x \cos 2x$
$\cos 4x = 1 - 2\sin^2 2$ or $2\cos^2 2x - 1$ or $\cos^2 2x - \sin^2 2x$ ett

Half Angle

- $\sin \frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}}$
- $\cos \frac{x}{2} = \pm\sqrt{\frac{\cos x + 1}{2}}$
- $\tan \frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$

Formulae with $t = \tan\left(\frac{x}{2}\right)$:

- $\sin x = \frac{2t}{1 + t^2}$
- $\cos x = \frac{1 - t^2}{1 + t^2}$
- $\tan x = \frac{2t}{1 - t^2}$

- Proof: Use double angle

$\cos 2x = 2\cos^2 x - 1$ | $\cos 2x = 1 - 2\sin^2 x$

$\cos x = 2\cos^2 \frac{x}{2} - 1$ | $\cos x = 1 - 2\sin^2 \frac{x}{2}$

$\cos \frac{x}{2} = \pm\sqrt{\frac{\cos x + 1}{2}}$ | $\sin \frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}}$

Triple Angle & Powers of Trig

$\sin 3x = 3\sin x - 4\sin^3 x$ ↳ $\sin^3 x = \frac{1}{4}(3\sin x - \sin 3x)$

$\cos 3x = 4\cos^3 x - 3\cos x$ ↳ $\cos^3 x = \frac{1}{4}(3\cos x + \cos 3x)$

$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

- Proof: write as $\sin(2a + a)$ and use addition formula etc.
- Other powers of trig by finding $\sin 4x$, $\sin 5x$ etc:
  $\sin^4 x = \frac{1}{8}(\cos 4x - 4\cos 2x + \frac{6}{2})$ and $\cos^4 x = \frac{1}{8}(\cos 4x + 4\cos 2x + \frac{6}{2})$
  $\sin^5 x = \frac{1}{16}(\sin 5x - 5\sin 3x + 10\sin x)$ and $\cos^5 x = \frac{1}{4}(\cos 5x + 5\cos 3x + 10\cos x)$

Cofunction

- $\cos x = \sin(90 - x)$ and $\sin x = \cos(90 - x)$
- $\tan x = \cot(90 - x)$ and $\cot x = \tan(90 - x)$
- $\sec x = \operatorname{cosec}(90 - x)$ and $\operatorname{cosec} x = \sec(90 - x)$

These are useful when you want to swap $\sin$ to $\cos$ and vice versa. Using $\sin x = \pm\sqrt{1 - \cos^2 x}$ $\cos x = \pm\sqrt{1 - \sin^2 x}$ from Pythag identities make things more complicated.

Even Odd

- $\sin(-x) = -\sin x$ and $\cos(-x) = \cos x$
- $\tan(-x) = -\tan x$ and $\cot(-x) = -\cot x$
- $\sec(-x) = \sec x$ and $\operatorname{cosec}(-x) = -\operatorname{cosec} x$

Addition

- $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
- $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
- $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

If you need to turn RHS into LHS
$r\sin(x \pm \alpha)$ or $r\cos(x \mp \alpha)$: $r = \sqrt{a^2 + b^2}$, $\alpha = \tan^{-1}\left(\frac{+b}{+a}\right)$.

Sum to product

- $\sin A + \sin B \equiv 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ and $\sin A - \sin B \equiv 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$
- $\cos A + \cos B \equiv 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ and $\cos A - \cos B \equiv -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

Product to sum

- $\sin x \sin y = \frac{1}{2}[\cos(x - y) - \cos(x + y)]$ and $\cos x \cos y = \frac{1}{2}[\cos(x - y) + \cos(x + y)]$
- $\sin x \cos y = \frac{1}{2}[\sin(x + y) + \sin(x - y)]$ and $\cos x \sin y = \frac{1}{2}[\sin(x + y) - \sin(x - y)]$

- Used mainly when you want to integrate products
- Proof: Add or subtract 2 pairs or addition angle formulae and solve simultaneously

Small Angle

$\sin\theta \approx \theta$ $\cos\theta \approx 1 - \frac{\theta^2}{2}$ $\tan\theta \approx \theta$

Useful when finding limits as $x \to 0$

Inverse

Notation: $\sin^{-1} x = \arcsin x$ and $\sin^{-1} x$ is equivalent to $\sin\theta = x$ (etc for other trig function)

- $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ and $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$ and $\operatorname{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2}$
- $\sin(\sin^{-1} x) = x$ and $\sin^{-1}(\sin x) = x$ (etc for other trig functions)
- $\tan^{-1} x \pm \tan^{-1} y = \tan^{-1}\left(\frac{x \pm y}{1 - xy}\right)$

	$\sin^{-1} x$	$\cos^{-1} x$	$\tan^{-1} x$
Domain	$-1 \le x \le 1$	$-1 \le x \le 1$	$-\infty \le x \le \infty$
Range	$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$	$0 \le x \le \pi$	$-\frac{\pi}{2} < x < \frac{\pi}{2}$

Differentiation

$\sin f(x) \Rightarrow f'(x)\cos f(x)$

$\cos f(x) \Rightarrow -f'(x)\sin f(x)$

$\tan f(x) \Rightarrow f'(x)\sec^2 f(x)$

$\sec f(x) \Rightarrow f'(x)\sec f(x)\tan f(x)$

$\operatorname{cosec} f(x) - f'(x)\operatorname{cosec} f(x)\cot f(x)$

$\cot f(x) \Rightarrow -f'(x)\operatorname{cosec}^2 f(x)$

$\sin^{-1} f(x) \Rightarrow \frac{f'(x)}{\sqrt{1 - (f(x))^2}}$

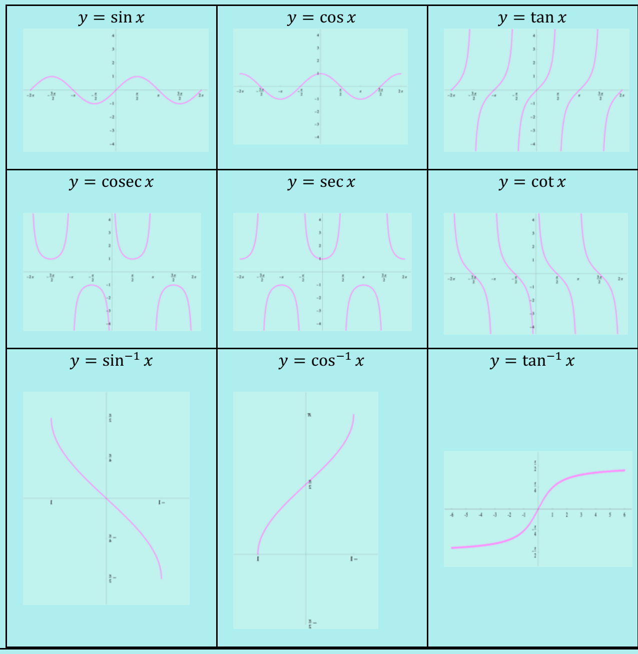
$\cos^{-1} f(x) \Rightarrow -\frac{f'(x)}{\sqrt{1 - (f(x))^2}}$

$\tan^{-1} f(x) \Rightarrow \frac{f'(x)}{1 + (f(x))^2}$

$\sec^{-1} f(x) \Rightarrow \frac{f'(x)}{f(x)\sqrt{(f(x))^2 - 1}}$

$\operatorname{cosec} f(x) = \Rightarrow -\frac{f'(x)}{f(x)\sqrt{(f(x))^2 - 1}}$

$\cot^{-1} f(x) \Rightarrow -\frac{f'(x)}{1 + (f(x))^2}$



Integration

$\int f'(x)\sin f(x)dx \Rightarrow -\cos f(x)$

$\int f'(x)\cos f(x)dx \Rightarrow \sin f(x)$

$\int f'(x)\sec^2 f(x)\,dx \Rightarrow \tan f(x)$

$\int f'(x)\sec f(x)\tan f(x)\,dx \Rightarrow \sec f(x)$

$\int f'(x)\operatorname{cosec} f(x)\cot f(x)\,dx \Rightarrow -\operatorname{cosec} f(x)$

$\int f'(x)\operatorname{cosec}^2 f(x)\,dx = -\cot f(x)$

$\int \operatorname{cosec} f(x)\,dx = -\ln|\operatorname{cosec} f(x) + \cot f(x)|$

$\int \sec f(x)\,dx = \ln|\sec f(x) + \tan f(x)|$

$\int \tan f(x)\,dx = \ln|\sec f(x)|$

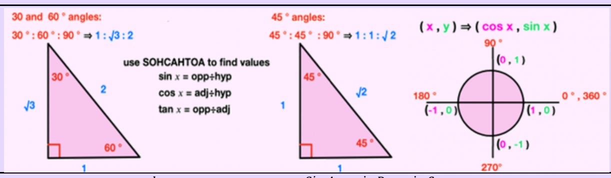
$\int \cot f(x)\,dx = \ln|\sin f(x)|$

$\int \frac{1}{\sqrt{a^2 - (bx)^2}}dx \Rightarrow \frac{1}{b}\sin^{-1}\left(\frac{bx}{a}\right)$

$\int \frac{1}{-\sqrt{a^2 - (bx)^2}}dx \Rightarrow \frac{1}{b}\cos^{-1}\left(\frac{bx}{a}\right)$

$\int \frac{1}{a^2 + (bx)^2}dx \Rightarrow \frac{1}{ab}\tan^{-1}\left(\frac{bx}{a}\right)$

Special Angles



	0	30°	45°	60°	90°	180°	270°	360°
$\sin x$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
$\tan x$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	∞	0

Triangle/Sector

- Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ or $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ if finding angle
- Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$ or $A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$ if finding angle
- Area= $\frac{1}{2}ab\sin C$ or $\sqrt{s(s - a)(s - b)(s - c)}$ where $s = \frac{a + b + c}{2}$
- Arc length= $r\theta$, Arc area= $\frac{1}{2}r^2\theta$ when θ is in radians

- We use these for **non**-right angled trig. For right angled trig we can use SOHCAHTOA $\sin x = \frac{O}{H}$, $\cos x = \frac{A}{H}$, $\tan x = \frac{O}{A}$ and area= $\frac{1}{2} \times base \times height$
- Use sine rule when we have a complete pairing of a side and an angle, meaning we have the numerical value of angle and its side opposite. If we don't have this we use cosine rule